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WITH TABLES, ILLUSTRATIONS, AND RECORDS

BY

LOWIS D'A. JACKSON, A.M.I.C.E.

AUTHOR OF
'HYDRAULIC MANUAL AND STATISTICS' 'CANAL AND CULVERT TABLES'
'THE SIMPLIFICATION OF WEIGHTS AND MEASURES'



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PREFACE.

Errors and Omissions

P. 13, line 6, *for* $A' = (a + f + d') \cos \phi$ *read* $A' = (a' + d') \cos \phi$
,, 108, ,, 14, *for* 1128 *read* 1120
,, 171, ,, 5, *omit* 1·
,, ,, ,, 11, *for* 12·18 *read* 13·18
,, ,, ,, 14, *for* 16·65 *read* 12·65

deal with the setting-out of the railway engineer, but often more actually devoted to the design of complicated cross-over roads, others intended to apply to the special work of preliminary deposited plans of Parliamentary surveys for Railway Bills, while some seemed rather to aim at presenting the student with a large set of nicely drawn plans for him to contemplate as models in drawing rather than at supplying him with information useful in surveying, and others aimed at providing the designer rather than the surveyor with a convenient set of trigo-



PREFACE.

THE object of this small work on Survey-Practice for all ordinary purposes is to supply a special want, which manifested itself in the inability of the author to find any single published work which would answer the purposes of his pupils and serve as a general book of reference for methods, formulæ, and forms of record.

He found many books on Surveying, some dealing exclusively with chain-surveys of small plots of ground, some dealing purely with large Trigonometrical surveying operations carried out with enormously expensive and refined instruments and appliances, some intended to deal with the setting-out of the railway engineer, but often more actually devoted to the design of complicated cross-over roads, others intended to apply to the special work of preliminary deposited plans of Parliamentary surveys for Railway Bills, while some seemed rather to aim at presenting the student with a large set of nicely drawn plans for him to contemplate as models in drawing rather than at supplying him with information useful in surveying, and others aimed at providing the designer rather than the surveyor with a convenient set of trigo-

nometrical, logarithmic, and numerical tables for use in office calculations.

Each and all of these works, though perhaps excellent in their way and for some special purpose, failed to serve as a general guide in survey-practice, while their generally too comprehensive titles sometimes were deceiving to the purchaser; there remained therefore the two alternatives of getting a small library for reference on each subject or of giving written instructions to pupils for their use; the latter plan having been adopted, the notes have been since condensed into the following pages.

This book is of course a compilation in the general sense of the word; all books are either compilations from recorded or observed facts, habits, customs, events, methods, or of thoughts and ideas occurring to persons under different circumstances and moulded into some special shape; but this book, not having been compiled from other books, is not a compilation in the strictly just meaning of the word; occasionally other books or persons, and their methods, are quoted and referred to with names attached, but, on the whole, the work is a compilation deduced from practice or experience, based on verbal and exemplified instruction received in early days, the results of which were subsequently modified and altered wherever any improvement seemed practicable.

While stating this, the author wishes to express his gratitude to his former instructors, and to acknowledge them fully as his practical guides up to a certain period of life in various branches of work; they were—

Mr. Charles Green, civil engineer.
Mr. Millais, civil engineer.
Captain Henry Scott.
Mr. Crawford Campbell, civil engineer.
Mr. William Morton, civil engineer.

It is not within the author's power to state how far the instruction received was based on practice already then recorded in books, or on modifications introduced by the above gentlemen. There are always plenty of persons ready to declare that there is nothing new under the sun, to depreciate every improvement, and to imagine their own genius augmented by treating matters and persons in a contemptuous style; such persons are not by any means to be envied, and it is not the author's wish to imitate them.

A large portion of this book is reprinted from the second edition of the author's Curve-Book, published in 1869, the greater part of which confined itself to the setting-out and designing of curves of railways, canals, and roads; but which also contained an Appendix on Errors in Levelling; the former being chiefly the result of experience on the Metropolitan Railway, the latter being called forth by levelling operations on the Tumbaddrā Canal, which had in some parts a fall of only four inches per mile, and required special care and consideration, necessitating reference to the errors recorded in the extended levelling operations of the Great Trigonometrical Survey of India. The additions in this book consist of Part I. On Surveying, and Part IV. On Route-Surveys and minor astronomical observations in connection with them; both of these parts are based on

notes, experience, and observations made by the author on land and at sea, with the exception of two examples of longitude-observation at sea worked out by him on data recorded by others. Part I. was recently condensed from notes and written for the use of pupils. Part IV. was condensed in the same way for the use and convenience in reference of the author himself on the occasion of his services being accepted as surveyor to an expedition intending to proceed from India by land route to China under Captain Smyth. This eventually was countermanded at the advice of the British representative at Pekin, who expected a recurrence of the fate of the former expedition (whose officers were tortured and partially burnt alive), and wished to avoid political difficulties of an inconvenient nature. The condensed notes consequently became useless for that particular object, but may yet answer the similar purposes of travellers generally. The observations at sea were subsequently taken and have been since added to this portion of the book.

The subjects more specially avoided are, descriptions of instruments generally which are in common use and can be seen, information and tabular matter more specially useful to the designer, and information and models of plans for the draughtsman to follow or imitate; these are subjects distinct from those more specially useful to the surveyor and setter-out. The adjustments of instruments are, however, explained, and the necessary knowledge of the requirements of the draughtsman or designer from the surveyor or setter-out acting in accordance with him is also indicated in occasional remarks.

The object of the book is thus extremely limited in one respect, though extensive in another.

The Surveys shown in the Plates and Records have been mostly executed by the author at various times; but in some cases, when his note-books did not supply sufficient record of any Survey suited to the special purpose of a book of this sort, the want was supplied by others, whose kindness is therefore here acknowledged with thanks. Among the Route Surveys, the observations at sea were his own, and the Thibetan Routes Record was obtained through the influence of others.

The particular portions of the book containing matter for which the author is not indebted to anyone in any way are, the two systems of ranging curves on six-point polygonal methods, the corresponding mode of ranging a compound curve, some of the notes on telemetrical operations, and some of those on errors in levelling, more especially the notice of reflex refractive action in hot climates. In other respects the book is, as proposed, an account of known or of modified methods applicable in practice; it is intended to be useful in occasionally refreshing the memory of those thoroughly conversant with all of them, and in shortening their labour by the tables and formulæ, while it may be of greater use to those whose survey-practice has been limited to some one or two special branches, or to none.

With reference to its use to this latter class of students, it must be frankly confessed that this book was not written with the object of making a Surveyor out of an utterly inexperienced person by its perusal alone. No

book can ever do this, or go beyond assisting in doing it. Surveying is practical work of a mechanical description, that is generally acquired by imitating other persons, and by personal practice and experience; it requires but little skill or science, as it merely demands a thorough practical and theoretical knowledge of method, and a competent facility of execution, combined with care and judgment.

The knowledge of method may be obtained from a book, if a suitable one can be found, but the facility of execution, the mechanical manipulation of instruments and appliances, and general management in the field would be very tediously acquired by any beginner that entirely dispensed with personal guidance; hence the necessity for following and imitating some experienced person at the onset, and at the same time learning the motives of his procedure.

Such a person should evidently neither be a purely practical nor a purely theoretical man; for the former may perform his work automatically, or as a conjurer does his tricks, and leave his pupil to become a mere imitator, a very probable case when the purely practical man either plumes himself on his ignorance of theory, or is unable to explain his motives; on the other hand the purely theoretical man, that has had little or no practice, cannot possibly serve as a model for guidance in actual work in which he is inexperienced; the best guide, and generally also the best Surveyor, is hence a person both thoroughly conversant with all theory bearing on the subject, and also fully competent in practice, whose experience has been gained in several branches of

the subject, and whose ideas and habits are free from narrow-minded maxims and traditions; a guide of limited experience should be followed in his special branch only.

In learning to survey, as in learning many other things, the first, and also the most important point, is to do it well, the second to do it rapidly; both exactitude and dexterity are inherent individual qualities which may be cultivated to a certain degree, and generally, though not always, to the extent sufficient to make a fairly good Surveyor; a want of precision, resulting in slovenly and useless work is fatal to good surveying, while an amount of exactitude in excess of the requirements of the case, which causes waste of labour, is merely a result of bad judgment, and may be more easily remedied. Rapidity, estimated by the time spent on a survey, is only to a certain extent dependent on individual dexterity or rapid mechanical manipulation; it also depends on the means and methods employed, the general arrangements for work, and on many fortuitous circumstances, such as the possibility of obtaining sufficient men, instruments, appliances, poles, and pegs at the immediate instant and place where they are required, and on the freedom from mistakes and delays of chainmen and subordinates; extreme dexterity is hence not so absolutely necessary in surveying as good management and judgment, while a moderate want of dexterity may be well compensated by the latter qualities combined with a knowledge of expedients: hence accuracy and knowledge may be considered indispensable, and dexterity very advantageous, in survey-practice.

Of these requirements, the knowledge is the only thing that a book can supply ; it was with this limited object that this work was written.

The attainment of the capability for carrying out ordinary general and route surveys, treated in this volume, involves but little special technical knowledge, and may rather be now regarded as one of those acquirements which, like drawing, should be invariably comprised in the ordinary education of an English gentleman, as it may be useful at any time either in England, at the tropics, or in any foreign country or colony.

Keeping the before-mentioned purposes in view, this book has been arranged accordingly, special care having been taken that no exclusively local terms should be introduced ; while such operations as are exclusively confined to Great Trigonometrical Surveys, which involve the use of three-feet theodolites and compensating bars, the construction of towers and scaffolds 40 and 50 feet in height, the clearance of hill-tops, and miles of visual ray for enormous triangulations, have been referred to merely as far as they give indications useful for guidance in ordinary Surveying.

L. D'A. J.

Royal Institution, Albemarle Street.

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AID
TO
SURVEY-PRACTICE.

PART I.
GENERAL SURVEYS.

CHAPTER I.
INSTRUMENTS AND CALCULATIONS.

Section 1. GENERAL EXPLANATION.

ALL Geodetical operations may be said to be generally comprised under the three heads of Surveying, Levelling, and Setting out, or locating.

Surveying generally consists in taking the measurements and observations on which a plan, chart, or map of the natural and artificial features of a tract of land or water may be based; although this does not strictly apply to some special branches, such as geological surveys; the term itself is also sometimes used to include levelling. General surveys may be classified in three ways: either according to the nature and scope of the ultimate object, or in accordance with the instruments used in the work, or the scale of the intended plan.

Survey work of the highest and most perfect type is known as Great Trigonometrical Surveying, which

may extend over many degrees of latitude and longitude, and even include a large region: as for instance, that of India, of Germany, France, or the British Isles. The utmost precision being necessary in such work, the general management of it is entrusted to scientific men, with whom rest the initiation or adoption of improved methods, instruments, and appliances; while the carrying out of the detailed routine work, its checking and superintendence, is usually delegated to inferior men that happen to be naturally well fitted to adhere to a rigid system of work not requiring the exercise of much judgment or discretionary power. These systems and appliances are fully described in books devoted to that special subject, which is beyond the scope of this work; they will hence be merely referred to here in so far as they serve as guides, or afford indications useful in ordinary practice. The least perfect type of Survey-work is termed Military Surveying. It is conducted on the principles of ordinary practice, but in a rough manner, mostly with pocket-instruments, aided greatly by good guess-work; the permissible amount of error in it is large, while extreme rapidity is of paramount importance. For such surveying no additional training or special instruction is necessary, although a knowledge of strategical requirements is a valuable adjunct.

All ordinary general survey-practice lies between these two extreme types, and may be thus classified on the first-mentioned principle.

1. *District Surveys*, of extended tracts of country. These being of large scope, and requiring great precision, are carried out in style and method after the model of operation originally adopted in Great Trigonometrical Surveys, although less exact and cumbrous

instruments and appliances are used, and a very much less amount of refinement and exactitude, as well as time, is expended on them.

2. *Parish and Estate Surveys*, being more limited in scope, require less exact instruments and appliances, but follow the general principles adopted in the filling in of Great Trigonometrical Survey work.

3. *Town Surveys*, which ordinarily cover about the same extent of land and range of limits as those of the last class, differ from them in requiring greater exactitude, as they have to be plotted to a much larger scale, and in involving a greater amount of labour.

4. *Engineering Surveys.* These having for their special object the future location of some work of public or private utility, such as a railway, canal, waterworks, drainage works, or factories, are generally confined either to a long narrow strip of country or a small tract of country or plot of land; the amount of exactitude necessary being liable to vary all over the ground in accordance with its bearing on future requirements and the nature of the intended works.

5. *Route-Surveys* have in view the limited object of producing a map or plan of a route taken by an explorer or traveller either by land or sea; in the latter case it is usually termed navigation, and consists in laying down the ship's course: the position of natural and artificial features of the country, or of detached points of land, in the immediate vicinity of the route, are also recorded in these surveys.

6. *Mining and Subterraneous Surveys* are limited by the extent of actual or possible future mining operations, tunnels, passages, or works of construction; they generally involve a record of position or distribution of

minerals in the soil, and of other geological and hydro-geological data.

7. *Hydrographic and Marine Surveys*, of rivers, lakes, estuaries, marshes, and coasts, comprise not only a general survey of the land in the immediate neighbourhood of the water, as it may appear under various conditions of flood and tide, but also such a record or delineation of the subaqueous land, of currents, soundings, and depths, also in some instances, of quantities of water, that the subaqueous formation under any conditions may be as thoroughly described in plan as the former. Such surveys are subdivided into the two classes of Nautical and Engineering Hydrographic Surveys, the former being of greater extent, the latter of greater precision but within narrower limits.

All such surveys may be perfectly independent, and may be so executed as to suit any required scale in the final plan; but in practice they are frequently based on, or joined on to, the results of previous surveys in existing maps and plans; in such a case it is best to adopt a scale that has some convenient ratio to that of the existing plan. A list of scales for reference will be given in Section 1 of Chapter II.

The following are the operations which either separately or in combination are practised in surveys of every description.

1. Direct measurement of distance, and of height and depth.

2. Angular measurement and taking magnetic bearings.

3. The calculation of distances, heights, depths, and angles.

4. The selection and demarcation of survey points and lines.

5. Recording results in such a form and manner that a draughtsman may utilise the results without doubt or difficulty.

6. Astronomical observations for determining time, latitudes, longitudes, and azimuths.

The various means and appliances used in direct and angular measurement on surveys will be mentioned in the two next sections, the special principles involved in their construction and use, the limits of their application, and the preferable methods of manipulation and adjustment, occasionally entered into in detail; the corrections to be applied to results of instrumental observation, under various special conditions will also be fully explained and deduced; but definitions of terms, descriptions of easily visible instruments and appliances, and practical directions for commonplace manipulation, have generally been excluded from this work as unnecessary.

The two following sections will treat of the selection and demarcation of survey-points and lines in the field, and give a collection of formulæ commonly used in survey calculations; the less frequent or more special formulæ will be found attached to their appropriate subject or instrument. The subject of astronomical observations, the adjustments and corrections to be applied to the Transit instrument, and the necessary formulæ for all such purposes, are included in the Part devoted to Route-Surveys. The ordinary forms and modes of recording survey operations are fully exemplified in a series of practical field-records suited to surveys of every sort, which are collected at the end of the book, though their corresponding sketches or reduced plans are interspersed with the text bearing on the subject.

The modes of procedure and methods of operation adopted in surveying with various instruments are described in Chapter II.

Section 2. Instruments for Measuring Distance.

The instruments and appliances most generally used in measurement of distance, also sometimes for height and depth, are the following:

1. Steel or iron chains of various patterns, plain, painted, or galvanized, used for all purposes.
2. Wooden rods or bars, now mostly used only in short distances, as offset staves or foot-rules, or in short measurements of height and depth, as graduated level staves, and vane staves.
3. Continuous steel tapes, and woven tapes strengthened with wire, or otherwise.
4. Wheel pedometers.
5. Pacing, with or without the aid of passometers.
6. Direct telemeters for observation, with or without the aid of micrometers, on staves of known length whether graduated or otherwise.
7. Reflecting telemeters for observation to any fixed points, dependent on the length of the instrument as a base of triangulation.

1. *The steel chain* is the ordinary appliance of the surveyor for measuring long distances and for setting out on works of engineering. In large District Surveys the 100-feet chain is most convenient, more especially for measuring base lines; it also conduces to the advantage of keeping distances as well as heights in feet; in such work where accurate measurement is necessary,

the chain may be supported on planks or courses of brick, and in hot climates thermometrical observation is also necessary to enable some allowance to be made for expansion. In Parish Surveys the 66-feet chain, subdivided into 100 links, is used, as it is more easily handled by two men, and more convenient for calculating acreage; also in Railway Surveys, where miles and furlongs are sometimes obligatory under the Standing Orders of the Houses of Parliament, from its convenience in being the 80th part of a mile in length. In other surveys where these latter points are not of consequence the 50 feet chain is to be preferred.

Ordinary chain-measurement is generally far from being so exact as is usually imagined. Although the chain itself may be originally of correct standard length, and compared daily with some temporary standard marked on a building, or kerb-edge of a footway, or checked by comparison with level staves, and straightened or shortened as required; yet, even with chainmen practised in keeping the chain straight and level, putting the pins plumb, and holding the ends of the chain true to them, Mr. William Burt found the average error was $\frac{1}{500}$th, and sometimes as much as $\frac{1}{220}$th on using two sets of chainmen; though it was only $\frac{1}{1000}$th on using the same set of chainmen and same appliances at about the same time. (See Key to the Solar Compass, page 35.) In ordinary practice the amount of error must comparatively be higher, for inferior careless chainmen, as among men of every description, are the rule and not the exception; while in foreign countries there are additional natural difficulties with them; the surveyor himself, who attends to the recording, and perhaps to instrumental

and other work also, cannot, after the first few starting chain-lengths, personally attend to all the minutiæ of the chaining, or see that each end of every chain, and its whole length, is correctly placed; he is hence dependent on his chainmen to a very great extent, and this must cause in addition to the above a high percentage of error, unless carefully guarded against. It is much to be wished that telemetrical observation should take the place of the greater part of the chain measurement in long distances, which is both laborious and rather uncertain, even when unaccompanied by hurry. In setting out, chaining cannot be dispensed with, it is true; but in ordinary surveys the only difficulty seems to be the expense and weight of an instrument. For soundings less than 100 feet the chain with a weight attached is generally used; there is however the liability to rust if it is not galvanised. Chains are marked with brass distinctive appendages at every 10 links or 10 feet, most frequently giving numbers 1, 2, 3, 4, from each end up to the middle where there is a round brass; this method of marking is liable to produce error: it is better never to let the same mark recur, but to number it from the beginning to the end with distinctive tallies. A description of the method to be adopted in ordinary chaining will be given in the paragraph on Chain-Surveys. For very special cases, where extreme exactitude in the measurement of a base line may happen to be necessary, a standard steel-chain, having a known rate of expansion for temperature, should be used, and the method formerly adopted in the Great Trigonometrical Survey, before compensating bars were used, imitated with more or less fidelity. In that practice the chain was

100 feet long, and consisted of 40 links, each half an inch square; it was kept constantly strained with a 56-lb. weight, and laid in coffers supported by trestles; at each chain's length a post, carrying a scale moved by a screw, was set up to obtain an exact mark from which to measure after removal of the chain onwards; thermometers were also used to ascertain the temperature of the chain during the work.

2. *Wooden rods or bars* were at one time used in the Great Trigonometrical Survey for base-measurement, and were also much used in ordinary surveying in measuring long distances; but as they expand or contract with damp, which renders them objectionable, and are cumbersome to manage, they are now superseded by the chain, except in cases where a chain cannot be readily obtained. The offset staff, having feet or links marked on it alternately painted black and white up to 6 feet or 10 links, used for measuring short offsets, is a useful remnant of this system. The level staff, with its various patterns, will be described in the chapter on levelling; it is a wooden staff painted and graduated to feet, tenths and hundredths, generally used either for giving heights or depths direct, or by observation on it with the horizontal wire of a levelled telescope; distances may also be obtained by its aid by observing on it with two fixed horizontal wires in a level-telescope, thus getting two readings, the difference between which is a measure of the distance. By exact setting of the wires the ratio may be made convenient for a rapid calculation of the required distance from the intercepted height, as, a chain to a foot, or 100 feet to a foot.

3. *Tapes.* The steel tape being continuous is slightly more accurate than the common land-chain,

and being much lighter is more convenient for measurements made at any height from the ground; it is also useful for long offsets; but its liability to twist and kink renders it easily broken, while few persons can anywhere be found to repair it. With care one may be safely used for a long time, but the surveyor should not allow it to leave experienced hands even for a moment. Woven tapes strengthened with cords of catgut or wire are in common use for the same purposes, but are far less accurate, and are unsuited for measuring more than two tapes-length in one continuous distance without appreciable error. Common measuring tapes are altogether untrustworthy; they stretch to an extent visible to the naked eye, and shrink after wetting.

4. *Wheel-pedometers* record on a dial-face the number of revolutions of, or distance passed over by, a wheel rolling on the surface of the ground; on very smooth and even ground the results are moderately fair, the error being principally due to slipping.

5. *Pacing and passometers.* Simple pacing is, next to the guessing of a bad estimator, the worst mode of obtaining distances; the inequality of the paces under various conditions combined with the errors in counting render it most inaccurate, while the monotony of perpetual counting makes it a positive torture.

A passometer can be used to register the number of paces taken; the paces themselves can be best taken by some one that has been trained to take equal paces, or has been under military training. The pacing adopted should be even, but natural, and not strained with the mistaken object of conforming to any arbitrary length; some paced distance, on the flat, uphill and downhill, should be tested by careful measurement to

obtain the ratio for reduction to yards, feet, or miles, under these three conditions of any future pacings; the ratio should again be frequently checked. The passometers that effect the reduction to miles are inconvenient for distances less than a mile, and do not yield very accurate results in any case.

Guessing or judging distance may, with continued practice and checking, be carried to an accuracy that is surprising; the main points to be noticed are the rise or fall of the ground and the direction from which the light falls on any object at the distant point. Some persons estimate in yards, others in their own paces, which are more readily available for testing such guesses.

6. *Telemetrical observation on graduated staves.* A graduated staff is held vertically at the required distant point, and its refracted image is observed in the eye-lens of the telescope, where it appears cut by two horizontal threads or wires; the height of the intercepted portion determines the horizontal distance of the staff from the vertical axis of the instrument. The practical development of this principle was chiefly due to Porro, a Piedmontese military engineer. Telemeters have been used in the United States Coast Survey, in a topographical survey in Schuykill county from 1862 to 1865, and in a geological and mining survey at Cape Breton from 1863 to 1866; the error given by them in a single sight of 660 feet being determined to be only four inches, or $\frac{1}{2000}$th, this mode of determining distance compares favourably with chain measurement under ordinary conditions where the error is about $\frac{1}{100}$th. The theory and principles of direct telemetric observation of this nature are best explained together with the history

of its development. The following account of Porro's improvements has hence been extracted from an article by De Sénarmont in the 'Annales des Mines,' 4th series, vol. xvi. page 383.

To investigate the principles involved,

First when the telescope and its line of sight is truly horizontal,

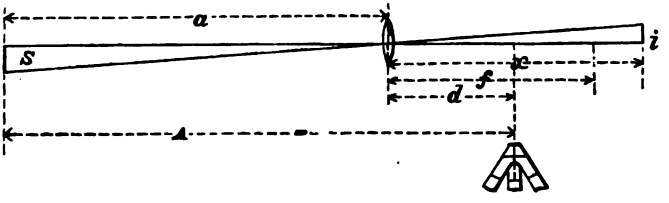


FIGURE 1. DIRECT TELEMETRY.

Let s be the intercepted height on the staff,

i the height of its image,

a the distance of the staff from the object-glass,

x the distance of the image from the object-glass,

f the focal length,

d the distance of the axis of the instrument in rear of the object lens.

Then by proportion $\frac{a}{x}=\frac{s}{i}$, and by the general formula for the foci of lenses, $\frac{a}{x}+1=\frac{a}{f}$, hence $a=f\frac{s}{i}+f$;

and if A be the distance of the staff from the axis of the instrument

$$A=a+d=f\frac{s}{i}+f+d$$

in which $f+d$ is a constant for each instrument, varying from about 12 to 24 inches.

Secondly, when the line of sight is inclined at a slight angle ϕ to the horizon, the required distance becomes

$$A' = (a+f+d)\cos\phi$$

$$= f\frac{s}{i}\cos^2\phi + (f+d)\cos\phi$$

$$= f\frac{s}{i}\cos^2\phi + f + d \text{ very nearly, as } \phi \text{ is small.}$$

Thirdly, when the graduated staff is not truly vertical,

Let s be the intercepted height when truly vertical,
s' be the intercepted height when inclined at an angle θ,
then $s' = s\cos\theta$,

and the previous formula becomes

$$A'' = f\frac{s}{i}\cos^2\phi\cos\theta + f + d,$$

hence the resulting error in the distance, due to the inclination of the staff to the vertical, becomes equal to

$$A' - A'' = f\frac{s}{i}\cos^2\phi(1-\cos\theta),$$

$$= \frac{fs}{i}\cos^2\phi.\frac{\theta^2}{2} = A'.\frac{\theta^2}{2} \text{ very nearly;}$$

Now, if we take θ roughly in terms of the circular measure of the angle,

when $\theta = 1°$, $\frac{\pi}{180} = \frac{1}{60}$, and the error $= \frac{A'}{7200}$,

when $\theta = 2°$, $\frac{2\pi}{180} = \frac{1}{30}$, and the error $= \frac{A'}{1800}$;

hence 2° may be considered as the extreme limit for the deviation from verticality, which shows the necessity of invariably using the plummet with the staff.

Porro's modifications of the simple telescope with a two-wire diaphragm were intended to obtain an increase of field and of power in the telescope, and to make the resulting distances count exactly from the vertical axis of the instrument.

In the first place, instead of using a single large object-lens, which would require a focal length of twelve times its diameter to eliminate excessive spherical aberration, he used a compound lens of two separate achromatic object-glasses placed one behind the other,

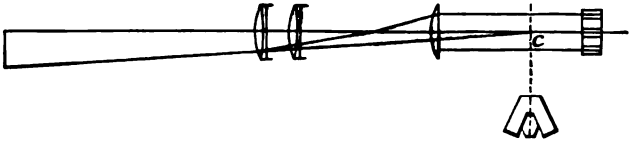


FIGURE 2. PORRO'S LENSES.

thus getting a bright clear object. Secondly, in order to avoid the spherical aberration that might result from observing with one eye-lens only when the threads were wide apart in the field of view, he used three eye-lenses, each placed opposite to a corresponding thread or wire, of which he used three; and besides, placed them farther apart without involving error. In the third place, he introduced between the object-glass and the threads an additional lens whose focus was at the same point as that of the object-glass, thus obtaining as a result that all rays after passing through the additional lens become parallel, while the sizes of all objects subtending the same angle at the point

c, the centre of anallatism, or unchangeableness, are proportional to their distance from *c*. The point *c* being then set in the exact vertical axis of the instrument, the required objects were attained, besides the additional advantage in observations having the line of sight inclined to the horizon, that the requisite correction on this account is exactly effected simply by multiplying the observed distance by the square of the cosine of the inclination. The compound telescope above described was only 2½ inches in diameter, with a focal length of 15 inches, but had a magnifying power of from 60 to 80 times. Porro also used 5, or 7, horizontal hairs or threads in some instances instead of three, in order to check errors in reading the graduations on the staff, and to help in their subdivision by the eye, as well as to assist in cases where some readings were obscured by casual obstruction.

The maximum error with such an instrument was $\frac{1}{2000}$th in distances less than 660 feet, and $\frac{1}{1000}$th in distances more than 660 feet up to 1320 feet, beyond which distance the sights become useless for practical work on account of increased error. The instrument admitted of testing in all its parts, but the threads were permanently fixed; it is evident that such an instrument would require a good maker, and perhaps be expensive.

The further employment of the telemeter on the Schuykill topographical survey resulted in the following practical conclusions, indicated by Mr. Lyman in a paper read before the Franklin Institute in 1868.

1st. That the additional lenses and complications adopted by Porro, to cause the centre of anallatism to

fall in the exact vertical axis of the instrument, were needless, as the inconvenience from adding to every distance observed a constant quantity, equal to the sum of the focal distance of the object-glass and the distance of that glass in front of the vertical axis, rarely exceeding one foot, is comparatively trifling.

2nd. That three horizontal hairs or wires are sufficient for all purposes, as the reading of the middle wire affords sufficient check on the other two; that fixed are preferable to moveable wires with protected adjusting screws; and that the fixed wires should be so set that the visible height on the staff intercepted between the middle and either outer wire should bear some exact ratio to the distance, such as 1 foot to 100, thus avoiding calculation.

3rd. That the staff should be graduated to hundredths of a foot.

4th. That, in combination with the above arrangements, a telescope magnifying only 20 times, and reading to the 200th of a foot at a distance of 660 feet, will produce results as correct as those of Porro's larger and more complicated instrument.

5th. That the errors arising from spherical aberration may under these circumstances be neglected in angles of less than 10° on either side of the focal axis.

Distance-wires. From these conclusions, it appears that a good level-telescope with three distance wires and a 16-feet level-staff are sufficient for all ordinary purposes, in distances limited to 660 feet if the telescope will read correctly to one graduation or $\frac{1}{100}$th of a foot at that distance. The length of staff utilised would be $\frac{1}{50}$th of the distance, or 13·2 feet, and the error in reading, lying within a half of a graduation or $\frac{1}{200}$th

of a foot at this range, would thus cause an error in distance of three inches, or about $\frac{1}{2500}$th of the distance. Using the same telescope, which would give readings at increased distances up to 1320 feet with half of the exactitude before mentioned, the maximum error in sights between 660 and 1320 feet would then be double the former. If however a telescope of superior power be used, capable of reading to hundredths of a foot at a distance of 1000 feet, which every superior 18-inch level telescope should do in clear weather, the maximum error in distance, about 5 inches, will as before be only $\frac{1}{2500}$ up to that distance. The same ratio of maximum error to distance could be attained up to 1320 feet distances with a still more powerful telescope, magnifying about 25 times.

Under these circumstances it seems surprising that everyone possessing a level with a good telescope does not have distance wires added to the diaphragm by a good maker, which can be done at the small cost of half-a-guinea, and that anyone should buy a new level without such distance wires. It would, however, be very much preferable to have the distance lines marked on a lens, if this could be done without destroying several in the attempt and thus causing much expense. Before applying distance wires to a telescope for telemetrical purposes it is as well to ascertain its magnifying power; this can be best effected by direct practical observation on a graduated staff with the aid of Pouillet's reflecting tube. This tube being fitted and clamped on to the eyepiece of a telescope enables the reflected real image of the staff to be seen in coincidence with its direct magnified image, and by a comparison of the two apparent sets of graduations, the magnifying power of the

telescope is simply and correctly determined. It then becomes a mere matter of calculation, as before shown, to ascertain within what limits of distance the telescope is correctly suitable to purposes of telemetry under a favourable condition of the atmosphere. The constant error for any such instrument can be determined by a series of observations combined with accurate measurements to various distances.

Telemetrical observation on staves of known length.— The next development of the principle of telemetrical observation was carried out in Eckhold's Omnimeter (which like Porro's Omnimeter also serves as a theodolite), having for result that graduations on the staff

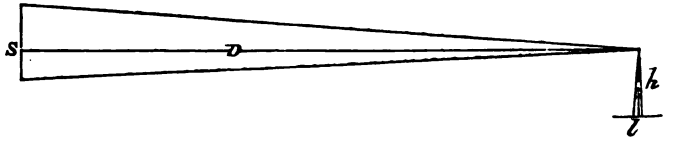


FIGURE 3. ECKHOLD'S TELEMETRY.

become needless, two correct marks being alone necessary. In this instrument a telescope and a microscope are permanently attached to each other at right angles, both turning together in the same vertical plane, and have their common turning axis at a known fixed height above a horizontal scale, placed parallel to the telescope, on which the microscope reads. When the telescope is directed first to the top or upper mark of the distant staff, a reading is taken on the scale with the microscope, a second reading is taken in the same way when the telescope is directed to the bottom or lower mark of the distant staff; the difference of these two readings made with the help of a micrometer is the measure of the required distance.

Thus if S=the height observed on the staff, which is generally fixed at 10 feet,

D=the distance of the staff from the instrument,

h=the height of the telescope axis above the scale,

l=the length or difference of readings on the scale,

it is evident that the two triangles described through the motions of the telescopic and of the microscopic lines of sight are similar; hence if the distance is required,

$$D=\frac{Sh}{l} \text{ or } \frac{10h}{l}$$

generally, and if the distance be known, any unknown height (s) observed at that distance, is

$$s=\frac{lD}{h}$$

Also the height of any point at that distance, either above or below the level of the axis of the instrument, can be obtained; for when the telescope is set truly level, the microscope reads exactly on a zero or point of departure on the scale; thus this instrument acts as a level as well as a telemeter, in the same operation. The readings on the scale, which is only about four inches in length may be made to six figures, of which two are read with the microscope and four with the aid of the micrometer disc. The mode of operating with this instrument is thus described by Mr. Eckhold in an article in Spon's 'Dictionary of Engineering.'

1. Place the staff vertically at one end of the distance to be measured, and the instrument properly

adjusted at the other, care being taken that the micrometer be set to zero.

2. Point the telescope to the upper line of the staff, clamp it, and read on the scale the number of divisions shown by the cross lines of the microscope. Suppose it to be 67 and something more, this is therefore more than 670 000 parts of the whole scale of one million parts; the micrometer disc is then turned either way until the next division of the scale is brought exactly between the cross lines that indicate the centre of the microscope, when the vernier of the micrometer may give an additional reading of 2035; the total reading thus becomes 672 035; but in case of the microscopic reading having been more than $67\frac{1}{2}$, the total reading would be 677 035, as the micrometer vernier subdivides only the half of one of the numbered divisions of the horizontal scale.

3. Unclamp the telescope and the microscope, and observe similarly on the lower marked line of the staff, but in the same vertical plane, obtaining a second reading which we will suppose is 660 015.

4. Notice also the reading for the point of departure on the scale when the telescope is levelled; suppose it to be 500 010.

The mode of calculation.

From these three readings all the necessary results for distance, altitude, and level are obtained by the before-mentioned formulæ or proportion; where if $S=$ 10 feet; $h=6$ inches $=1\,500\,000$ parts of the scale, and $l=672\,035-660\,015=12\,020$ parts, then the distance

$$D=\frac{hS}{l}=\frac{1\,500\,000\times 10}{12\,020}=1247{\cdot}92 \text{ feet.}$$

And to obtain the altitude of the lower point on the

staff, here we have D given, and the value of l in this case being the difference of readings between that of the lower point and the point of departure = 160 005, we obtain

$$s = \frac{lD}{h} = \frac{160\,005 \times 1247{\cdot}92}{1\,500\,000} = 133{\cdot}115 \text{ feet.}$$

Should the reading for the lower point on the staff be less than that of the point of departure, the altitude thus obtained will be negative. The value of s may also be obtained by another proportion, thus,

$$s = \frac{160\,005 \times 10}{12\,020} = 133{\cdot}115 \text{ feet.}$$

To avoid dividing the constant dividend, a small table may be made from which the distances may be obtained by inspection.

Other points of importance.

The following shows the amount of exactitude that may be obtained with this instrument.

Distances of 100 feet exact to 0·001 foot.
" 1000 " " 0·05 "
" 3000 " " 1· "

Heights at a distance of 300 feet exact to 0·0005 foot
" " 1000 " " 0·002 "

It is easily seen that the divisions of the scale may be of any convenient corresponding length, as they are merely required to represent ratios; and that the base scale may be of any length, instead of exactly four inches, so long as it is divided into 200 equal parts, and that the micrometer vernier divides each of these into 5000 equal parts; hence the instrument admits of modification without inconvenience.

To determine the value (h), or as it is called the *base line* of the instrument, measured from the point of rotation of the microscope perpendicular to the scale; measure very accurately a distance, say 100 feet, from the instrument to the 10-feet staff, and take the readings giving l as a difference, hence obtain

$$h = \frac{l \times 100}{10}.$$

But before thus obtaining this value, it must be made sure that the optical axis of the microscope and telescope are in the same vertical plane, or rather in parallel vertical planes, and perpendicular to each other: to effect this adjustment, first sight a 10-feet staff, and next a shorter one, say of 5 feet, at the same distance, and try with the acquired data whether the base line of the instrument remains in each case proportional to the distance or not; should this not be the case, the cross-lines of the microscope must be moved by the motion of their adjusting screw.

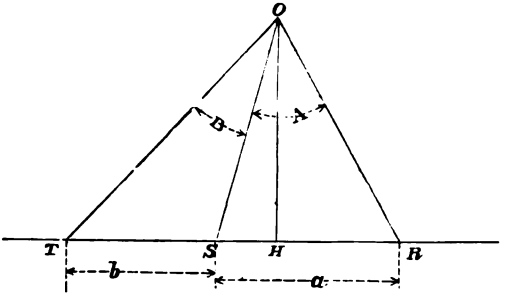


FIGURE 4.—INSTRUMENTAL CORRECTION.

To determine mathematically the height of the base line of the instrument, and the position of the point of

departure on the scale, shown as OH and RH in the figure.

Let $RS=a$, any distance measured on the scale, subtending the known angle A; and $ST=b$, any convenient distance subtending the known angle B.

Put the unknown angle $ORH=X$, and the unknown side $OS=z$.

Then since $a : z :: \sin A : \sin X$

and $z : b :: \sin (A+B+X) : \sin B$

$\therefore$ $a : b :: \sin A . \sin (A+B+X) : \sin X . \sin B$

but $\sin (A+B+X)=\sin (A+B)\cos X+\cos (A+B).\sin X$

$$\therefore \sin (A+B) \operatorname{cotg} X=\frac{a \sin B}{b \sin A}-\cos (A+B)$$

$$\text{or } \operatorname{cotg} X=\frac{a \sin B}{b \sin A \,.\, \sin (A+B)}-\operatorname{cotg} (A+B)$$

whence X becomes known.

and since $RO=\dfrac{(a+b) \sin (A+B+X)}{\sin (A+B)}$ in known terms,

and $OH=RO \sin X$; and $RH=RO \cos X$; the values of these two required quantities can be obtained.

The following is the form of Field-book or record for this work recommended by Eckhold.

Station	Base line = 1,500,000				Departure Point at 500,010						Remarks
	Staff = 10 feet		Difference	Distance	Difference	Altitudes				Reduced Level	
						Back sight		Fore sight			
	Upper	Lower				+	−	+	−		
				Feet							

The numerous advantages of such an instrument require little or no explanation, and the exactitude of result being undoubted, the necessity of measuring and depending on carefully-measured base lines in large surveys is entirely obviated; levelling as well as measuring can be done at very long distances on inclined planes; and having a constant starting point for levelling there is nothing further to do with the collimation of the optical axis of the telescope. This instrument has been used in railway surveys in America and on revenue surveys in India.

Micrometer Eyepieces.—In the same way that the Porro principle with its complicated eyepieces has resolved itself into the simple application in practice of using extra wires in level telescopes of high power, correspondingly the Eckhold principle of using both microscope and micrometer may reduce itself in practice to the simple application of micrometers to the eyepieces of theodolites. The use of moveable micrometer-eyepieces was first publicly exemplified in survey work on the topographical survey of Abyssinia. The six-inch transit theodolites used had eyepieces furnished with a pair of micrometers fitted into a rectangular frame, which was capable of being turned round on a collar, so as to bring the micrometer wires parallel either to the horizontal or the vertical wires of the fixed diaphragm plate; a division of the micrometer was equal to about two seconds. This construction enables the angle subtended by a pole of known length, set up either in a vertical or a horizontal position, to be measured by the micrometers; distances can thus be deduced with considerable accuracy, either in base lines or in traversing; while in astronomical observations for time and longi-

tude, either by zenith distances or meridional transits, the two additional wires, which may be used in either position, afford increased accuracy of result.[1]

7. *Reflecting Telemeters.*—While in the adoption of either of the two former principles, a known distant height is required for observation, in the one case taking the form of a level staff graduated to hundredths of a foot, and in the other case a simple vertical rod with two marks on it exactly ten feet apart, a further simplification in this respect is effected by the adoption of the horizontal or reflecting telemeter, when a simple unmarked vertical staff, or vertical line or edge of a building, becomes sufficient to observe on. The constant quantity in this case is the length of the horizontal hollow bar or tube constituting the telemeter, and the angle observed in operation being that subtended by this constant length at the distant point or staff, the distance is a trigonometrical function of this angle and is obtained from inspecting a table con-

[1] The principle of the eyepiece micrometer is very simple, its manipulation easy, and it is free from the difficulties involved in using microscopes. In applying it to the measurement of the horizontal angle subtended by a marked length on a bar held up horizontally at an unknown distance, the micrometer is set horizontal by means of the collar. The field of view then shows two parallel vertical threads, a horizontal serration, and the object with its two marks: the fixed thread is first set exactly to one of the object marks by motion around the axis of the instrument, through its clamping and tangent screws, and the moveable vertical thread is set to the other object-mark by turning the micrometer screw. The reading is effected by counting the number of intercepted teeth in the serration for units, and reading the number of divisions on the micrometer screw opposite its index for tenths and hundredths, the sum being an angular measure of the length observed. The total value of the reading is then reduced to seconds, in accordance with the known sum of the micrometer, and from this subtended angle the distance of the object can be calculated, or obtained from a table. In vertical measurement the mode is the same. (The chaining of long, and even moderate distances, can thus be entirely superseded.)

structed to suit the constant length of the instrument used.

The instruments of Professor Piazzi Smyth, Lieut.-Colonel Clerk, Otto Struve, and the optician Adie were respectively 60, 72, 73·5, and 36 inches long; there is also a small 18-inch instrument made by the latter suited to range-finding, military surveys, and rough purposes generally, but Adie's 36-inch instrument is perhaps that best known to the general public.

Its principle of operation is that of taking two simultaneous observations on the distant vertical line or object from the two extremities of the instrument where there are two object-glasses and two reflecting prisms, the one fixed so as to reflect the object in the direction of the tube, the other moveable about its vertical axis by a tangent screw, so that its reflected ray may be transmitted along the axis of the tube in a reverse direction: the reflected rays from these end prisms are received by a pair of central prisms in the field of the telescope which are reversed in position and divert the rays into the line of vision of the single eyepiece used. A horizontal separation in the field of view, similar to that in other reflecting instruments, in conjunction with the movement of the tangent screw, enables the two reflections of the vertical line or distant object to be superimposed, so as to form one continuous vertical line, when the scale-reading will give the required angle. Each division on the fixed scale is two minutes, and each one on the vernier is two seconds, the intermediate minute, if odd, being shown by the coincidence of the verniers. The index error requires occasional verification, as in the case of all reflecting instruments (see Section on Angular Measurement), and if it should be required to

effect an index adjustment, the screws under the end-prisms may be very slightly moved with a screw-driver until it is correct; this is necessary after cleaning the instrument. The eye and object glasses, and the end and central prisms, can all be easily got at for purposes of wiping and cleaning. Any distance may be once measured with this instrument in about half a minute, but a mean of several readings is to be preferred. The maximum error in distance with the 36-inch instrument, is 3 inches at 300 feet, $7\frac{1}{2}$ feet at 1500 feet, and 30 feet at 3000 feet.

The advantages of instruments of this type are, the doing away with the necessity of any graduated or marked distant staff of vertical position; and that in long sights the effect of varied refraction is annihilated in results. The defects of telemeters based on this principle are—That the twice reflected images in long sights lose their sharpness of outline, and that the two images being in very close proximity, if not overlapping, excessive nicety of management is required to measure the required interval. That any unequal expansion of the tube, arising from alteration of temperature, by direct solar heat, causes curvature and alters the angles of reflection, thus producing considerable error.

It seems hence to be a reasonable conclusion that such instruments become practically useless for very accurate survey work, both in hot climates and in very hot weather anywhere, although outer cases and sun-shades may be used to mitigate ill effects; and that even in temperate climates the amount of accuracy in result with instruments as short as 36 inches is insufficient; although longer instruments of 60 inches

might answer all practical purposes for distances up to about 700 feet or even more.

While instruments of this type might certainly be used in conjuction with reflecting instruments for angular measurement, it must be remembered that when telemetry is accurate, rapid, and easy, triangulation can be carried on without the need of any angular measurement; while if triangulation is required simultaneously with levelling operations, telemeters on the vertical refracting principle before mentioned are much to be preferred.

The means, appliances, and instruments more specially used in levelling will be treated in the chapter devoted to that subject.

Section 3. Instruments and Appliances commonly used in Angular Measurement.

As the object of a survey is to produce a delineation of the country, district, town, route, or piece of land on a horizontal plane, the angles mostly required in surveying are horizontal angles; vertical angles are less required, from the fact that elevations on the earth's surface being comparatively small, the angles commonly subtended by them are also small, vitiated by atmospheric refraction, and hence less capable of direct observation with sufficient minuteness; while levelling operations for the determination of direct height or depth can be carried out with greater accuracy of result.

Surveying instruments for angular measurement generally fall under one or other, or some modification or combination, of the three following types.

1. The Altitude and Azimuth, or Theodolite type; for measuring horizontal and vertical angles.

2. Repeating Circles and Reflecting Instruments for measuring angles in any desired single plane.

3. Magnetic Instruments, for obtaining magnetic bearings, or angles dependent on them.

I. *Instruments of the first type* are made in many sizes and of various patterns, from the 3-feet theodolite, reading to fractions of seconds and weighing about two hundredweight, of the Great Trigonometrical Survey, down to the 4-inch theodolite reading to one minute which is used by surveyors that prefer a light instrument. The most common size used in England is the 5-inch theodolite, which, with the aid of a vernier, gives horizontal angles correct within half a minute; the 6-inch theodolite reading to a third of a minute is also very common; while the 8-inch theodolite reading to 10″ is used only on very extended or extremely accurate work, in which case it is generally of the transit-theodolite pattern, and fitted for rough astronomical purposes. The patterns in most common use are the Y theodolite, having a semicircular vertical arc; the Everest, having two small portions of vertical arc; and the Transit-theodolite, which has the advantage of the reversibility of the telescope in a vertical plane, and is made either with or without any vertical arc: the first pattern being the most common, the second the lightest, the third the most preferable in some respects. In important subterraneous surveys and tunnel work, transit-theodolites, with a hollow bearing-axis, are generally considered necessary in order to obtain sufficient light from a lamp on the cross-wires. They are generally set up over any required point or spot simply by the help of a plummet hanging from a hook below the axis of the instrument, and by moving the legs of the tripod until exactly in

position; but in some there is a special appliance fitted to the lower part of the instrument, by means of which a slight motion may be given to it in either direction, thus ensuring greater exactitude in this respect, as well as saving time in getting the required position. A further advantage may be obtained by having the instrument fitted with an eyepiece-micrometer, as it then can be used for direct observation of distance in telemetry; besides that greater exactitude is afforded in astronomical observations for time, latitude, &c.; this has been already noticed in the section on telemetry.

The setting up of a theodolite, which consists in getting it into position, levelling it, and focussing the eye and object lenses of the telescope; and the observation of an angle, which consists in directing the telescope suitably focussed first to one object and then to another, and taking the difference of the two corresponding readings on the horizontal graduated limb, are two very simple operations best learnt by imitation.

There are, however, two very common sources of error that require attention. 1st. Although the focussing may have been carefully attended to, and thus all parallax impossible, yet the cross-wires of the diaphragm may be slightly out of their proper position, which is the exact axis of the telescope. To test this, observe on any well-defined distant object, and then either roll the telescope half round in its Y's, or reverse it laterally in its bearings (according to its construction) and observe again; should the intersection of the cross-wires not be exactly on the same distant point in each case, and this be proved the second time by careful repetition, the cross-wires must be moved by means of their respective

pairs of diaphragm screws, until they are perfectly correct and stand the test. Or, if it be preferred to leave the cross-wires unmoved, each angle taken must be twice observed, the second time with the telescope reversed; the mean will be freed from this error. 2nd. If there happen to be any inaccuracy of division in the verniers or the graduation of the horizontal plate, the error from this cause may be nearly eliminated by Borda's method of repetition, which distributes the effect of these errors all round the circle, or throughout the length of the vernier, and nearly neutralises them. It is thus. Having observed an angle in the usual way from zero, and noted the position of the index, clamp the two horizontal plates firmly together, loosen the lower plate, and turn them, still fastened together, until the line of sight coincides with the direction first taken, or first side of the angle; now clamp the lower plate and loosen the upper, turning it until the telescope sights again the second side of the angle; next clamp the upper plate also, and notice the position of the index, which will then give a reading double the required angle as first observed. This repetition should be carried on to any number of times until the one plate has nearly made one, two, or more complete turns with regard to the other; the number of turns being noted, multiplied by 360°, and added to the angle finally read. The total divided by the number of observations of the angle gives a mean angle very nearly freed from this error as well as from the first of the two following sources of error; and, if necessary on the score of collimation error, the same process can be repeated with the telescope reversed.

A theodolite, either badly made or damaged from

wear, is liable to two additional sources of error, on account of which a correction must be applied to any angles observed with it.

1. From want of perfect parallelism of the horizontal plates.

2. From the vertical limb not moving exactly in a vertical plane.

The Horizontal Limb.—To test the accuracy of the horizontal limb of the instrument, level the horizontal plates very carefully, for four positions of the telescope, across, reversed, and reversed across, as well as in the original position. Do the same again, and in each case where needful bring the bubble of the telescope-level to the centre of its run, half by means of the tangent screw of the vertical and half by means of a pair of the thumb-screws; now, if both the plates be loosened and turned completely round, the bubble will retain its position, provided that the horizontal plates are perfectly parallel; otherwise the axis is faulty, and an instrument-maker is necessary to put it right. In the meantime, however, the instrument may be used if the following correction be made on this account.

Let ZO, figure 5, represent the position of the vertical axis of the upper plate $EABD$, and $EA'B'D'$ the position of the lower plate; then if OA and OB be the two positions of the index when an angle is observed, the angle $A'OB'$ will be read instead of AOB. Let the values of these be represented by C' and C, and the zenith distances $A'Z$ and $B'Z$ by ζ and ζ', and since the angle included between the two planes ZOA' and $ZOB' = C$, we have

$$\cos C = \frac{\cos C' - \cos \zeta . \cos \zeta'}{\sin \zeta . \sin \zeta'}.$$

Again, supposing that when the index is at E the zenith distance ZE happens to be exactly a right angle; let the angles $EOA=D$, $EOB=D'$, $EOA'=\delta$, $EOB'=\delta'$ then since $\sin 90°=1$, and $\cos 90°=0$,

$$\cos D=\frac{\cos \delta}{\sin \zeta}, \text{ and } \cos D'=\frac{\cos \delta'}{\sin \zeta'}.$$

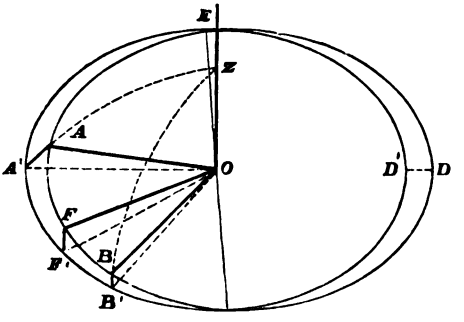


FIGURE 5. INSTRUMENTAL CORRECTION FOR THEODOLITE.

But as in the triangles AEA', BEB', A and B are right angles,

$$\cos AA'=\frac{\cos \delta}{\cos D}=\sin \zeta; \text{ and } \cos BB'=\frac{\cos \delta'}{\cos D}=\sin \zeta',$$

hence also $\sin AA'=\cos \zeta$; and $\sin BB'=\cos \zeta'$;

Hence putting $AA=\theta$ and $BB'=\theta'$, and substituting, the original formula becomes

$$\cos C=\frac{\cos C'-\sin \theta \,.\, \sin \theta'}{\cos \theta \,.\, \cos \theta'}$$

$$\therefore \sin \frac{C}{2}=\sqrt{\left(\frac{\sin \tfrac{1}{2}(C'+\theta-\theta') \,.\, \sin \tfrac{1}{2}(C'-\theta+\theta')}{\cos \theta \,.\, \cos \theta'}\right)}$$

a formula adapted for logarithmic computation. The value of θ, with the aid of which any other value θ' may be easily deduced, can be obtained thus.

Make the upper plate truly horizontal, and sight some distant object with the telescope; turn the lower plate through 180°, and then the upper plate with the telescope back again through 180°, the resulting deviation of the line of sight will be double the angle of error AOA'. Repeat this proceeding around the whole of the circle, until the exact position is found where the error becomes a maximum, which is supposed to be at FOF', and obtain a value of FF' at this point; then since the arcs are small

$FF' : BB' : AA' :: \sin C + C' : \sin C' : \sin C$; hence $\theta = FF' \sin C$, and $\theta' = FF' \sin C'$.

If the instrument happens to be so faulty in this respect as to render it necessary to reduce this error not for a single case, but for a large amount of pressing work that may utilise any part of the circle; it is better to start from a position where $\delta = 0$, and calculate a tabular set of values of δ' for position of every 5 degrees all round the circle up to the maximum, which may be interpolated by proportion; the formula for correcting the observed angle will then be

$$\sin \frac{C}{2} = \sqrt{\left(\frac{\sin \frac{1}{2}(C' - \delta') . \sin \frac{1}{2}(C' + \delta')}{\cos \delta'}\right.}$$

2. *The Vertical Limb* may not move exactly in a vertical plane, either from having the bearing axis of the telescope inclined to the horizon, or from any of the attachments or supports being ill-made, damaged, or bent. To test this, adjust the instrument very carefully in every respect, and observe in the vertical plane of the instrument on any well-defined object, as the top of a steeple, or a star, and its reflection in still water or in an artificial horizon; if the reflected image is not

exactly in the vertical plane of the instrument, when in adjustment, there is an important source of error, which will affect any horizontal angles taken with the instrument.

Should the mounting of the telescope be of the transit theodolite pattern, the telescope bearings or trunnions may be a little worn, and this deviation of the turning axis from the horizontal may be the sole cause of error; in some of these instruments this may be rectified, or nearly so, by using the adjusting screws made for this purpose; in larger instruments of this kind a striding level may be used for checking this error, and the deduced level correction applied to all angles taken, on the principle adopted with transit instruments in astronomical observation. (Refer to Chapter on Route Surveys, article on the Corrections of the Transit Instrument.) Under other circumstances and with instruments of other patterns the source of error may lie in any part of the attachments; if the results are very marked, it is certainly best to reject the instrument entirely until altered by an instrument-maker.

To investigate the correction to be applied on account of this defect.

Let a = the inclination of the bearing axis to the horizon.

And $90° - a$ = the inclination to the horizon of the incorrect vertical plane of motion of the telescope.

Let ϕ = the horizontally projected angle error in an angle observed on two objects differing in height,

θ = the vertical angle of elevation or depression, in the same case.

then we have in the right-angled spherical triangle

$$\tan \overline{90^\circ - a} = \frac{\tan \theta}{\sin \phi}$$

$$\text{or } \sin \phi = \tan a \tan \theta$$

$$\phi = a \tan \theta, \text{ as } a \text{ and } \phi \text{ are very small,}$$

where if ϕ is positive its value must be subtracted from, and if negative added to, the observed angle. In this formula θ will be positive when it is an angle of elevation, and negative when an angle of depression; a will be positive when the numbering is arranged from left to right, and the turning axis of the telescope is higher on the left than on the right, but negative when higher on the right than on the left; and ϕ will be positive when a and θ are of similar sign, and ϕ will be negative when they are of different sign.

Hence if we suppose C to be the true horizontal angle after correction, C' to be the observed horizontal angle obtained by two readings on the horizontal arc c_1 and c_2, and ν_1 and ν_2 to be the two readings above and below zero on the vertical arc,

$$C \text{ will} = c_1 - c_2 - a(\tan \nu_1 + \tan \nu_2).$$

To obtain the value of the remaining unknown quantity a, let us suppose that, from observing on a plumb-line, the turning axis of the telescope is found to be higher on the left than the right, and a is positive when the angular readings are c_1 and c_2 as above; if we now take a second observation of angle on the same objects, after reversing the telescope and turning the upper plate round 180°, and let c_3 and c_4 be the two readings in this case; then the effect of reversal of the turning axis will be to make the second pair of readings

give an angle that will be as much too small as the first pair gave too large, hence

$$C \text{ will} = c_3 - c_4 + a\ (\tan \nu_1 + \tan \nu_2)$$

and as also $C = c_1 - c_2 - a\ (\tan \nu_1 + \tan \nu_2)$

we obtain from these practical observations, a value of a

$$a = \frac{c_1 - c_2 - c_3 + c_4}{2\ (\tan \nu_1 + \tan \nu_2)}$$

which can be recorded as suitable to the instrument; from which the value of any error $\phi = a \tan \theta$ may be obtained for any subsequent observation in accordance with the circumstances above referred to.

For example. If $a = +5$ minutes; and a horizontal angle be observed when the first line of sight is directed at an angle of elevation of $+5° 27'$, and the second line of sight is at a depression giving an angle $-2° 51'$, then

$$\phi = +5' . \tan (+5° 27') - 5' . \tan (-2° 51')$$
$$= +\cdot 4749 - \cdot 2486 = 0\cdot 2263 \text{ minutes}$$

As the difference in elevation has so important an effect on errors of this class, the index error of the vertical arc, if there be any, should be remembered and occasionally tested by levelling the telescope.

II. *Repeating and Reflecting Instruments* for angular measurements in any plane or direction. Borda's repeating circle has been preferred to the theodolite by the French and Swedish Great Trigonometrical surveyors, both on the score of its simplicity and portability and from mistaken ideas with regard to excessive accuracy as obtained by repetition.

The repeating circle used by Delambre had a diameter of about 7 inches, each of the two telescopes attached being about 24 inches long; the circle was

graduated to minutes from right to left around the whole circumference, on its upper side only, the telescopes were capable both of independent motion, and of motion with the circle by the use of clamping screws, and a stand was used to hold the instrument in any required plane.

The principle of repetition, or of observing the same angle an even number of times and then obtaining the mean was justly depended on for diminishing errors of graduation, and neutralising errors of observation; but it was also then imagined that by frequent repetition up to 20 times, any angle might be determined within a second; as however the graduations only read to minutes, and the telescopes were not sufficiently powerful to discern or divide to a quarter of a minute, this seems impossible.

The method of repetition is as follows.

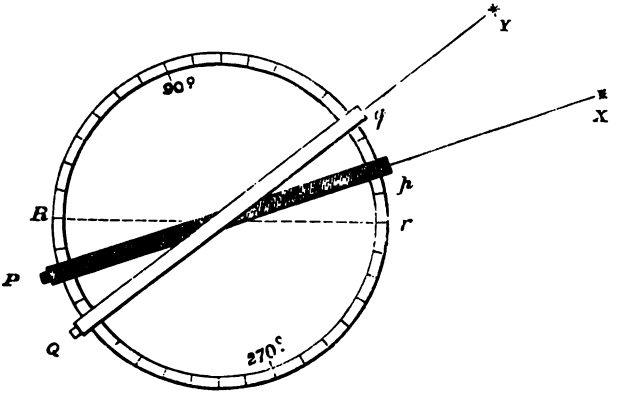


FIGURE 6. PRINCIPLE OF REPETITION.

The instrument having been brought into the plane of the two objects X and Y, the upper telescope Pp is set to zero and directed to X, the lower telescope Qq is

directed to Y, and the two telescopes are then clamped, thus making one observation of the required angle: its value however cannot immediately be obtained as the lower side of the circle is not graduated. The instrument is then turned onwards in its present plane until Qq be directed to X, when Pp will fall into the position Rr; and the instrument is then fixed in position. The telescope Pp is then unclamped, removed from its position Rr, directed to Y, and clamped. The reading on the circle will now be double the value of the angle. The whole circle must then be turned until Pp points to X, and the previous process repeated any even number of times, the circle being always turned to the right through the arc pq, and the two telescopes alternately to the left through the arc rq, or double the former. The final reading is divided by the number of observations to obtain the mean.

The angle having been observed in an oblique plane, the altitudes or elevations of the two objects are then observed or obtained, and the oblique angle is reduced to the horizon by tables prepared for this purpose, or by calculation. (See pp. 52 and 59 for formulæ and example.)

The advantages of this principle of repetition are obtained in the Reflecting Repeating Circle of Troughton, an exceedingly convenient instrument for measuring astronomical altitudes and distances in arc; it must however be noticed that there is always a constant error which cannot be removed by any number of repetitions.

The Surveying- or Box-Sextant is used by surveyors, as a light substitute for a theodolite, for obtaining a few horizontal angles in a small survey, or for one or two casual altitudes, such as heights of buildings, and station-marks; but it is not a favourite instrument with them,

owing to the disadvantages inseparable from reflecting instruments and the necessary reduction for obtaining horizontal angles from the oblique angles observed; although its accuracy of reading in comparison with magnetic instruments is greatly in its favour.

The principle of this instrument being, like that of the nautical sextant, so well known, it is only necessary to mention its adjustments and tests. They are:

1. That the index-glass shall be truly at right angles to the plane of the instrument.

2. That the horizon-glass shall be truly at right angles to the plane of the instrument.

3. That the line of collimation of the telescope, or line of sight, shall be truly parallel to the plane of the instrument.

The index-glass being a permanent fixture, any error in its position should be rectified by an optician; but its adjustment may be tested by setting the index to 60°, and looking at the reflection of the limb of the instrument in the index-glass, which should appear continuous with the real limb.

The position of the horizon-glass may be corrected by means of the adjusting screws. It can be tested by clamping the index at zero, and observing on a distant well-defined object, whose real and reflected image should coincide or appear perfectly continuous; if it should not be so, a slight motion given to the tangent screw of the index will effect this; the index error resulting should then be noted, and applied to all subsequent observations; this is negative in sign when its amount has to be added as a correction to an angle, and is called positive when its amount has to be subtracted.

The telescope-bearing or attachment being a perma-

nent fixture, the correctness of its adjustment may be tested by observation on two objects subtending a right angle and causing the plane of the sextant to revolve slowly around the axis of observation on the reflected object; this should not disturb the coincidence of the two images.

In surveying sextants there is frequently a supplementary arc and small mirror below the index glass for observing angles above 120°; the size of the brass box constituting the frame of the instrument is generally three or four inches in diameter; the latter is to be preferred, it reads to thirty minutes or to twenty seconds with the aid of the vernier, and with well-defined objects, and by using the small telescope, which can generally be dispensed with, can be employed to a range of three-quarters of a mile. The manipulation of these, as of all reflecting instruments, requires some dexterity and practice; the reduction of oblique angles may often be dispensed with by judicious use of a plumb-line.

Troughton's reflecting circle is the best instrument of this type in use in England; its principle and its glass-adjustments are similar to those of the sextant; but as it gives readings to seconds, and has three branches to its index, and admits of reading backwards and forwards, and repetition, the errors of the simple sextant become reduced in it to one-sixth; it will also measure an angle up to 150°.

Its collimation adjustment, or the setting of the line of collimation parallel to the plane of the instrument, is performed with two screws, which also fasten the collar of attachment of the telescope to the upright stem on which it is mounted. The collimation is tested by observing on the sun and the moon, when they are about

130° or more apart, and bringing their limbs in contact at the outside of the wire next to the circle, and afterwards examining under similar conditions at the outside of the other wire. If it answers this test the adjustment is correct.

III. *Magnetic Instruments.*—The principle applied in using compass bearings for directions in route and small traverse surveys, and in mining operations, and in using differences of bearings as angles for the triangulation and detail of small surveys of any sort, requires no explanation. The forms of compass used are the pivoted needle, and the suspended needle; the former being the most common and convenient generally, while the latter is more accurate and suited to scientific purposes. The graduated arc, on which the needle reads, is, for surveying purposes, divided into degrees and parts from 0° to 360°, in such a way that a reading of 90° may be the bearing of an object lying to the magnetic east of the observer. Sights for obtaining the accurate direction of any distant points, and telescopes, may be applied in very various forms. A magnifying prism is frequently used for sighting and reading the bearing simultaneously, in which case the graduations and numbering are reversed in order; and in pivoted compasses a stop or lever may be used for checking the vibration of the needle or throwing it off its pivot to prevent wear. The forms of case or compass box used are also very various. Formerly compasses were rigidly attached to, or formed part of, most theodolites and surveying levels; but as they were seldom used with them, and added to their weight, they are now made to be readily detached or used independently when required. The stand used for

a surveying or miner's compass or circumferenter is generally a light tripod; in the latter case, low for convenience in galleries: in soft ground a spiked rod about five feet long surmounted by a square flat top or resting table is often preferred; and again the compass is sometimes fixed permanently on the top of a surveyor's crosshead, which is used on a spiked rod; while in other instances stands are entirely dispensed with, the box or compass being held in the hand as level as possible.

The four-inch box-compass gives readings to a half or to a third of a degree, larger compasses to a quarter; the many possible causes of disturbance render a further exactitude in reading unnecessary. The neighbourhood of iron, whether in the form of lodes or disseminated iron in surrounding rocks, an iron bridge or structure, or keys and instruments carried about the person, may vitiate observed bearings to a very important extent. Magnetic changes and storms also affect bearings; while the diurnal change itself has been known to amount to as much as a quarter of a degree. Under these circumstances a three-inch compass may be sufficiently accurate.

When a higher degree of accuracy is required, the manifest objections to the supported pivot are obviated by adopting Gauss's principle of suspension, which was applied to surveying or portable purposes by Porro. In Porro's compass the needle is suspended in a tube by a silk thread, which also is in a tube at right angles to the former; a reflector partaking of the movement of the needle is fixed at right angles to it; a half mirror superimposing a scale, seen through the transparent portion of it, shows the reflection of a permanent wire or fine thread, whose oscillation and final position determines

the azimuth. A telescope for viewing the field, and a lens focussed on the graduated scale also form part of this instrument. Considering the motions of the needle to be referred to rectangular planes, they may be said to consist of two pendulous movements, a pitching, a rolling, and an azimuthal movement; of these one of the pendulous movements and the rolling do not affect the position of the reflected image, while the other pendulous movement and the pitching impress on the image a swinging motion from front to rear, and do not prevent it from retaining a position over the fixed graduated scale; the azimuthal motion is hence practically unaffected by the other movements. As to subdivision of reading, Porro claims that with a needle of 40, and a thread of 15, millimètres, a bearing may be read within three minutes.

The use of such an instrument for surveying purposes must necessarily be accompanied with a knowledge of the laws of magnetic disturbances, and with full information of magnetic occurrences obtained from the nearest magnetic observatory.

In any case the variation of the compass should be known; if this must be determined by practical observation, one of the methods explained in the Chapter on Astronomical Observations in Route Surveys should be adopted.

The Portable Transit-instrument is occasionally used by the surveyor or engineer to range in long-distant and intermediate points in any required direction with exactitude and precision. Its adjustments are explained in the chapter on Astronomical Observation in Route Surveys.

Section 4. FORMULÆ FOR TRIGONOMETRICAL CALCULATIONS.

While the operations of the surveyor aim at obtaining a sufficient number of distances. angles, and heights, from which the required plan can be made, the whole of these are not necessarily measured, as some may be calculated from others. Again, calculation may also be adopted for checking the correctness of measurement. The greater part of the calculations of the surveyor are based on the principles of simple plane and spherical trigonometry, a few mensuration formulæ only being required for calculating areas of estates. A knowledge of these principles and their application forming part of ordinary education, the reader of this book will be assumed to possess this knowledge, while the formulæ more useful for the purposes of surveyor's triangulation will be found in the following collection. Occasional formulæ suited to particular instances will be given throughout the book attached to the case or special subject on which they bear.

Collection of Trigonometrical Formulæ.

I. *Functions of any two angles.*

$$\sin (A+B)=\sin A \cos B+\cos A \sin B$$
$$\sin (A-B)=\sin A \cos B-\cos A \sin B$$
$$\cos (A+B)=\cos A \cos B-\sin A \sin B$$
$$\cos (A-B)=\cos A \cos B+\sin A \sin B$$
$$\tan (A+B)=(\tan A+\tan B)\div(1-\tan A.\ \tan B)$$
$$\tan (A-B)=(\tan A-\tan B)\div(1+\tan A.\ \tan B)$$
$$\operatorname{cotg} \overline{A+B}=(\operatorname{cotg} A.\ \operatorname{cotg} B-1)\div(\operatorname{cotg} A+\operatorname{cotg} B)$$
$$\operatorname{cotg} \overline{A-B}=(\operatorname{cotg} A.\ \operatorname{cotg} B+1)\div(\operatorname{cotg} B-\operatorname{cotg} A)$$

$$\sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\sin A + \sin B = 2 \sin \frac{A-B}{2} \cos \frac{A+B}{2}$$

$$\cos A + \cos B = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\cos B - \cos A = 2 \sin \frac{A-B}{2} \sin \frac{A+B}{2}$$

$$\tan A + \tan B = \sin (A+B) \div (\cos A . \cos B)$$

$$\tan A - \tan B = \sin (A-B) \div (\cos A . \cos B)$$

$$\operatorname{cotg} A + \operatorname{cotg} B = \sin (A+B) \div (\sin A . \sin B)$$

$$\operatorname{cotg} B - \operatorname{cotg} A = \sin (A-B) \div (\sin A . \sin B)$$

$$\sin^2 A - \sin^2 B = \sin \overline{A-B} . \sin \overline{A+B}$$

$$\cos^2 A - \sin^2 B = \cos \overline{A-B} \cos \overline{A+B}$$

$$\tan^2 A - \tan^2 B = \sin \overline{A-B} . \sin \overline{A+B} \div (\cos^2 A . \cos^2 B)$$

$$\operatorname{cotg}^2 B - \operatorname{cotg}^2 A = \sin A - B . \sin A + B \div (\sin^2 A \sin^2 B).$$[1]

II. *The solution of triangles.*

Let a, b, c represent the three sides, and A, B, C, the three angles opposite to them of any plane triangle,

[1] The algebraic signs of sines and cosecants of supplements are the same as those of the original angles; in the other quadrants the signs are given in the following table.

	0°		90°		180°		270°		360°
Sine . .	0	+	R	+	0	−	R	−	0
Cosecant . .	∞	+	R	+	∞	−	R	−	∞
Cosine . .	R	+	0	−	R	−	0	+	R
Secant . .	R	+	∞	−	R	−	∞	+	R
Tangent . .	0	+	∞	−	0	+	∞	−	0
Cotangent . .	∞	+	0	−	∞	+	0	−	∞
Versine . .	0	+	R	+	$2R$	+	R	+	0

and let $s=\frac{a+b+c}{2}$, then the relations between the functions of three angles are,

$$\sin A=\sin \overline{B+C};\ \cos A=-\cos \overline{B+C};$$

$$\tan \frac{A}{2}=\operatorname{cotg} \frac{B+C}{2}, \text{ and } a=\frac{b.\sin A}{\sin B};$$

$$\tan \frac{A-B}{2}=\frac{a-b}{a+b}\operatorname{cotg} \frac{C}{2};$$

$$\cos \frac{C}{2}=\surd\left(\frac{s.\overline{s-c}}{ab}\right);\ \sin \frac{C}{2}=\surd\left(\frac{\overline{s-a}.\overline{s-b}}{ab}\right).$$

Small angles are obtained through their sines, tangents or cosecants, and angles near 90° through the other functions. In the ambiguous solution, refer to the field record as to whether the angle was obtuse or acute.

A right-angled triangle forms a particular case of the above.

The following formula affords a check on the correctness of the observed inward angles of any irregular polygon of traverse.

Let N=the number of angles of the polygon.

Then in the particular case of the angles being all salient, the sum of all the observed inward angles should be $=2N\times 90°-360$.

And if there be any re-entering angles the sum of all the salient inward angles should be equal to the sum of all the re-entering outward angles.

III. *The calculation of areas.*

Triangle; area $=\frac{ad}{2}=\frac{bc.\sin A}{2}=\surd\overline{s.\overline{s-a}.\overline{s-b}.\overline{s-c}}$,

where d is the perpendicular from A on a.

Parallelogram; area $=bd=bc.\sin A$,

where d is the perpendicular on b.

Trapezoid; $\text{area}=(a+b)\frac{d}{2}$,

where a and b are parallel at a distance apart $=d$.

Regular polygon; $\text{area}=\frac{na^2}{4}\operatorname{cotg}\frac{180^\circ}{n}$,

where $n=$number of sides, each of which $=a$.

Irregular figure; $\text{area}=b\left(\frac{p_0}{2}+p_1+p_2+p_3+\&c.\ +\frac{p_n}{2}\right)$,

where $n=$the number of the equidistant ordinates

$b=$their common distances apart.

Irregular figure slightly curved (parabolic)

$$\text{area}=\frac{b}{3}\left(p_0+p_n+2.\overline{p_2+p_4+\&c.}+4.\overline{p_1+p_3+\&c.}\right),$$

in this case n must be an even number.

IV. *The solution of geodetic or large triangles on the earth's surface.*

1. For limited purposes of this description, up to triangles of 40 miles in length of side, the earth is generally considered an exact spheroid, having the ratio of $\frac{99666}{100000}$ for its polar and mean equatorial diameters, or an ellipticity of about $\frac{1}{299}$, and a mean radius of 20 900 086 feet; on this assumption,

Let $R=$the mean radius of curvature of the earth in feet,

$P=$the area of the large triangle (calculated as plane) in square feet,

$E=$the spherical excess in seconds,

Then $E=\frac{P.648\,000}{\pi R^2}$ or $\log E=\log P-9{\cdot}325871$.

2. To apportion the errors of observation among the angles.

Having obtained the spherical excess, and using mean values of A, B, and C, then $A+B+C$ should $=180°+E$; and any small error indicated should be divided equally among the three angles, if the number of observations of each have been equal.

But if otherwise, Gauss's rule must be used to apportion the error (T) among the angles.

Let n = the number of recorded observations of A,
l, l', l'', &c. the seconds of reading of the several observations,
m = the mean value or average of them,
a = the relative coefficient of error in A,
x = the actual amount of error in A,

Then $$a=\frac{(m-l)^2+(m-l')^2+(m-l'')^2+\&c.}{\frac{1}{2}n^2}$$

And β and γ the corresponding quantities may in the same way be found for the angles B and C; and their sum $a+\beta+\gamma$ obtained.

Then if y and z are the actual errors in B and C

$$x=T.\frac{a}{a+\beta+\gamma}\ ;\ y=T.\frac{\beta}{a+\beta+\gamma}\ ;\ z=T.\frac{\gamma}{a+\beta+\gamma}$$

If a single angle (C) has been only once observed, an arbitrary value is assumed for γ.

3. For the computation of the sides of such a triangle, reduce each angle after correction by one-third of the spherical excess thus calculated, but keep any measured side unaltered in length; the remaining two sides may then be calculated by the formulæ given for plane triangles, without any error whatever.

4. For larger triangles, even having sides up to 200 miles in length, and ill-conditioned in form, the same method of Legendre may be adopted without error, but for such cases the local mean radius of curvature R' used in the formula for spherical excess must be more exactly determined in accordance with the mean latitude of the triangle.

The formula for obtaining the local mean radius of curvature (R') in feet at any latitude L is

$$R' = \frac{\text{equatorial radius} \times (1 - Q^2)}{1 - Q^2 \sin^2 L}$$

where Q = the mean ellipticity of the earth

$$= \frac{\text{equatorial radius} - \text{polar radius}}{\text{equatorial radius}} = \tfrac{1}{295}.$$

The following are the results of Colonel Clarke's computations of the dimensions of the earth.

	Feet	Mètres
Length of polar axis . .	41 706 858	12 712 136
Longer equatorial axis in longitude 15° 34′ E.	41 853 700	12 756 588
Shorter equatorial axis in longitude 105° 34′ E.	41 839 958	12 752 701
Length of meridian quadrant of Paris	32 813 524	10 001 472
Length of minimum quadrant 105° 34′ E.	32 808 772	10 000 024

Colonel Clarke adopts a very short value of the mètre at 32° Fahrenheit, namely 3·280 86933 feet, while even Captain Kater adopted 3·280 899 ; he computes that the

earth's ellipticity in the longitude of Paris is $\frac{1}{286}$, and that the earth's mean ellipticity is $\frac{1}{293}$, which when corrected for local attraction becomes $\frac{1}{295}$.

The mean length of a degree of latitude in the longitude of Paris hence is 364 591 feet or 69·05 miles; the mean diameter of the earth is 41 800 173 feet or 7916$\frac{2}{3}$ miles; and the mean circumference of the earth is 23 871 miles.

The arc of the meridian subtending one second is 101·36 feet; that subtending one minute is 6081·6 feet; and that subtending one degree is 365 896 feet.

It may, however, be noticed that these data, based on the scientific equivalent of the mètre at 32°, are much less than they would be if the commercial value at 62°, about 3·2818 feet, were used

V. *Formulæ occasionally required in geodetic calculations.*

1. If a geodetic base line (c) consist of two portions, a and b, slightly inclined to each other, forming an angle θ,

$$\text{then } c = a + b - \frac{ab.\theta^2}{2(a+b)}.$$

2. Reduction to Sea Level.—If the corrected length of a base line (c), situated at an elevation (h) above sea level, is to be reduced to its value (d) at sea level,

$$d = c - \frac{ch}{r} \text{ where } r \text{ is the radius of the earth.}$$

3. Well-conditioned Triangles.—If one side of a triangle (a) be known, and the three angles remain to be fixed and observed, and it be desired to arrange them so that the sides b and c may be least affected by the errors

of angular observation represented by α, β, γ, the resulting error in the side c will be equal to

$$\alpha \cdot \left(\frac{\sin B}{\sin^2 A} \cdot \gamma + \frac{\sin C \cdot \cos A}{\sin^2 A} \cdot \beta \right)$$

and that in the side b will correspond to this in form; hence A should be less than 90°, and B and C should be nearly equal.

4. The reduction of a slightly-oblique to a horizontal angle.

If h and h' be the altitudes of the two objects in seconds, and D=the observed slightly-oblique angle, then approximately the correction

$$x = \frac{1}{4} \left\{ (h+h')^2 \cdot \tan \frac{D}{2} - (h-h')^2 \operatorname{cotg} \frac{D}{2} \right\} \operatorname{cosec} 1''$$

or more correctly

$$\cos (D+x) = \frac{\cos D - \sin h \cdot \sin h'}{\cos h \cdot \cos h'}.$$

5. The supplementary or satellite station.

Let A be the axis of the signal or mark observed.

„ B and C the two neighbouring stations of observation.

„ S the satellite station or position of the instrument.

„ m the measured horizontal distance AS.

„ b and c the calculated distances, AC and AB,

If $BAC = A$; $BSC = S$; $ASB = \beta$; $ASC = \gamma$;

$$\text{Then } A = S - 206\,264''{\cdot}8 \times m \left(\frac{\sin \beta}{c} - \frac{\sin \gamma}{b} \right)$$

when S lies to the right of both AB and AC.

But when S lies to the left of AC, sin γ changes its sign; and when S lies to the left of AB, sin β changes its sign.

6. The Distant Triangle.

Let A, B, C be the three angles, a, b, c the three sides of the distant triangle; and P any point at which the angles β subtended by b, and α subtended by a, are observed; then the angles required are $CAP=\theta$, and $CBP=\phi$; whence PA, PB, PC can be obtained.

Let $Z=360°-(\alpha+\beta+C)$; and $\phi=Z-\theta$;

$$\text{Then } \operatorname{cotg}\theta=\operatorname{cotg} Z\left(1+\frac{b.\sin\alpha}{a\sin\beta.\cos Z}\right)$$

Where, if $Z<90°$ but $>0°$, $\cos Z$ is $+$, and $\operatorname{cotg} Z$ is $+$,
$<180°$ „ $>90°$, „ $-$, „ $-$,
$<270°$ „ $>180°$, „ $-$, „ $+$,
$<360°$ „ $>270°$, „ $+$, „ $-$.

This problem becomes indeterminate only when all the points P, A, B, C fall on the circumference of a circle.

7. The determination of a distance or base line that is nearly meridional, by the difference of the two observed latitudes of its extremities.

Let $L-L'$ be the difference of latitudes in seconds,
„ M be the true azimuth of the base line,
„ d be the reduced length of the base line in feet,

Then $d=101{\cdot}36.\sec M\times(L-L')$.

8. The calculation of latitudes, longitudes, and azimuths.

Let L be the given latitude of any station, A and C the colatitude.
L' be the required latitude of any station B, and C' the colatitude.

R and Q, the local radius of curvature in feet and mean ellipticity of the earth.

M is the azimuth of B as seen from A.

d=the reduced length of the arc AB in feet.

$$\text{Then } L-L'=\left\{\frac{d\cos M}{R\sin 1''}+\frac{d^2.\sin^2 M.\tan L}{2R^2\sin 1''}\right\}(1+Q^2\cos^2 L)$$

And if G=the difference of longitude in seconds,

N=the azimuth of A as seen from B,

$$\text{Then } G=\frac{d.\sin M}{R\cos L'\sin 1''}$$

$$\text{and } N=180°-M-G.\sin 1''\times\frac{\sin\frac{1}{2}(L+L')}{\cos\frac{1}{2}(L-L')}.$$

9. The following however are approximations frequently used when the arc AB is less than one degree.

Let AB be reduced to seconds of arc, and let n be the difference of azimuth.

$$\text{Then } C-C'=AB.\cos M-\frac{AB^2}{2}\sin^2 M.\tan L.\sin 1'';$$

$$\sin G=\frac{\sin AB.\sin^2 M}{\cos L'}\text{ ; or } G=\frac{AB.\sin M}{\cos L'}\text{ nearly ;}$$

$$\sin N=\frac{\cos L.\sin M}{\cos L'}\text{ ; or } \sin\frac{n}{2}=\sin\frac{G}{2}.\sin\frac{1}{2}(L+L').$$

VI. *The general formulæ of spherical trigonometry.*

1. Properties of any spherical triangle, having angles A, B, C, and sides a, b, c; where $s=\frac{1}{2}(a+b+c)$.

1. Any side $a<\pi r$; and any angle $A<180°$.

2. $a+b+c<2\pi R$; and $A+B+C<3\pi$ and $>\pi$.

3. $a+b>c$; $a-b<c$; and $A+B>180°-C$.

4. If $a+b=180°$, then will $A+B=180°$.

5. If a is $>=$ or $<b$, then will A be $>=$ or $<B$.

6. The polar triangle corresponding to a right-angled triangle will always have at least one side $=\frac{\pi}{2}$.

II. Fundamental formulæ.

$$\cos A=\frac{\cos a-\cos b.\cos c}{\sin b.\sin c};\ \sin A=\frac{\sin B.\sin a}{\sin b}.$$

In right-angled triangles, when $A=90°$.

1. $\cos a=\cos b.\cos c$; and $\cos a=\operatorname{cotg} C.\operatorname{cotg} B$.
2. $\sin c=\sin a.\sin C$; and $\sin b=\sin a.\sin B$.
3. $\sin c=\tan b.\operatorname{cotg} B$; and $\sin b=\tan c.\operatorname{cotg} C$.
4. $\cos B=\operatorname{cotg} a.\tan C$.

III. Napier's rules for circular parts.

The parts being the two sides, the complements of the angles, and of the hypotenuse.

The sine of the middle part=product of tangents of adjacent parts.

The sine of the middle part=product of cosines of the opposite parts.

IV. Formulæ adapted to logarithmic computation for any spherical triangle.

1. If two sides and the included angle (C) be given,

$$\tan\frac{A-B}{2}=\frac{\sin\frac{a-b}{2}}{\sin\frac{a+b}{2}}.\operatorname{cotg}\frac{C}{2}\quad\ldots.\ (1)$$

$$\tan\frac{A+B}{2}=\frac{\cos\frac{a-b}{2}}{\cos\frac{a+b}{2}}.\operatorname{cotg}\frac{C}{2}\quad\ldots.\ (2)$$

$$\sin^2\frac{c}{2}=\sin^2\frac{a+b}{2}-\sin a.\sin b.\cos^2\frac{C}{2} \quad \ldots\ldots (3)$$

$$\operatorname{cotg} A=\operatorname{cotg} a.\sin b.\operatorname{cosec} C-\cos b.\operatorname{cotg} C.$$

2. If two angles and the included side (c) be given,

$$\tan\frac{a+b}{2}=\frac{\cos\frac{A-B}{2}}{\cos\frac{A+B}{2}}.\tan\frac{c}{2} \quad \ldots\ldots\ldots (4)$$

$$\tan\frac{a-b}{2}=\frac{\sin\frac{A-B}{2}}{\sin\frac{A-B}{2}}.\tan\frac{c}{2} \quad \ldots\ldots\ldots (5)$$

$$\sin a : \sin b : \sin c :: \sin A : \sin B : \sin C \quad \ldots\ldots (6).$$

3. Three sides given.

$$\cos^2\frac{C}{2}=\frac{\sin s.\sin\overline{s-c}}{\sin a.\sin b.};$$

$$\text{and } \sin^2\frac{C}{2}=\frac{\sin\overline{s-a}.\sin\overline{s-b}}{\sin a.\sin b} \quad \ldots\ldots\ldots (7).$$

4. Three angles given.

Apply the last formula to the corresponding polar triangle.

5. Two angles and an adjacent side given.

These are ambiguous cases; use formulæ (6) and (1) or (3).

Formulæ specially suited to astronomical purposes will be given in the Section on Astronomical Observations in Route Surveys.

Section 5. SOLUTIONS AND EXAMPLES.

The greater portion of the foregoing formulæ are so well known as not to require proof, and are so simple in application to the general purposes of calculating distances as not to require examples. A few of them however, that are not so often used, require illustration by example for the convenience of reference. (See pages 49 and 50.)

The solution of a large triangle; taking an example from the 'Encyclopædia Britannica.'

In a triangle ABC, at a mean latitude of 55° 40′ the arc $c = 352\,037{\cdot}62$ feet, and the following are the observations of the angles.

A. 56° 43′ 29″·97	B. 79° 42′ 28″·69	C. 43° 34′ 38″·36
27·04		35·43
28·72		

Means, 56° 43′ 28″·58 79° 42′ 28″·69 43° 34′ 36″·89
and the local mean radius of curvature of the earth is assumed = 20 946 862 feet.

First. To determine the spherical excess.

Computing approximate values of a and b,

$a = c.\sin A.\operatorname{cosec} C = 426\,970$ feet

$b = c.\sin B.\operatorname{cosec} C = 502\,480$ feet

and the area of $A.B.C = \frac{1}{2} b.c.\sin A = 73\,945\,000\,000$ sq.ft.

$$\therefore \text{ the spherical excess } E = \frac{\text{area} \times 648\,000}{\pi . (20\,946\,862)^2} = 34''{\cdot}76.$$

Secondly. Let it be required to apportion the errors of observation.

Using the mean values of the angles before given, $A + B + C = 180°\ 0'\ 34''{\cdot}16$, but the spherical excess $+ 180° = 180°\ 0'\ 34''{\cdot}76$, hence T, the total error to be apportioned is $= -0''{\cdot}60$.

Then using the formula with reference to the angle A,

$$a=\frac{1\cdot39^2+1\cdot54^2+0\cdot14^2}{\frac{1}{2}\times 3^2}=0\cdot961,$$

$\beta=10$, as there is only one observation,

$$\gamma=\frac{1\cdot47^2+1\cdot46^2}{\frac{1}{2}\times 2^2}=2\cdot416,$$

and $a+\beta+\gamma=13\cdot37$.

$$\therefore\ x=\frac{0\cdot60\times 0\cdot961}{13\cdot37}=0''\cdot04, \text{ and } A=56^\circ\ 43'\ 28''\cdot62,$$

$$y=\frac{0\cdot60\times 10}{13\cdot37}=0''\cdot46, \text{ and } B=79^\circ\ 42'\ 29''\cdot15,$$

$$z=\frac{0\cdot60\times 2\cdot416}{13\cdot37}=0\cdot10, \text{ and } C=43^\circ\ 34'\ 36''\cdot99,$$

these values of A, B, and C being the corrected values.

Thirdly. To compute the true values of a and b.

These corrected values of the three angles, when reduced by one-third of the spherical excess, viz. $\frac{34\cdot76}{3}=11''\cdot59$, become,

$$A'=56^\circ\ 43'\ 17''\cdot03$$
$$B'=79^\circ\ 42'\ 17''\cdot56$$
$$C'=43^\circ\ 34'\ 25''\cdot40$$

And using these as for a plane triangle, having the given side $c=352\ 037\cdot62$ feet, we obtain

$$a=c.\sin A'.\operatorname{cosec} C'=426\ 974\cdot06 \text{ feet.}$$
$$b=c.\sin B'.\operatorname{cosec} C'=502\ 504\cdot42 \text{ feet.}$$

The proofs or solutions of the formulæ by which this example is worked, are beyond the scope of this work; they consist of principles adopted by Gauss and Legendre, and since then in common application everywhere.

The reduction of oblique angles to the plane of the horizon. (See page 52.)

Solution. Let D be the observed oblique angle,
$D+x$ the corresponding horizontal angle,
h and h' the two small angles of altitude,

Then as $\cos \overline{D+x} = \dfrac{\cos D - \sin h \,.\, \sin h'}{\cos h \,.\, \cos h'} = \dfrac{\cos D - h \,.\, h'}{\cos h \,.\, \cos h'}$

nearly

and as $\cos \overline{D+x} = \cos D \cos x - \sin D \,.\, \sin x$
$= \cos D - x \,.\, \sin D$ nearly

and as $\dfrac{1}{\cos h \,.\, \cos h'} = \dfrac{1}{\left(1 - \dfrac{h^2}{2} + \&c.\right)\left(1 - \dfrac{h'^2}{2} + \&c.\right)}$

$= \dfrac{1}{1 - \frac{1}{2}(h^2 + h'^2) + \&c.} = 1 + \frac{1}{2}(h^2 + h'^2)$ nearly.

$\therefore x \,.\, \sin D = \cos D - (\cos D - h \,.\, h')\{1 + \frac{1}{2}(h^2 + h'^2)\}$
$= hh' - \frac{1}{2}(h^2 + h'^2) \cos D$
$= \frac{1}{4}\{(h+h')^2 - (h-h')^2 - [(h+h')^2 + (h-h')^2]\cos D\}$

$\therefore x = \dfrac{1}{4 \sin \theta}\left\{(h+h')^2(1 - \cos\theta) - (h-h')^2(1 + \cos\theta\right\}$

$= \dfrac{1}{4}\left\{(h+h')^2 \tan \dfrac{D}{2} - (h-h')^2 \operatorname{cotg} \dfrac{D}{2}\right\}$

when x is taken in circular measure; but if taken in seconds,

$$x = \frac{\operatorname{cosec} 1''}{4}\left\{(h+h')^2 \tan \frac{D}{2} \,.\, -(h-h')^2 \operatorname{cotg} \frac{D}{2}\right\}$$

Example, using the more precise formula

$$\cos \overline{D+x} = \frac{\cos D - \sin h \,.\, \sin h'}{\cos h \,.\, \cos h'}$$

Let $D = 50°$, $h = 1°$, $h' = 1°30'$.

l cos 50°	$\bar{1}\cdot808\,0675$	0·642 78⁄6
l sin 1°	$\bar{2}\cdot241\,8553$	
l sin 1°30′	$\bar{2}\cdot417\,9190$	
	$\bar{4}\cdot659\,7743$	0·000 4569
	$\bar{1}\cdot807\,7587$	0·642 3307
log sec 1°	0·000 0662	
log sec 1° 30′	0·000 1488	
l cos. 50° 0′ 37″	$\bar{1}\cdot807\,9737$	

hence $D + x = 50°\ 0'\ 37''$.

The supplementary station:—or the reduction to the centre. (See page 52.)

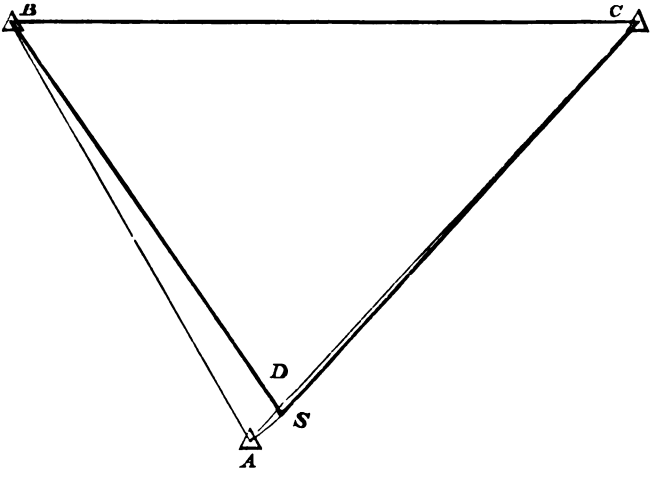


FIGURE 7. SUPPLEMENTARY STATION.

Solution. Let A be the position of the station mark observed from B and C, and S the position of the instrument, or supplementary station, then if A, B, C be the

three angles, a, b, c, the three sides corresponding, and B and C be observed, one value of A is obtained $=180°-\overline{B+C}$, and if a is known, b and c can be calculated on this assumption. To verify the value of A, measure $AS=m$ a very short distance, also the angles $BSC=S$, $ASB=\beta$, $ASC=\gamma$; and suppose the very small angles ABS, ACS to be represented by x and y.

Then as $A=BDC-x=S-x+y$,

and as the values of x and y in circular measure are

$$x=\sin x=\frac{m.\sin\beta}{c};\quad y=\sin y=\frac{m\sin\gamma}{b};$$

$$\therefore A=S-\frac{m\sin\beta}{c.\sin 1''}+\frac{m\sin\gamma}{b.\sin 1''}.$$

Example. Let $BC=a=2000$ feet; and let the observed angles $ABC=B$ and $ACB=C$, be respectively 60° and 50°; let the distance of the satellite station S from A be 10 feet; and the angles observed at S be $ASB=90°$, $BSC=70°\ 14'\ 45''$; required the angle BAC.

Now using the value of $BAC=180°-\overline{B+C}=70°$

the side $c=\sin 50°\ \mathrm{cosec}\ 70°\times 2000=1630{\cdot}42$

the side $b=\sin 60°\ \mathrm{cosec}\ 70°\times 2000=1843{\cdot}21$

And $A=S-10\ \mathrm{cosec}\ 1''\left\{\frac{\sin 90°}{1630{\cdot}4}-\frac{\sin 19°\ 45'\ 15''}{1843{\cdot}2}\right\}$

the logarithmic calculation of which is

l cosec $1''$	5·314 4251	l. cosec $1''$	5·314 4251
log 10	1·	log 10	1·
log sin 90°	0·	l. sin 19°45′15″	1·528 8976
log 1630·42	3·212 2982		5·843 3227
1265·	3·102 1269	log 1843·21	3·265 5748
		378	2·577 7479

Hence $A = 70° 14' 45'' - 1265'' + 378''$
$= 70° 14' 45'' - 14' 47''$
$= 69° 59' 58''$, thus checking the above value 70°

The distant triangle. (See page 53.)

Solution.—Using the terms given with the formulæ.

$$\sin \phi = \frac{PC \cdot \sin \alpha}{a}; \text{ and } \therefore \frac{\sin \phi}{\sin \theta} = \frac{b \cdot \sin \alpha}{a \sin \beta}$$

$$\sin \theta = \frac{PC \cdot \sin \beta}{b}$$

$$\text{Since } \operatorname{cotg} \overline{\theta + \phi} = \frac{\operatorname{cotg} \theta \cdot \operatorname{cotg} \phi - 1}{\operatorname{cotg} \theta + \operatorname{cotg} \phi}$$

$$= \frac{\sin \theta \cdot \sin \phi \cdot (\operatorname{cotg} \theta \operatorname{cotg} \phi - 1)}{\sin \overline{\theta + \phi}}$$

$$= \frac{\operatorname{cotg} \theta \sin \theta \cos \phi - \sin \theta \cdot \sin \phi}{\sin \overline{\theta + \phi}}$$

$$= \frac{\operatorname{cotg} \theta \cdot \sin \overline{\theta + \phi} \cdot - \operatorname{cotg} \theta \cos \theta \sin \phi - \sin \theta \sin \phi}{\sin \overline{\theta + \phi}}$$

$$\therefore \operatorname{cotg} \theta = \operatorname{cotg} \overline{\theta + \phi} + \frac{(\cos^2 \theta + \sin^2 \theta) \sin \phi}{\sin \overline{\theta + \phi} \cdot \sin \theta}$$

$$= \operatorname{cotg} \overline{\theta + \phi} \left\{ 1 + \frac{b \cdot \sin \alpha}{a \cdot \sin \beta \cos \cdot \overline{\theta + \phi}} \right\}$$

Example. Let the given triangle ABC have its three angles, $C = 70°$, $A = 60°$, $B = 50°$, and its sides $c = 2000$ feet, $b = 1630{\cdot}42$, $a = 1843{\cdot}21$; and the angles observed at P be respectively, α (subtended by a) $= 11° 9' 10''$ and β (subtended by b) $= 23° 40' 0''$

Then $\theta + \phi = 360° - 70° - 34° 49' 10'' = 255° 10' 50''$.

The calculation with the formula for the angle CAP is

log 1630·42	$= 3\cdot212\ 2982$
log sin 11° 9′ 10″	$= \bar{1}\cdot286\ 5144$
	$2\cdot498\ 8126$
log 1843·21	$= 3\cdot265\ 5748$
log sin 23° 40′ 0″	$= \bar{1}\cdot603\ 5897$
log cos 75° 10′ 50″	$= \bar{1}\cdot407\ 8561$
	$2\cdot277\ 0206$
$-1\cdot66645$ . . .	$0\cdot221\ 7920$
1·	
$-0\cdot66645$ . . .	$\bar{1}\cdot823\ 7676$
log cotg 75° 10′ 50″	$\bar{1}\cdot422\ 5480$
$-$cotg 80°, or $+$cotg 100°	$\bar{1}\cdot246\ 3156$

And as the last term will be negative, involving a cosine of an angle greater than 180°, and less than 270; and as the whole expression also becomes negative, the angle θ will be 100°, as calculated and shown by sign,

and $PC = b.\sin 100°.\operatorname{cosec} 23°\ 40' = 4000$ feet.

PA and PB may hence be easily obtained.

The calculation of latitudes, longitudes, and azimuths. (See page 54.)

Example. From the English G. T. Survey, ii. p. 88.

In the triangle ABC,

Given, L the observed latitude of $A = 50°\ 37'\ 7\cdot3$ North
the observed longitude of $B = 1°\ 11'\ 36''$
M, the azimuth of B, as seen from $A = 84°\ 54'\ 52\cdot5$ NW.
and d, or the arc $AB = 314\ 397\cdot5$ feet.

Required the latitude of B, the longitude of A, and the azimuth of A as seen from B.

In this case R, the mean radius of curvature at A, is assumed to be $=20\,963\,000$, and $1+Q^2\cos^2 L=1{\cdot}002\,57$ and the logarithmic calculation for difference of latitude is

$\log(1+Q^2\cos^2 L)$		$=0{\cdot}001\,1600$
$\log d$		$=5{\cdot}497\,3548$
$\log\cos M$		$=8{\cdot}947\,6336$
	Sum	$4{\cdot}446\,1484$
$\log R$		$7{\cdot}321\,4534$
$\log\sin 1''$		$4{\cdot}685\,5749$
	Sum	$2{\cdot}007\,0283$
	Comp.	$7{\cdot}992\,9717$
	$274''{\cdot}87$	$2{\cdot}439\,1201$

$\log(1+Q^2\cos^2 L)$		$=0{\cdot}001\,1600$
$\log d^2$		$=10{\cdot}994\,7096$
$\log\sin^2 M$		$=9{\cdot}996\,5742$
$\log\tan L$		$=0{\cdot}085\,7300$
	Sum	$1{\cdot}078\,1738$
$\log R^2$		$=4{\cdot}642\,9068$
$\log 2.\sin 1''$		$=4{\cdot}986\,6049$
	Sum	$9{\cdot}629\,5117$
	Comp.	$9{\cdot}370\,4883$
	$28''{\cdot}10$	$1{\cdot}448\,6621$

$$L-L'=-274''{\cdot}87+28''{\cdot}10=-4'\ 6''{\cdot}77$$
$$L'=50^\circ\ 37'\ 7''{\cdot}3+4'\ 6''{\cdot}77=50^\circ\ 41'\ 14''{\cdot}07.$$

and the logarithmic calculation for difference of longitude is

$$\frac{d\sin M\sec L'}{R\sin 1''}$$

$$\begin{aligned}
&\log d && = 5{\cdot}497\,3548\\
&\log \sin M && = 9{\cdot}998\,2871\\
&\sec L' && = 0{\cdot}198\,2168\\
&\text{Co log } R \sin 1'' && = 7{\cdot}992\,9717\\
&\text{(G) } 4862''{\cdot}2 && 3{\cdot}686\,8304
\end{aligned}$$

or the difference of longitude is $1^\circ\ 21'\ 2''{\cdot}2$

the required longitude is $1^\circ\ 11'\ 36'' + 1^\circ\ 21'\ 2''{\cdot}2$

$= 2^\circ\ 32'\ 38''{\cdot}2.$

In this case the value of G found from the survey was $1^\circ\ 2'\ 46''{\cdot}4$; the error of $15''{\cdot}8$ in the above calculation being due to erroneous assumption in the length of the degree, which makes all the longitudes on the southern coast of England too small.

The logarithmic calculation for azimuth is

$$G \cdot \sin 1'' \cdot \sin \frac{L+L'}{2} \sec \frac{L-L'}{2}$$

$$\begin{aligned}
&\log G && = 3{\cdot}686\,8304\\
&\log \sin \tfrac{1}{2}(L+L') && = 9{\cdot}888\,3650\\
&\log \sec \tfrac{1}{2}(L-L') && = 0{\cdot}000\,0001\\
&3760''{\cdot}06 && 3{\cdot}575\,1955
\end{aligned}$$

$N = 180^\circ - 84^\circ\ 54'\ 52''{\cdot}5 - 1^\circ\ 2'\ 40''{\cdot}06 = 94^\circ\ 2'\ 27''{\cdot}44.$

The observed angle in this case was $94^\circ\ 2'\ 22''{\cdot}75$, the discrepancy of $4''{\cdot}69$ being due to the error in longitude before mentioned.

Adopting the same data for exemplifying the approximate formulæ.

Difference of latitude

$$= AB \cos M - \frac{AB^2}{2} \tan L \cdot \sin^2 M \cdot \sin 1'',$$

here arc $AB = \dfrac{d}{101{\cdot}36} = \dfrac{314307{\cdot}5}{101{\cdot}36} = 3101''$,

C and C' are the colatitudes,

and the logarithmic calculation is

log AB	$=3{\cdot}491\,49$	log AB^2	$=6{\cdot}982\,98$
log cos M	$=8{\cdot}947\,63$	log $\sin^2 M$	$=1{\cdot}996\,58$
$274''{\cdot}87$	$2{\cdot}439\,12$	log sin $1''$	$=4{\cdot}685\,57$
		log tan L.	$=0{\cdot}085\,73$
		sum	$1{\cdot}750\,76$
		log 2	$0{\cdot}301\,03$
		$28''{\cdot}66$	$1{\cdot}449\,73$

$\therefore$ the required difference of latitude $=274''{\cdot}87-28''{\cdot}66$
$=246''{\cdot}21.$

$L'=50°\ 37'\ 7''{\cdot}3+4'\ 6{\cdot}21=50°\ 41'\ 13''{\cdot}51.$

And for difference of longitude,

Difference $=AB\sin^2 M.\sec L'$

log AB	$=3{\cdot}491\,49$
2 log sin M	$=9{\cdot}996\,58$
log sec L'	$=0{\cdot}198\,22$
$4856''$	$3{\cdot}686\,29$

or the difference of longitude is $1°\ 20'\ 56''$,
and the required longitude is $1°\ 11'\ 36''+1°\ 20'\ 56''$
$=2°\ 32'\ 32''.$

The approximate calculation of difference of azimuth

$$\text{Sine half difference}=\sin\frac{G}{2}.\sin\frac{1}{2}(L+L')$$

here $\frac{G}{2}=0°\ 40'\ 28''$; and $\frac{1}{2}(L+L')=50°\ 39'\ 10''{\cdot}68$

l. sin	$0°\ 40'\ 28''=8{\cdot}070\,81$
l. sin	$50°\ 39'\ 11''=9{\cdot}888\,36$
l. sin	$0°\ 31'\ 18''=7{\cdot}959\,17$

hence $N=180°-84°\ 54'\ 52''{\cdot}5-1°\ 2'\ 36''=94°\ 2'\ 31''{\cdot}5.$

The exemplification of the use of the approximations

proves that they have been applied to a case slightly beyond their powers, as laid down by the limit given with the formulæ, for the arc exceeds one degree.

Section 6. DEMARCATION OF SURVEY POINTS.

The selection of survey stations, base lines, and survey lines, has necessarily to be suited to the description of survey contemplated; while the form of record must again be in accordance with the results obtained with the various instruments that may be adopted: information on these points will be given attached to the various sorts of survey afterwards described, and models or examples of forms of record also given in every requisite case.

The mode of demarcation of the various survey points must be in accordance with the time the marks will have to last, the nature of the ground on which they have to be placed, and their liability to removal or disturbance.

The temporary marking of intermediate chained distances, offset points, and such details is effected by pins, or skewers, which are best if accompanied with numbered tallies indicating the chainage for the end of any complete chain; or by 'whites,' which are cleft-sticks holding little bits of white paper, on which any pencilled number can be written. If the soil is not soft enough for these to stick in it, a cross is scratched to indicate the exact mark, and the pin or stick is laid down with its point at the mark.

The distant points required for ranging the direction of measurement, or for angular observation, are painted poles shod with iron; if the distances are not very long

eight-feet poles, and for longer distances 12- and even 20- or 25-feet poles may be necessary, which then require rope stays. The shorter poles are painted in alternate lengths of black and white, the longer have the top painted and a red flag attached. They should be set truly vertical, and so fixed as to remain so. The loss of time from often having to send men long distances to reset poles on windy days may be very great. For very distant points in large triangulations, a signal pole with flag attached may be set up in a tree, and firmly bound to two or three branches; the station point corresponding to it is then carefully plumbed down to the ground and marked, if angular observation from that point is necessary. Spires and towers do not generally answer such purposes, as they involve satellite stations of observation; but these as well as other permanent well-defined objects, such as corners of buildings, are well suited as distant ranging points; and more important survey lines may sometimes be conveniently ranged in the direction between two such objects. When survey lines cross fences or walls, the crossing points may be notched with a chisel, or painted.

Survey station-points are marked with pegs or stakes, if they are likely to be required after removal of the temporary poles; their position with reference to surrounding objects is measured and noted in the field-book, or failing these means, four pegs are so placed and driven in that the intersection of the diagonal measurements between them may fall at the station point. For precision the exact point may be pencilled by a cross-mark on the top of the stake and marked with a nail. If any such marks are required to remain permanently the pegs should be driven flush with the ground; or for greater

permanence, a marked tile, brick, or small block of stone should be buried flush.

All faulty or incorrect survey marks should invariably be removed or completely obliterated.

Several sets of marks may have to be made over the same country, as, for instance, first, the survey marks; secondly, level marks; and if proposed works follow, thirdly, setting-out marks for position; and, fourthly, setting-out marks for level. To prevent confusion, sufficient distinction should be made in pegs or stakes belonging to the various series; some may be round, some square, and again the sectional dimensions may be made to vary in each set; or, again, they may have their tops diversely painted.

Permanent floating stations are marked by buoys having some special mark or short pole. As they deviate with the tide and wind, however well they may be moored, the amount of possible deviation should be recorded in each case. Temporary or intermediate floating stations are left unmarked; their position is generally determined by three or four angles taken with the sextant to surrounding permanent marks; these should, if possible, be observed simultaneously.

Very distant stations in survey, that happen to be rarely visible, are best observed at night, lamps being set for this purpose; or, in bright weather, the heliostat must be used in the daytime with the same object. A three-inch mirror, set on a tripod stand over the exact station-point, is sufficient to answer the purpose of giving the flashes.

CHAPTER II.

SURVEYING OPERATIONS.

Section 1. METHODS OF PROCEDURE.

THERE are four principal methods of procedure in surveying by which the position of a number of points, requisite to obtain a sufficiently close and adequate representation of the features of the country, and the natural and artificial objects there existing, may be determined for record, and finally reduction to plan.

These modes of procedure may be adopted either severally or in conjunction with each other, whether measurement or calculation or both happen to be the means of carrying out these modes. They are:

1. By distances, or distances and offsets.
2. By triangulation, or a network of triangles.
3. By traversing, or a continuous combination of distances with angles or bearings.
4. By determining the latitudes, longitudes, and azimuths of detached points.

To explain the application of these four methods in their simplest forms, there must necessarily be some starting survey point, and sometimes also some starting direction of some survey line in every survey which may be considered as a relative datum point and datum line of departure. It need not necessarily be the point or line from which the surveying operations commence in order

of time, nor need it always be the most important point or line, as the whole of the results of any complete survey are mutually dependent; but this datum point and datum line may be considered as the starting point and starting line for the purpose immediately under consideration, when the relative position of some single unknown point is required.

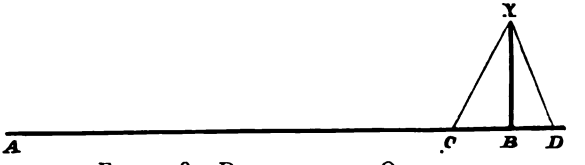


FIGURE 8. DISTANCES AND OFFSETS.

1. *Adopting the first mode of procedure*, let A be the datum point of known or assumed position, and X the unknown or required point; then if AX happen to be in the direction of any datum line of the survey, AB, the distance AX is all that is required to fix the position of X apart from checking; but if X is not in that direction, two distances, or a distance with one rectangular offset, such as AB and BX, become necessary. If, again, the means of setting out a correct right angle are not available, recourse must be had to two distances in the same direction, AD, AC, and two or even three oblique offsets, DX, CX, by which the position of X is determined; but this then becomes a limited case of triangulation combined with distance.

Should X be so far distant from, or so inconveniently placed with regard to A that this mode of procedure cannot be adopted in this the simplest form, it may be arrived at through an intermediate series of distances and offsets on the same principle, or through intermediate steps based on other modes of procedure.

2. *The method of triangulation.*

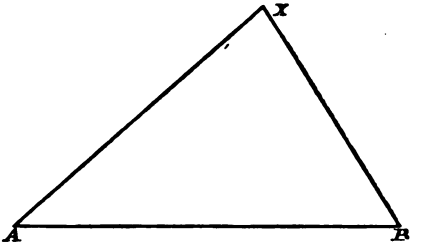


FIGURE 9. TRIANGULATION.

As before, let A be the datum point, and X the point whose relative position is required. If AB is also a datum line, X may be arrived at through one triangle only; in which case either AB, BX, and AX are all measured, or if AB only be measured, and BX and AX be calculated from angular measurements of any two of the three angles, the relative position of X is determined. This may be further checked by measuring a tie-line in the one case, or the third angle in the other.

Should X be so far distant from, or so inconveniently placed with regard to A, that it cannot be arrived at through one triangle, it may be arrived at through a series or network of triangles, or through a combination of triangulation with distances and offsets, as shown in the figure following.

3. *The method of traversing.*

As before, let A be the datum point, AB the datum line, whose direction is either known or assumed, and X a distant point whose relative position is required. If it happens to be either impracticable or inconvenient to arrive at X by any direct route, and if the method of triangulation be considered too tedious, a

method more circuitous than the former and less tedious than the latter can be adopted. Should there happen to be some road or stretch of open country convenient for measuring operations nearly between *A* and *X*, it

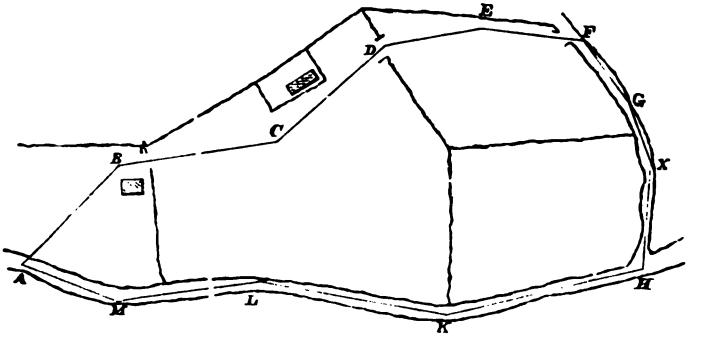


FIGURE 10. TRAVERSING.

may be best to conform partly to that course; supposing this to be the case, let such a course lie to the left of the imaginary line *AX*. Then if *AB* be measured, a mark set up at any point *C*, and the inward angle *ABC* measured; the distance *BC* measured, another mark set up at *D*, the inward angle *BCD*, and the distance *CD* measured; and so on, measuring every distance and every inward angle on a route going through a series of points, until the last of them, *G*, is so placed that *GX* can be measured, the point *X* is determined relatively to *A*. This process is called traversing, and *ABCDEFGX* is termed an unclosed traverse. If, however, the traverse is continued by some other route *XHKLMA* back to *A*, the starting point thus forming a complete irregular polygon, it becomes a closed traverse; and the correctness of the observed

inward angles may then be checked by the formulæ given at page 48. The correctness of the measured distances, however, can only be checked by check bearings or angles on convenient lateral conspicuous objects, or on traversed points not adjacent

4. *The method of latitudes and longitudes* is generally adopted as a check on triangulations or on traverses of very large extent, such as the triangulation of a large Topographical Survey, or the route-surveys of travellers and navigators. In the former case the latitudes and longitudes of a certain number of distant points are determined by observation with the zenith sector, and with the aid of sets of chronometers transported from place to place, or simultaneous chronometrical observation of a series of signals. From these, with the aid of any distances and azimuths laid down or obtained in the triangulation work, the latitudes and longitudes of any other intermediate survey points and the azimuths of any other survey lines may be calculated in accordance with the formulæ given in the general collection (page 54). In the latter case, that is, on route-surveys of travellers and navigators, the modes of observation practised are rather different, this observation being much more frequent and less precise. An account of these, with the necessary formulæ of reduction, is given in the Chapter on Astronomical Observation in Route Surveys; while the formulæ for calculating the latitudes and longitudes of intermediate points are rarely necessary; when they are so, the approximations in the general collection (page 54) are sufficiently accurate in most instances.

The above methods are, for brevity, described in a

form having for object the determination of the relative position of any single distant point; but it will be evident that the same principles apply to the practical determination of position of any number of points, which is the object of surveying operations.

A skeleton groundwork of points, distances, angles, &c., is the substratum of the final delineation of roads, hedgerows, rivers, buildings, quarries, bridges, tunnels, rocks, shoals, outcrops, capes, and harbours, &c., that may be required; and though this final representation should be so far correct that no appreciable error should be possible, and nothing omitted that is essential to the object of the survey, yet the choice of modes of procedure, as of instruments to be used in the skeleton-work, is so much at the pleasure of the surveyor that abstract rules for guidance are perfectly impossible.

The special objects of the survey, the scale on which the plan is required, the amount of time, number of men, and the sort of instruments available, are the matters that guide the surveyor in laying down the general arrangements; while, again, methods and details of execution are best left to the discretion of those competent to be entrusted with the work, who are then more capable of producing good results.

Verification or checking.—There is, however, one principle or rule that may be considered an established maxim in all surveying, which is absolute under all circumstances. It is that some means of verification be forthcoming for every part of the work done, or that nothing be made entirely dependent on a single measurement of distance, or a single angular or other observation of any kind, in which an accidental mistake may occur without possibility of discovery, and thus

vitiate results. The element of uncertainty must be completely eliminated. If a certain number or amount of measurements and observations are just barely sufficient to enable the series of required points to be arrived at and delineated, as, for instance, in a single triangle, either two adjacent sides and the included angle, or one side and the two angles at its extremities are barely enough; then some additional measurement or observation becomes necessary for verification, which in this case would be effected by measuring an additional angle or side. The same principle applies to the extremities of a series of triangles or any extended skeleton work, where either a single verification or a series of checks become necessary. The only case in which checks are dispensed with is in short rectangular offsets to hedgerows and similar minor features, when the memory, or recollection of form and disposition is most generally entirely relied on for verification of result, but in some cases an extra oblique offset to the same point is absolutely necessary as a check.

The various sorts of survey-work for general purposes being thus practically very much dependent on the sort of instruments chosen and used, they will be classified and treated in accordance with them in the following sections; while the surveys for special purposes will have their distinctive points subsequently mentioned.

The general classification will be as follows:

Surveys without instruments for angular measurement.

1. Chain surveys.
2. Telemeter surveys.
3. Plane-table surveys.

Surveys with small instruments for angular measurement.

4. Compass surveys. 5. Box-sextant surveys.

Surveys with larger instruments for angular measurement.

6. Theodolite surveys over small areas, or over large areas without much intricacy of detail.
7. Theodolite surveys over large areas, or over small areas with much intricacy of detail.

Limit of admissible error.

Whatever instruments be used, the object is to obtain a correct plan with the least amount of labour. The limit to the necessary accuracy or exactitude in surveying can be directly determined with reference to the scale of the plan, for the plan should have no visible error. Now any distance represented on a plan by a hundredth part of an inch is just fairly appreciable to the naked eye, and if we consider that $\frac{1}{200}$th of an inch is inappreciable, this amount in conjunction with the scale of the plan affords a limit to the exactitude required in actual measurement in the field, or extreme bounds admissible in any cumulative error or total discrepancy.

For instance, if the survey has for its object a correct plan to the scale of 60 feet to an inch, the limit $\frac{1}{200}$th of an inch on the plan will represent $\frac{6}{200}$th of a foot or about 3 inches on the ground; if the scale to be used is one mile to six inches, then $\frac{1}{2000}$th of an inch on the plan will represent about 4·4 feet on the ground.

The total error thus admissible on the ground should invariably be thus determined before commencing survey operations, as otherwise even a most careful survey may be either too inexact or too laborious in detail, in the one case causing a total waste of time and labour, and

in the other a partial loss. The limiting error can be thus computed in a moment from the scale.

The following is a list of scales commonly used in England.

Scale		Purpose
$\frac{1}{63360}$	or 1 mile to an inch.	Smallest scale Ordnance maps.
$\frac{1}{15840}$	„ $\frac{1}{4}$ mile to an inch.	Minimum allowed in public works deposited plans.
$\frac{1}{10560}$	„ $\frac{1}{6}$ mile to an inch.	Intermediate scale Ordnance maps.
$\frac{1}{6336}$	„ $\frac{1}{10}$ mile to an inch. „ 8 chains to an inch.	Convenient intermediate scale for public works.
$\frac{1}{4752}$	„ 6 chains to an inch.	Parliamentary deposited plans generally.
$\frac{1}{3960}$	„ 5 chains to an inch.	Plans of special alterations.
$\frac{1}{2376}$	„ 3 chains to an inch.	Tithe maps. Plans of estates.
$\frac{1}{1584}$	„ 2 chains to an inch.	Working contract-plans. Building ground.
$\frac{1}{4800}$	„ 400 feet to an inch.	Some enlarged plans of buildings.
$\frac{1}{2400}$	„ 200 feet to an inch.	Irish land or contract plans.
$\frac{1}{1200}$	„ 100 feet to an inch.	Sections of proposed public works.

Scale		Purpose
$\frac{1}{1056}$	or 88 feet to an inch.	Ordnance plans of ordinary towns.
$\frac{1}{528}$	„ 44 „ „	Ordnance plans of intricate towns.
$\frac{1}{480}$	„ 40 „ „	Engineers' enlarged working plans and sections.
$\frac{1}{360}$	„ 30 „ „	
$\frac{1}{240}$	„ 20 „ „	
$\frac{1}{120}$	„ 10 „ „	

8, 6, 5, 4, 3, 2, and 1 foot to an inch. Details of engineering plans.

1, $\frac{7}{8}$, $\frac{3}{4}$, $\frac{5}{8}$, $\frac{1}{2}$, $\frac{3}{8}$, $\frac{1}{4}$, $\frac{1}{8}$ inch to a foot. Details of builders' plans.

Decimal Scales.

For maps and charts.

Scale	
$\frac{1}{60000}$	or 1·056 inches to a mile or 0·9470 mile to an inch.
$\frac{1}{50000}$	or 1·267 inches to a mile or 0·7893 mile to an inch.
$\frac{1}{40000}$	or 1·584 inches to a mile or 0·6313 mile to an inch.
$\frac{1}{30000}$	or 2·112 inches to a mile or 0·4735 mile to an inch.

Scale	
$\frac{1}{25000}$	or 2·5344 inches to a mile or 0·3946 mile to an inch.
$\frac{1}{20000}$	or 3·168 inches to a mile or 0·3157 mile to an inch.
$\frac{1}{15000}$	or 4·224 inches to a mile or 0·2367 mile to an inch.
$\frac{1}{10000}$	or 6·336 inches to a mile or 0·1578 mile to an inch.

For plans.

$\frac{1}{5000}$	or 416·66 feet to an inch.	
$\frac{1}{4000}$	or 333·33 feet to an inch.	
$\frac{1}{3000}$	or 250 feet to an inch.	
$\frac{1}{2500}$	or 208·33 feet to an inch.	Large scale of Ordnance maps.
$\frac{1}{2000}$	or 166·66 feet to an inch.	
$\frac{1}{1000}$	or 83·33 feet to an inch.	
$\frac{1}{500}$	or 41·66 feet to an inch.	Ordnance plans of towns.

For details.

$\frac{1}{100}$	or 8·333 feet to an inch or 0·12 inches to a foot.

Scale						
$\frac{1}{50}$	or 4·166 feet to an inch or 0·24 inches to a foot.					
$\frac{1}{40}$	„ 3·333	„	„	0·30	„	„
$\frac{1}{30}$	„ 2·500	„	„	0·40	„	„
$\frac{1}{25}$	„ 2·08$\dot{3}$	„	„	0·48	„	„
$\frac{1}{20}$	„ 1·666	„	„	0·60	„	„
$\frac{1}{10}$	„ 0·83$\dot{3}$	„	„	1·20	„	„

The scales for distances on sections are identical with, or bear some convenient ratio to, those used on the corresponding plans. The scales for heights on sections are generally arranged to be 10 times the scale for distances, as this is often most convenient; another method is to use 10 times the number of chains in horizontal distances per feet in vertical; thus for 6 chains horizontal, 60 feet vertical, and so on; but the two scales of 100 feet to an inch, and 40 feet to an inch, are the minimum vertical scales laid down by Parliamentary Standing Orders for long and cross sections of proposed works with alterations of existing works. Records of height or depth made on plans are generally in feet and decimals, mètres and decimals, or fathoms and quarters.

The scales principally used in the Indian Surveys are:

4 miles to the inch.	Triangulation charts G. T. S. Indian Atlas sheets. Revenue survey of Northern India.

2 miles to the inch.	Maps giving heights, G.T.S.
1 mile to the inch.	Maps of coal-fields and mineral districts. Topographical surveys of certain provinces. Revenue maps of Southern India.

Besides these there are maps from Surveys of Rivers on various scales, 2 miles, 1 mile, ½ mile, 1000 yards and 500 yards, to the inch being most common; also railway maps or plans, which show greater variety as to scale.

Section 2. CHAIN SURVEYS.

General Remarks.—Work of this description, requiring the fewest and least expensive appliances, and the smallest amount of thought and knowledge, possesses the disadvantage of being monotonous and laborious. It may be correctly applied to portions of land, estates, or plots, not exceeding one square mile at a time; when used for larger tracts, the portions thus surveyed should be fitted on to, reduced, and verified by the skeleton work or distant points of some larger survey, such as the Great Trigonometrical or large Topographical survey of the country.

In estate-surveys, which are mostly surveyed with chain alone, the acreage of various holdings is often a more important consideration than the plan, hence the chain used for them is the 66 feet or Gunter's chain of 100 links, a convenient submultiple of the acre; the distances are measured in such chains and links to the entire exclusion of feet, and the acreage can then be more rapidly calculated.

In other surveys, in which acreage is comparatively

unimportant, and the introduction of Gunter's units might be troublesome, either the 100 feet or the 50 feet chain is used, and all distances are measured in feet; any mileage required on any part of the plan being subsequently marked or scaled off.

The appliances required on this work are:

First. The chain itself. (See appliances for measuring distance, Chapter I., pages 6 to 8.)—These are made of various patterns, and are galvanised, painted, or plain. The latter are most liable to rust, while the first are liable to be less correct from the additional process they undergo. The long-linked pattern, having each link a foot or a Gunter's link in length, is considered advantageous, from being lighter with the same amount of similar material, from an idea that its form shows most readily any accidental kink or derangement during chaining, and from its links being easily hammered straight after being accidentally bent. The curb-chain and such small-linked patterns are modifications introduced with the use of superior metal, the amount of which is correspondingly reduced; they can be made excessively light and convenient, but when damaged are less easily rectified than the old pattern. In any case the length of a new chain should not only be tested with a good standard when simply laid straight, but also again after stretching it by a weight. During the progress of survey work, the length of the chain should be daily tested by comparison with a temporary standard marked for the purpose and kept invariable. If a chain has to be mended or altered, the time and date should be recorded in the field-book of the survey.

The pins or skewers used with the chain should have numbered tallies attached to them; and the tallies at

every 10 feet or 10 links on the chain should read from the beginning to the end of the chain, not from both ends to the middle. The pins should be strung in order on a broad strap.

The other appliances necessary are an optical square or a cross staff for setting out right angles, a measuring tape for measuring long offsets, a painted offset staff for short offsets, and a set of ranging rods for marking stations. (See Demarcation of Survey Points, page 67.)

The optical square consists of two small silvered mirrors fixed in a brass box at a permanent inclination to each other of 45°, thus reflecting any object through an angle of 90°, the unsilvered portion of one mirror giving a direct view of the object, while the reflected and the direct object can be exactly superimposed in the field of view of the observer, when they are at right angles to each other. It requires occasional testing and adjustment, which is effected by a screw moving one mirror very slightly.

The cross staff, or cross head, has at least two pairs of sights fixed at right angles to each other on the upper end of a staff about five feet long, shod with iron; but generally these heads have four pairs of sights, which then enable half a right angle to be set out when required. They are made of various patterns; those least liable to damage and the lightest are preferable.

Either of these appliances may be easily tested by setting out four right angles from one spot.

The measuring tape and the offset staff, which is painted black and white in alternate lengths, should both be subdivided in accordance with the special unit adopted in the survey, either in feet or Gunter's links, but not with both of them, as that might cause mistakes.

Some cord for making stays to the ranging rods and tying them in bundles, one or two spare straps, and a few pegs complete the appliances necessary for chain survey work.

Mode of Chaining.—Unless the chaining is carried on in a systematic manner with chainmen drilled to their work under careful supervision, the results are sure to be faulty. The following is the mode recommended.

The starting point and the distant point being marked by ranging rods or other means, two men manage the chain—the leader who drags it and puts pins in the ground after each chain's length, and the follower who directs the leader to the distant point, picks up the pins when no longer required, and returns them to the leader when they amount to ten, informing the recorder of the fact. An eleventh special pin is used for putting down in place of the tenth for a few moments; this is both placed and withdrawn by the follower, who always retains it. In dragging the chain the leader walks straight towards the distant object till told by the follower to stop. When measuring the follower firmly holds the outside of his chain handle to the point or pin, and directs the leader by signs to the right or left until very nearly in alignment, telling him when he is so. The leader then takes the proper numbered pin in his right hand, grasping it with his chain handle, but keeping the remaining pins strung in order on a strap in his left hand; he tightens the chain and, keeping his eyes on his own pin and chain handle, obeys the verbal directions of the follower for moving it to the right or left. When the leader's pin is in true alignment, the follower says 'Mark,' and the leader marks the point by forcing the pin firmly and vertically into the ground, disen-

gages his hand from it, and draws away the chain so as to clear the pin.

The next chain-length is laid off in the same way, the follower being always responsible that the chain shall be tight, straight, and well-directed during measurement, and ready to inform the recorder of his own distance from the starting point whenever required.

Having thus measured ten chains' length, the leader on placing the tenth pin calls for more pins, when the follower runs up to him, places his special eleventh pin in the exact place occupied by the tenth, and hands over the ten pins to the leader, who counts them and arranges them in proper order on his strap. The recorder notes arrival at the tenth chain in his field-book, and the chaining proceeds as before. Such chaining, when directed on a single distant object or pole, is liable to a slight deviation from direction, which causes error; it is better, therefore, to establish the direction by the alignment of two or more distant or intermediate poles.

Most chaining should be carried on with the chain horizontal, not sloping down a declivity. When the ground falls, the uphill end should touch the ground, and the other end be raised up and held to a vertical rod placed at the exact spot. On very steep ground the chaining must proceed by half-lengths of chain, or even quarter-lengths.

The direct chaining thus requires two men in addition to the recorder, and most frequently also a third man for setting up and removing signal points, carrying rods, and helping to measure intermediate distances or take offsets with the tape or staff.

General Method of Procedure.—The skeleton survey work in chain survey consists of a series of triangles,

sometimes broken by trapeziums, while details are fixed by the method of distances and offsets along and from the sides of these triangles, all of which are measured. A check on each triangle is always obtained either by measuring a single tie-line from any angle to a known point in the opposite side, or by measuring two short angle-ties, one to each of any two angles, between two known points in adjacent sides near any angle. If some of these triangles are so large as to nearly include the whole of the extent to be surveyed, no further check is necessary; but if the triangles are comparatively small and joined on to each other, a through line going from one extremity of the survey to the other must also be measured, and its connection with the small triangles noted wherever they are crossed. In the event of there being troublesome continuous obstacles, such as large hedgerows or streams, which may prevent the network of triangles being perfectly continuous, a connection between two triangles, having sides nearly parallel, may be formed by a long trapezoid, whose short sides are two rectangular offsets taken at convenient spots; the tie-line then necessary will be a third rectangular offset taken between the long sides, which will check the connection formed by the trapezoid.

If large inclusive triangles are used, as before mentioned, all the detailed work will consist of a series of tie-lines and angle-ties on them, from which the detail points will be fixed by the method of distance and offset. This is the preferable plan when obstacles, the fall of the land, and so forth, admit of its being conveniently done; under other circumstances it may often happen that only one through line can be measured on tolerably flat unimpeded country.

In surveying minor details, such as blocks of buildings not admissible of measuring through, it becomes necessary to enclose them in a skeleton rectangle, from which offsets are taken to the corners and principal points.

Minor Operations and Calculations.—Under ordinary circumstances no calculations whatever are required in connection with chain surveys, the distances necessary being all measured, recorded, and laid down to scale on the plan; occasionally, however, some of the trigonometrical and mensuration formulæ given in the general collection at pages 46 to 48 may be required. Under special circumstances, special operations and special formulæ may be required.

1. To set out a right angle to a given direction with the chain only, at any intermediate point in a measured distance.

Measure a quarter of a chain both ways from the given point along the given line, thus obtaining two new points in it; have the two chain handles firmly held or fixed by pegs at these two points, stretch the chain out by holding its middle until it is tight, the middle point will then be in the required direction. This may be done also on the other side of the given line.

2. To set out a right angle to a given direction with the chain only at any point in a measured distance.

Measure a distance of 40 links or feet from the given point along the given direction, drive in a peg firmly, and put the chain handle over it; pick out the 80 link or feet mark on the chain, and have it held firmly at the given point; stretch the chain out in the required direction by holding it by its middle or 50 link or feet mark until it is tight; the middle mark will then be in the required direction.

The principle applied in the above operation of using multiples of the numbers 3, 4, and 5 for setting out a right-angled triangle may also be adopted with some other round numbers. To ascertain if any even given round number will answer the purpose as a side of such a triangle, let x be any such side, y the other, and z the hypothenuse, then putting

$$x=2mn,\ y=m^2-n^2,\ z=m^2+n^2$$

to satisfy the mathematical conditions, x must be capable of subdivision by two factors—m and n, and the sum and difference of their squares will give the values of z and y.

Thus, for example, let 28 be the even given round number; then if it will subdivide into factors, as it actually does, then

$$x=2\times 7\times 2,\ y=7^2-2^2=45,\ z=7^2+2^2=53;$$

and these three numbers satisfy the required conditions, as

$$28^2+45^2=2809=53^2.$$

If the given number happens to be odd, the same test may be applied to the double of that number; and in the event of its being suitable, the resulting numbers should then be halved.

3. To chain round any impassable obstacle in a survey line.

Obstacles to continuous chaining sometimes do not extend laterally to more than a short distance on one side or the other; under these circumstances there are two simple modes of chaining the *détour*—one using right angles, the other half right angles, set out with the cross-head.

By the first method, mark two convenient points one or two chains apart in the survey line just before arrival at the obstacle, lay off two equal rectangular offsets, thus obtaining through their extremities a line parallel to the original survey line that will clear the obstacle; continue the chaining on this new line, and revert to the original direction by setting off two rectangular offsets equal to the former; their extremities will be in the original direction. (See figure 11.)



FIGURE 11. DETOUR IN CHAINING.

Thus the operations consist in setting out *AB*, *ab*, using the new direction *BbC* for chaining, then setting out *CD* and *cd* to obtain the original direction, *Dd*, which will be in line with *Aa*. The length of the unmeasured distance *ad* then = *bc*. Further precision may be obtained by using three or four offsets on each side of the obstacle instead of two.

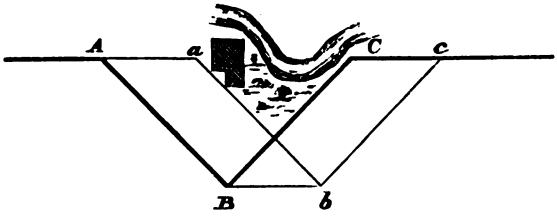


FIGURE 12. DETOUR IN CHAINING.

By the second method (see figure 12), having chosen similarly two convenient points, *A*, *a*, in the original line, set out half right angles at those points with the

cross-head, measure AB, ab, of equal length, check Bb, which should be equal to Aa, set out two rectangular offsets, BC, bc, equal in length to AB, ab; then Cc will be in the original direction, and the unmeasured distance $AC = ac = AB \times 1{\cdot}4142$.

Whichever of these methods be used, if the obstacle to continuous chaining admits of being seen over, as a pond or marsh, the resulting direction admits of verification by simple ranging.

4. To continue a survey line beyond an impassable obstacle when chaining round is impracticable.

First. When the obstacle can be seen over, as a stream or river, the direction of the survey line may be ranged in beyond it with ranging rods; and the length of unmeasured interval alone is required. (See figure 13.)

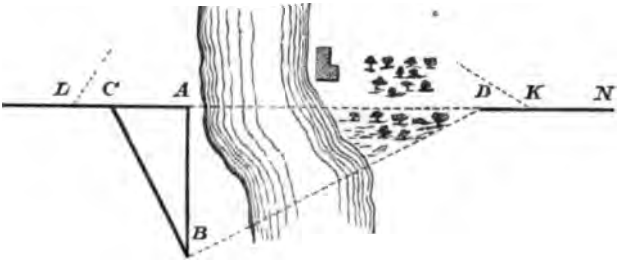


FIGURE 13. INTERVAL IN CHAINING.

Choose two convenient points A and C, one chain or two chains apart, in the original line; set out a rectangular offset AB to double the length of AC; at the point B set out the right angle CBD, having D fixed in the survey line beyond the obstacle.

Then the unmeasured interval $AD = 4\,AC$.

Or if AB is not fixed at exactly double AC, then $AD = AB^2 \div AC$.

A check on the accuracy of the point D may be obtained by setting out AB' to the left of the given direction CAD, making $AB' = AB$, and setting out the right angle $CB'D$ to obtain the identical point D given by the former operation; or if the ground does not admit of using equal triangles on each side of the line, unequal triangles may be used, and a second point K may be obtained.

Secondly. When the obstacle cannot be seen over in the original direction, as if a house or clump of trees should intervene on the bank of the stream.

Range a parallel line clear of the house by means of a pair of rectangular offsets, so that its continuation may be ranged in beyond the river; a second pair of offsets

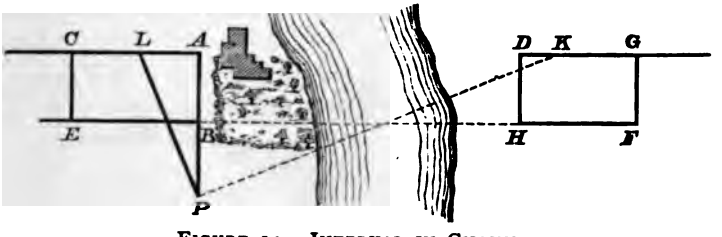


FIGURE 14. INTERVAL IN CHAINING.

will then give the original direction, and the mode just explained will give the unmeasured interval.

Thus in the figure 14. Let CA be the original direction, use two equal rectangular offsets, so that the parallel line $EBHF$ may be visibly fixed beyond the river; the original direction DG can then be regained by equal offsets HD, FG. Then let K be any point in DG visible from P; set out the right angle KPL, obtaining a point L, and measure AL; then the unmeasured interval $AK = AD^2 \div AL$.

A similar operation may be conducted to the left of the given direction so as to afford a check.

There are several other modes of carrying out such minor operations, based on the principle of similar triangles, but these described are the most simple, rapid, and convenient. Whenever more complicated measurements and operations become necessary in ordinary chain surveys the original survey line must have been badly selected, and is best abandoned.

5. To test by calculation the length of a tie line (x) of any triangle ABC, measured from the angle C to the side c, so as to divide c into two parts m and n, of which m is adjacent to the side b, and n adjacent to a.

$$\text{Then } x \text{ should} = \sqrt{\left(\frac{ma^2+nb^2}{m+n} - mn\right)}.$$

And if the tie line x is at right angles to the side c,

$$x = \sqrt{a^2-n^2} = \sqrt{b^2-m^2}.$$

6. Chaining on an inclination.

Sometimes, when there is a long piece of sloping ground of unvarying inclination to be chained and few offsets are required, the chaining is for the sake of rapidity measured on the declivity, instead of horizontally. In this case the difference between an inclined chain and a projected horizontal chain plumbed down must be measured with the offset staff and added to every chain thus measured.

For example, let AB be the chain lying on the declivity, mark A and B; then have the chain held truly horizontal as AC, and plumb down CD and mark the point D; measure BD with the offset staff or tape; BD is the correction to be added for each inclined chain

measured in order to obtain the correct chainage; and this correction should be measured with such exactitude that the final total may not be materially affected by error.

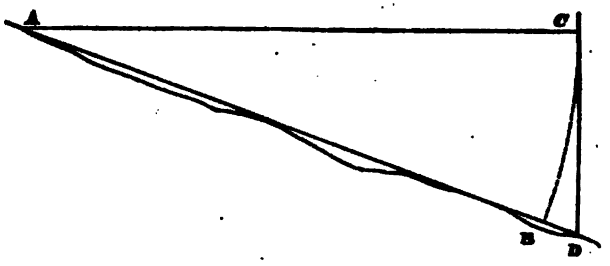


FIGURE 15. INCLINED CHAINING.

7. Traversing with the chain.

On the offset system.—Short pieces of traverse may sometimes be done with the chain and cross staff only; as for instance along a boundary, a footpath, or on one bank of a stream in comparatively open country, lying between two determined survey points.

Thus in Plate I., A and H are two fixed points, a stream lying to the left of AH, whose course must be surveyed. The right angle OAB is set out with the cross staff, and checked by measuring Aa, Aa', and aa'; the line AB is then used as a survey line, from which offsets to the stream can be taken. At B a half right angle bBC is set out with the cross staff, and checked by three measurements Bb, bb' and Bb'; the line BC then becomes a new survey line from which offsets to the stream can be taken. In the same way by setting out a succession of checked right angles and half right angles as required, the traverse is completed up to near the final point H, which is determined both by one rectangular offset GH and two oblique offsets, gH, $g'H$.

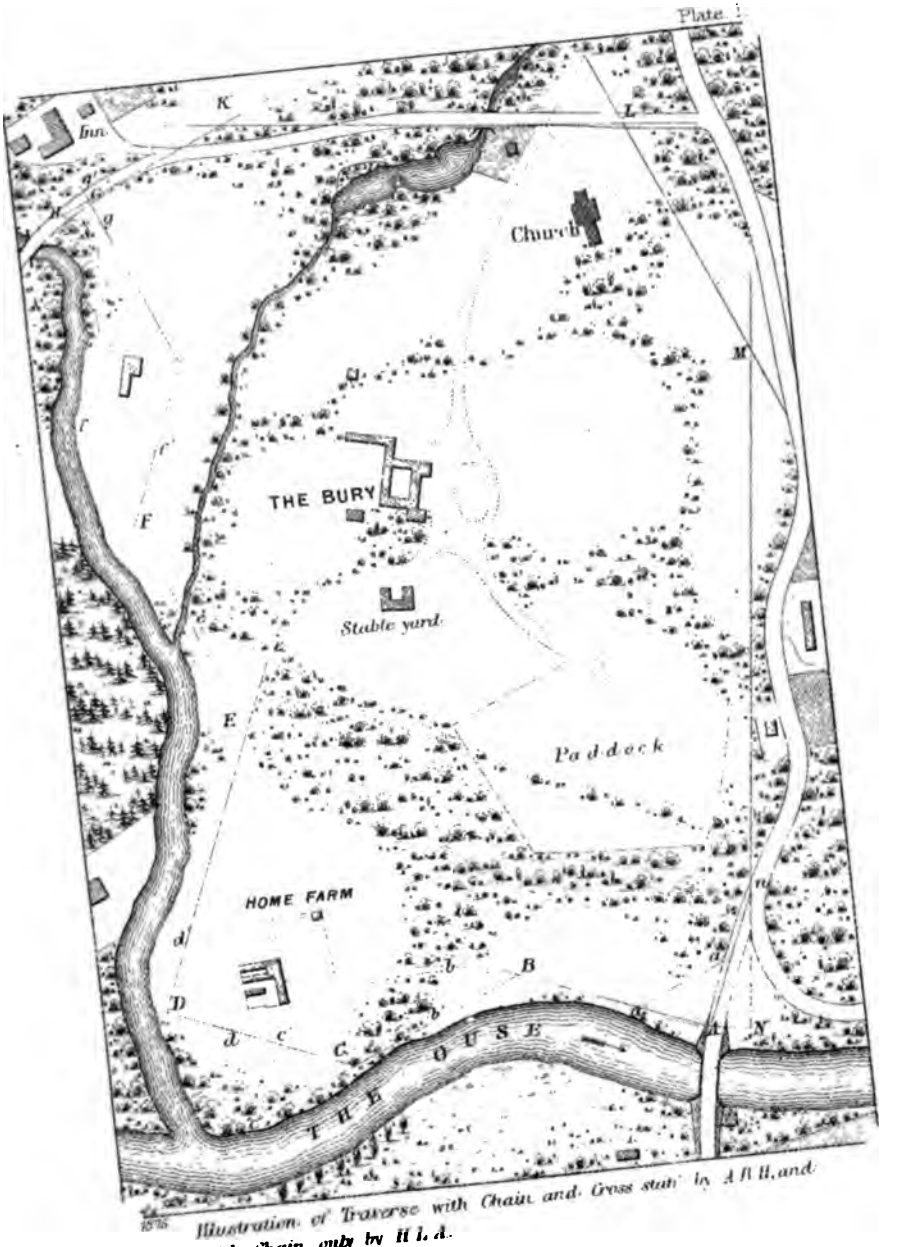


Illustration of Traverse with Chain and Cross staff by A.H.H. and with Chain only by H.L.A.

On the chord system.—A corresponding mode of traversing with the chain, which dispenses entirely with the cross-head, may be adopted by prolonging each traverse line in the same way, and at the extremity of one chain, setting out either half a chain, a whole chain, or a chain and a half, at its extremity as a chord; thus making practical use of another set of angles while merely dealing with their chords, and conducting the traverse in exactly the same way in every other respect. If this chord method is applied in further detail, using quarters or eighths of a chain, and the angle (β) corresponding to any chord set out should happen to be required, then

$$\sin \frac{\beta}{2} = \frac{\text{chord}}{2R};$$

but if the chord be set out at the extremity of two or three chains instead of one, then $2R$ or $3R$ must be substituted for R in the above formula.

This method must always be kept distinct from the rectangular offset system; it is not quite so rapid, as it requires three measurements instead of two, and for certainty requires also that both traverse lines should be prolonged both ways to repeat the measurements.

Record or field-book of chain survey.

In all cases the record of measurements should be accompanied by a sketch showing the skeleton survey-work, and the position of some of the important features of the tract surveyed, such as roads, rivers, marked buildings, churches, etc., with their names; the survey-stations and principal survey-lines should be distinctively designated by the letters or numbers used for them in the record.

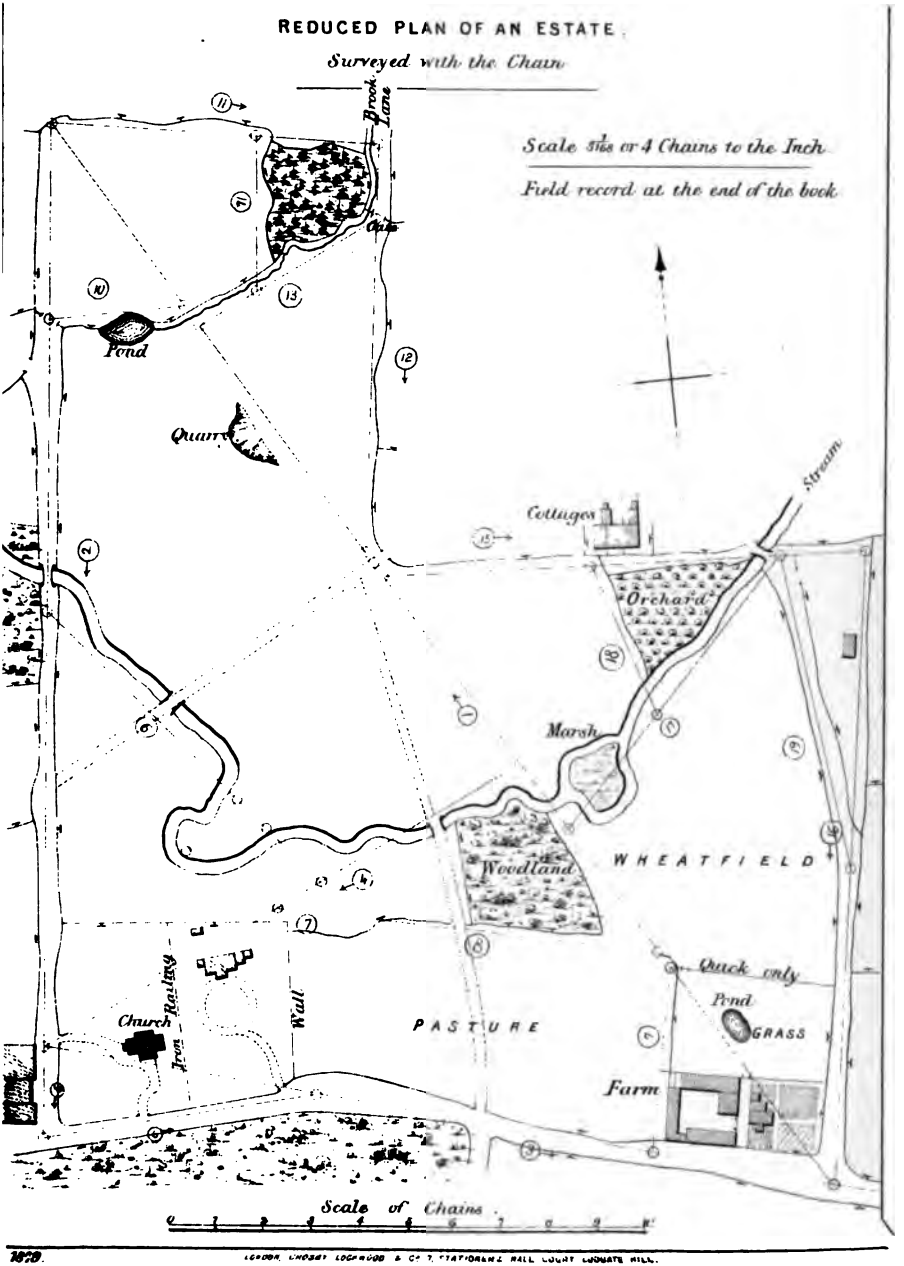
Some recorders use alphabetical letters to designate

station-points, and numbers for survey-lines in the order of their measurement; another method is to use numbers enclosed by circles for the stations, and to enter in the sketch the total distances between stations; the latter is more convenient for a stranger to plot from. Special sketches or diagrams of lines measured are required whenever intricate measurements, as of buildings, happen to occur, also for corners where more than two lines of measurement meet, and whenever the ordinary unaided form of record fails in conveying clear and sufficient explanation.

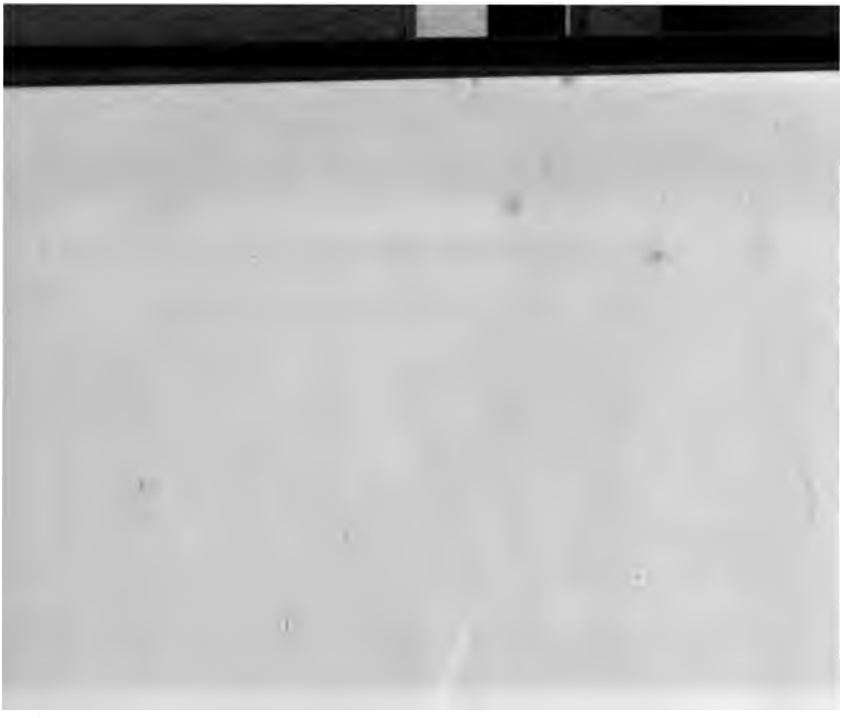
Field-records are best made with pencil in graphite, chalk, or indelible dry ink, as liquid ink is troublesome and liable to blot from rain; alterations in figures should never be made, corrections should be effected by obliterating or entirely striking out or through the error or erroneous portion, and rewriting it clearly.

The accompanying field-sketch, Plate II., together with Field-record No. 1, illustrate the system now in vogue in surveying estates when the chain alone is used. Chain surveys for engineering purposes are conducted much in the same way, and hence do not require any special illustration or notice.

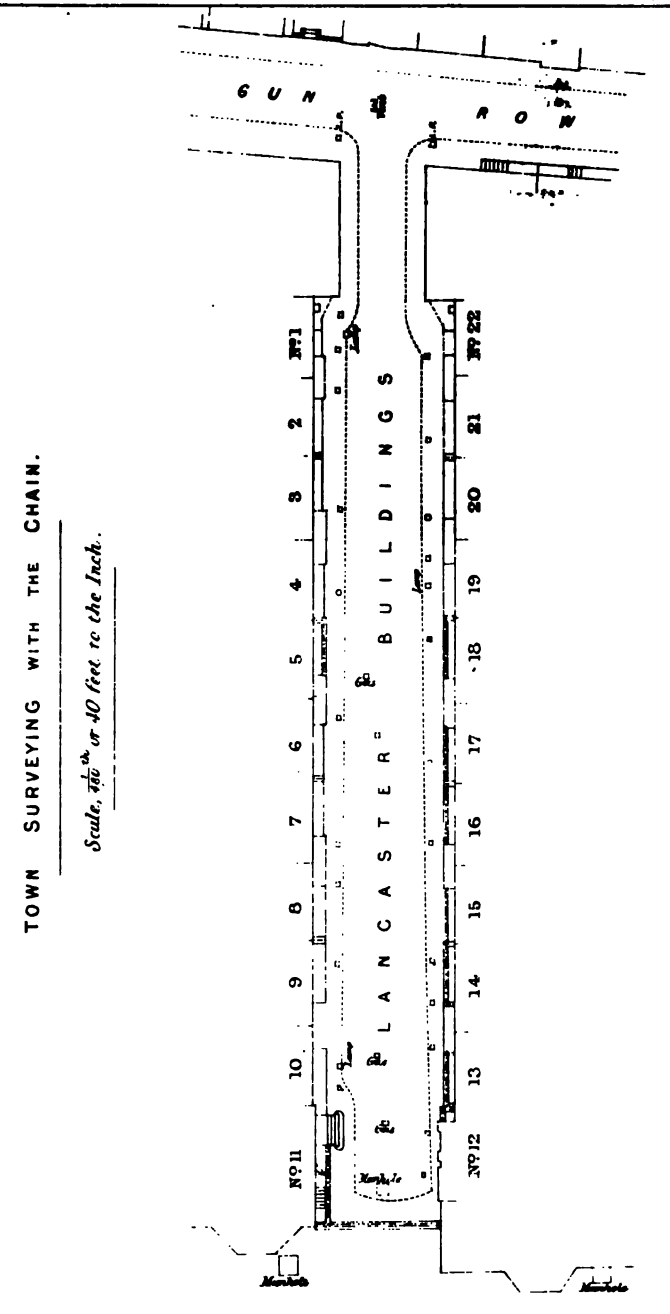
The next reduced plan, Plate III., together with Field-record No. II., illustrates a small portion of a town survey, comprising one street only, surveyed for plotting on the scale of $\frac{1}{120}$th or 10 feet to the inch. The parallelograms that form the network of this survey are double, each street having two lines of chain measurement. The offsets in this case are rectangular, and were taken with much care, but under such circumstances, wherever accuracy is very necessary, additional oblique offsets are imperative. For surveys of the same kind



1870. LONDON, LONGMAN GREEN, LONGMAN, ROBERTS & GREEN, PATERNOSTER ROW.







LONDON, CROSBY LOCKWOOD & CO'S STATIONERS HALL COURT LUDGATE HILL

for plotting on a smaller scale, as 40 or 44 feet to the inch, single lines would alone be necessary except in very wide streets. The scale of $\frac{1}{120}$th is generally the largest adopted in surveys of any description, all larger scales being used merely for details of engineering works; but as an exceptional case may be mentioned a small survey made for plotting to the scale of 3 feet to the inch, in which the offset measurements were taken correctly to $\frac{1}{8}$th of an inch, with steel tapes, instead of to full inches, as it was specially required to illustrate the accuracy of a few lengths of tunnel where a through meeting had been formed.

Section 3. TELEMETER SURVEYS.

Telemeter surveys differ from chain surveys in that the distances, within a range convenient to the telemeter used, are obtained by observation with instruments instead of by chaining (for Telemeters and Telemetrical Observation, see Chapter I., section ii., pages 10 to 24). For instance, if the power of the telescope and arrangement of the distance wires are such as to give correct distances in sights taken between 100 feet and 1000 feet, any distance exceeding 1000 feet has to be taken in portions, while any detached distance less than 100 feet must be measured with a tape or offset staff; correspondingly also when the powers of the instrument are limited to a range between 66 feet and 660 feet.

The general method of procedure, as regards triangulation and offsets, is substantially the same as that adopted in chain survey, unless the telemeter is accompanied by instrumental means for observing angles. The need of the occasional minor operations previously

explained on pages 87 to 94 is greatly reduced, as small brooks, sheets of water, etc., are not obstacles to telemetrical observation; and again, if Porro's method be adopted, the necessity for checking by tie-lines is also reduced, as each telemetrical observation with three wires affords a check on its own accuracy.

While telemetry possesses great advantages over chaining in reducing monotonous labour with at least an equal amount of precision, it must be remembered that it is more specially suited to surveys in which the distances are many and rather long, and the offsets comparatively few.

In this, as in all such substitutions of mechanical for manual work, the men employed must be more intelligent and capable. The staff-holders must not only be capable of holding the staff truly vertical, but of choosing the points necessary, ranging themselves correctly in any direction, taking the offsets well and truly at right angles to any direction or survey line, and also of roughly recording them, as their memories cannot be depended on for several points at a time; to do this well and without loss of time, they must also be capable of understanding and applying the elementary methods of procedure in survey work, and be worthy of some confidence. Work of this class is hence best entrusted to assistants and pupils in preference to the stray working men and boys generally employed in chaining.

The form of record for telemeter surveys is generally similar to that used in chain surveys, the principal difference being that the record itself is more lengthy and extended, from three telemetrical readings being entered in place of every single measured distance, whenever

Porro's principle is adopted, or two readings and a reduced distance when Eckhold's instrument is used; this is exclusive of short distances and offsets less than one chain in length.

Section 4. PLANE-TABLE WORK.

In work of this rather rough description, which is generally confined either to such details or filling in of the triangles of a topographical survey as can bear subsequent considerable reduction in scale, the survey is not recorded in forms, but plotted on the plane table itself in the field while the work is proceeding; angles are not measured, but laid down on the paper with a sighted ruler directed to any object, and thus the principles of triangulation are applied after measuring a single base line to start from, and laying it down to scale on the paper.

As each portion of plane-table work must be comprised within the limits of a sheet of paper of the size of the plane table, and subsequently be joined on to other sheets done in the same way, the errors will be greater at the points of junction farthest from the base line and will increase with the number of intervening sets of triangles; it is hence more usual to limit the sets of triangles on any single sheet either to one or to two, one directly dependent on the base line, the other being secondary, or dependent on the sides of the triangles of the first set.

The plane-table itself consists of a drawing-board, mounted on a light three-legged stand, and capable of motion on a universal joint and of clamping to any position of azimuth or altitude by means of screws. A paper is

strained on the board for scaling distances and ruling directions, and a ruler with sights is used for laying down the directions of any objects, such as survey marks, corners of buildings, etc.

The mode of using it is thus—

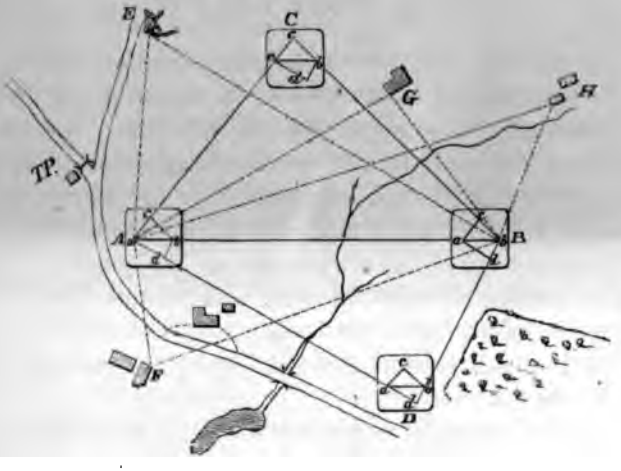


FIGURE 16. PLANE-TABLE WORK.

A convenient base line *AB* having been carefully measured is laid down to scale as *ab* on the paper; the plane-table carefully levelled is then set up at *A* with the line *ab* adjusted true to *AB*, and the directions of any number of points, *C*, *D*, *E*, *F*, etc., are sighted from *A* and ruled from *a*; the plane table is then set up at *B* with the line *ba* adjusted true to *BA*, and the directions of the same points, *C*, *D*, *E*, *F*, are sighted from *B* and ruled from *b*; the intersections of the two sets of ruled lines give the positions, *c*, *d*, *e*, *f*, on the paper, which can be checked by directions sighted and ruled from other

points subsequently used, or from the same points among themselves.

A needle stuck in the paper and board at the point corresponding to the station of observation is convenient in sighting with the ruler. It is also better to use a distinctive hard pencil for marking these sight lines, and to retain a softer one for the delineation of actual objects and details.

All points thus obtained by observation from the extremities of a measured base line are those of primary triangles; when again the plane-table is moved to any other point *C*, and *ca* is used as the fixed direction of departure, the secondary set of points, *K*, *L*, *M*, etc., is obtained in a similar way, but these are less accurately fixed. Smaller details are either measured by offsets and laid down to scale, or as is more frequently the case in work of this sort, guessed and sketched. The closing up of the sheets among themselves on sufficient points common to two of them, and the final closing on important well-defined points laid down in the triangulation of some larger survey, are the matters that demand the most care. The resulting sheets can finally be reduced by the draughtsman with a pantagraph, or otherwise, to any requisite scale, and transferred to a general plan comprising the whole.

The remaining appliances used in such work are mostly the same as those described for chain surveys; the amount of measurement is small, the work can be rapidly carried on by two men, one to set up and remove poles and marks, the other to manage the plane-table; but the results are necessarily rather coarse; in some cases pacing or taping is made to supersede chaining or telemetry, when the results are only suited to military

purposes. As such drawings executed in the field must generally involve some inaccuracy, it is usual to make them to double the scale of the intended final plan or map, and afterwards reduce them to half the scale, so as to entirely eliminate the inaccuracies.

Section 5. Compass Surveys.

Compass surveys have occasionally been executed on a large scale and with extreme precision, the angular observations in such cases being utilised to an exactitude of one minute, but as it then also becomes necessary to enter into magnetic observations, and verify the compasses used with a very high degree of refinement, such surveys involve particular care and special operations, and are hence comparatively rarely undertaken; their consideration is hence neglected in this work. The generality of compass surveys are very limited in extent, and may be said to be restricted to conditions under which an error of 30 minutes, due to diurnal and annual magnetic changes, or to local attraction, shall not affect the visible accuracy of the survey when delineated to the required scale: the actual compass readings are then taken to 15 minutes, or more frequently only to 20 minutes. (See paragraph on Magnetic Instruments, Chapter I., page 42.) Under these circumstances, as these surveys are based on the principle of triangulation (described in Methods of Procedure, page 72, and more fully explained under the head of Theodolite Surveys), it becomes evident that their extent is dependent on the size of the largest comprehensive triangles that can be used without final appreciable error in the delineation; for the smaller included triangles will be comparatively

free from error, while if an additional set of triangles were subjoined to the primary comprehensive triangles they would be vitiated by a double, or a further amount of error.

For instance, if we suppose that the longest side of one of the comprehensive primary triangles is 1000 feet, and that the liability to error in compass-bearing is 20 minutes; this will then, if roughly represented by its sine at that distance, be equal to about 5·8 feet. If also the scale of representation adopted renders such a distance practically inappreciable, as it would be either with a scale of 400 feet to an inch, or of $\frac{1}{5000}$th, or any smaller scale, then primary triangles of this extent may be correctly adopted; but if a subjoined set of triangles be applied to these, the errors in the second set, if equal in extent to the former, may amount to double those of the first, or may be 11·6 feet, which becomes appreciable on the scale of $\frac{1}{5000}$th.

It is hence best to limit such surveys to one set of comprehensive primary triangles, having their angles or extremities visible from each other, or at least from some one or two central points, while the extreme length of any side should be so limited that the error resulting from inaccuracy of chainage of the measured base, and that from angular observation, shall together be an inappreciable quantity on the scale of representation adopted.

It is also evident from the foregoing considerations that it is seldom advisable or possible to include in any such single compass survey an extent of more than half a mile of country: and that such work is best suited to the filling in of details of larger surveys on a comparatively small scale by means of pieces based on inde-

pendent triangulation determined with more accurate instruments. It is however for the same reason altogether unsuited to small engineering surveys to be delineated on a large scale with much precision.

Restricting therefore compass surveys to such limits, the ordinary modes of procedure adopted are as follows.

By triangulation.—When the whole of the details within any area have to be surveyed, the principle of triangulation is adopted; a base line is measured in some convenient open part of the country that is comparatively level and free from obstruction, its extremities being chosen so that most of the important points around may be visible from them, and so that the whole of the area may be included in one series of comprehensive triangles, which either have the base line for a side, or have one or other extremity of the base for an apex. Compass bearings are then taken from the two ends of the base to all visible important points; and check bearings are also taken from each of these points severally both on all the rest that happen to be visible and on the ends of the base. Smaller detail is interpolated in the same way by bearings from these points to minor objects, such as hedge corners, gates, corners of walls, and buildings; occasionally, also, short distances as well as offsets are measured with the tape to aid in completing the detail and minor triangulation. The next portion of survey is conducted in the same way, some of the extreme points of the first survey acting as points in the second portion; should these closing points of the two portions show appreciable error, bases of verification may be measured, similar to the measurement of the original base lines, between any of these closing points, and the plotting of the two portions of survey

may then be readjusted in accordance with the verified bases. In plotting this all unchecked bearings and faulty points have to be rejected.

A large extent of country may thus be surveyed in pieces, the portions being afterwards joined and fitted with care, provided that they be also fitted and adjusted to a series of well-defined and easily-obtained points occurring in some more comprehensive triangulated survey of the same country, such as a Great Trigonometrical or a good topographical survey; but without some such aid, the junction of a series of pieces of compass survey cannot be effected with much accuracy; and the pieces are then merely of value as independent portions.

For further information as to triangulation, and measuring and prolonging base lines, see paragraphs on such subjects in Chain Surveys and Theodolite Surveys

By traversing.—When survey details are merely required along or in the proximity of some line of route, however circuitous it may be, the method of procedure known as traversing, explained at page 72, is adopted in preference to triangulation. Each forward and each corresponding back bearing is obtained with the compass, and each distance between stations is measured or observed; and checks are afforded by taking additional bearings on lateral conspicuous objects, spires, buildings and towers. A compass traverse when adopted as a means of filling in details of land surveys to be plotted on a moderate scale, between $\frac{1}{5000}$th and $\frac{1}{20000}$th, should seldom exceed a mile in length; as the errors of observation in the route bearings may accumulate to a very seriously appreciable extent; and if the route be made up of a large number of short distances,

the liability to error becomes greater; hence the necessity for frequent closing on points determined by some more accurate means, as before mentioned in compass triangulation, or the need for completing or closing the traverse by continuing it by some other return route back to the starting point, unless the lateral objects have afforded sufficient check on its accuracy.

Closing a traverse.—A closed traverse in either of these cases forms an irregular polygon; for if any two closing points in it have been previously determined, their distance apart and azimuth is known, or can be obtained from a plan, and this becomes one of the sides of the polygon; while if the traverse is continued until it closes on its starting point, all its sides and angles are obtained from the measurements and bearings. To check the accuracy of the bearings, deduce from them all the inward angles of the complete polygon; distinguishing those that are re-entering from those that are salient, and taking the sum of the whole, check it by the formula given at page 48 in the collection of formulæ.

Reduction to a single meridian.—In plotting compass surveys, whether triangulation or traverse, the bearings are more usually laid down carefully on paper with a large protractor; there is, however, in traverse work much risk of undiscoverable error in so doing, for although the angles and consequently the bearings may be checked and found correct by the formula, any of the distances may be incorrectly scaled or laid down. To obviate this risk as well as to avoid protracting angles or bearings, the whole of the traverse may be referred to rectangular coordinates, or reduced to any single convenient meridian, such as one passing through two adjacent points of the traverse, or through the starting

point and the magnetic north: each inclined or measured distance is thus resolved into two parts, one taken along

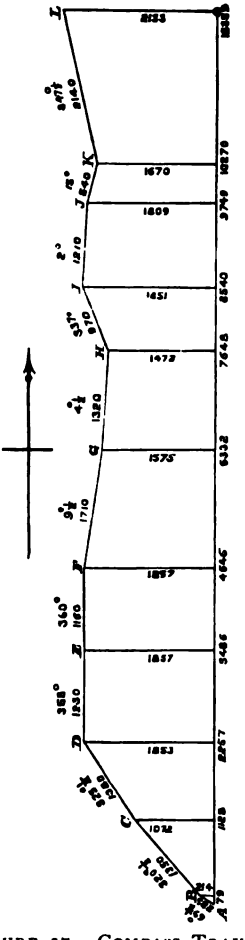


FIGURE 17. COMPASS TRAVERSE.

the assumed meridian, the other at right angles to it; the calculation being effected with the aid of a table of

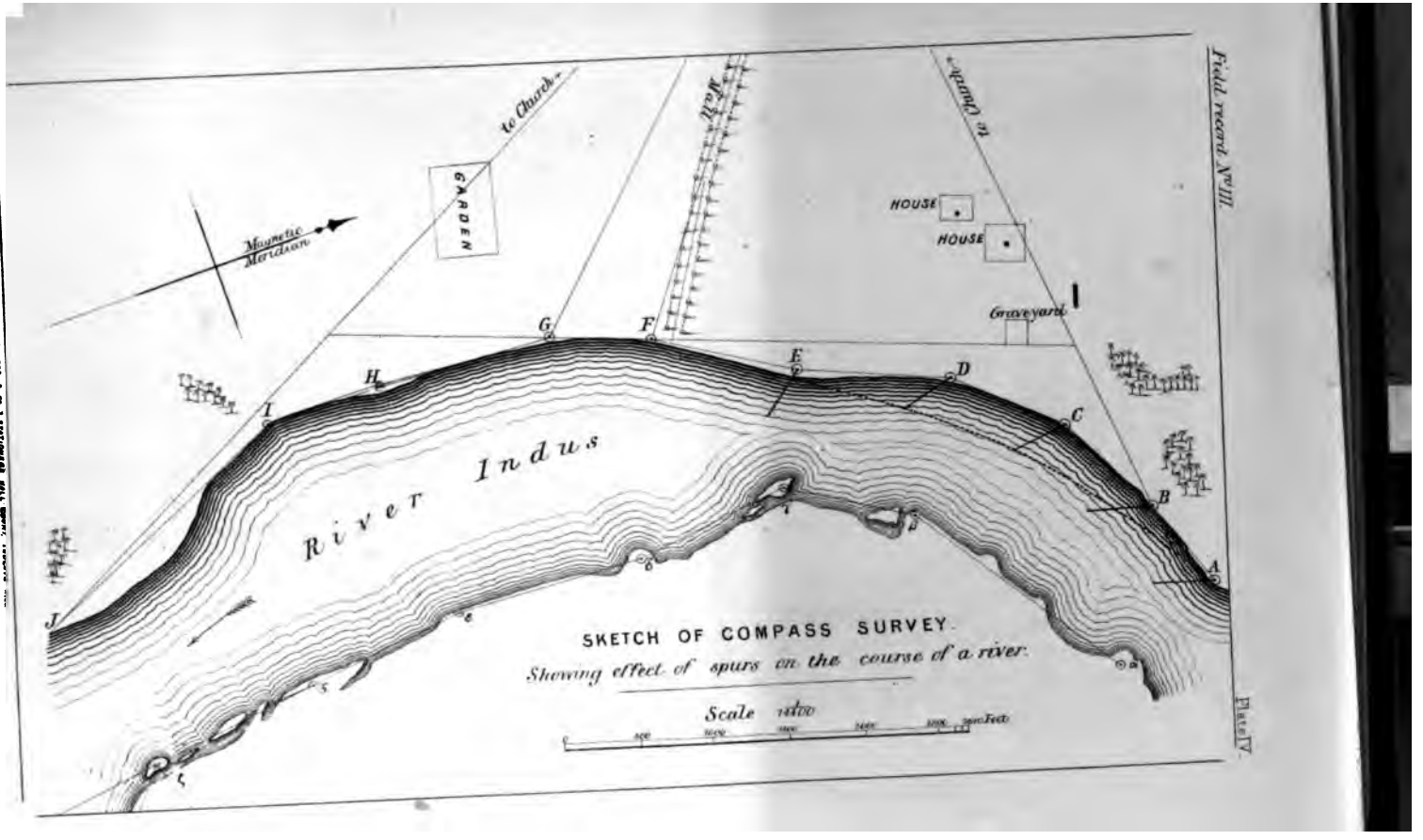
sines and cosines or with traverse tables. The algebraic sum of each of these sets of resolved distances should in any closed traverse be equal to zero, and this affords a verification of the accuracy of the whole before plotting it, and enables it to be plotted correctly without alterations.

The small unclosed traverse, shown in figure 17, is reduced to a single meridian in the following table:

REDUCTION OF TRAVERSE TO A MERIDIAN PASSING THROUGH A.

Stations	Observed Bearings	Distances	Reduced Bearings	Consecutive Co-ordinates Lat.	Dep.	Independent Co-ordinates N.	W.
A	69° 30′	225	69° 30′ NE.	79 N.	214 W.	79	214
B	320 30	1350	39 30 NW.	1041 N.	858 W.	1128	1072
C	325 30	1380	34 30 NW.	1137 N.	781 W.	2257	1853
D	358	1230	2 0 NW.	1229 N.	4 W.	3486	1859
E	360	1160	0 0 N.	1160 N.	0	4646	1857
F	9 30	1710	9 30 NE.	1686 N.	282 E.	6332	1575
G	4 30	1320	4 30 NE.	1316 N.	103 E.	7648	1472
H	337	970	23 0 NW.	892 N.	379 W.	8540	1851
I	2	1210	2 0 NE.	1209 N.	42 E.	9749	1809
J	15	540	15 0 NE.	521 N.	139 E.	10270	1670
K	347 30	2140	12 30 NW.	2089 N.	463 W.	12359	2133
		13235		12359 N.	2699 W.		
				0 S.	566 E.		
	Sum			12359 N.	2133 W.		

To obtain these reduced values; let P be the observed bearing at a preceding, F the observed bearing of a following station; if a the inward angle at the station be required, then $a = F - P \mp 180°$; using the negative sign when $P + a < 180°$, and the positive when it is greater. The reduced bearing R in any quadrant is thus obtained; when F lies between 0° and 90°, $R = F$ and lies NE.; when F lies between 90° and 180°,


Field record No III.
to Church
GARDEN
Marl
to Church
HOUSE
HOUSE
Graveyard
Magnetic Meridian
G
F
E
D
C
B
A
H
I
J
River Indus
SKETCH OF COMPASS SURVEY.
Showing effect of spurs on the course of a river.
Scale
Feet
Plate IV.

$R = 180° - F$ and lies SE.; when F lies between 180° and 270° $R = F - 180°$ and lies SW.; and when F lies between 270° and 360°, $R = 360° - F$, and lies NW.

The separate co-ordinates are thus obtained:

Each abscissa along the meridian

= corresponding distance × cosin R,

Each ordinate or departure

= corresponding distance × sine R;

these separate co-ordinates are added up or subtracted, in accordance with their signs being similar or different, from the starting point up to the point whose independent co-ordinates are required; that is to say, up to each one successively; and when the traverse is a closed one, round back to the starting point again where the final verification is shown by proving its co-ordinates through results of the summation to be equal to zero.

Compass traverses of large extent to be delineated on a very small scale, such as those of travellers and navigators, are to a certain extent based on the same principle or method of procedure. A chapter will subsequently be devoted to the special subject of route surveys.

Details of traverse work.—The above constitutes the skeleton of the traverse work; the adjacent details or filling in may be done by distances and offsets measured, or with the occasional aid of minor triangulation as before explained.

The verification of the compass.—The compass used in surveys of small extent requires merely rough verification; the magnetic meridian can only be indicated as accurately as the graduation of its circle will admit. The variation of the compass at Greenwich in 1879 is

18° 45′ W.; and the magnetic elements are given every year in Whitaker's Almanac. To determine the variation of any compass, used at any place and time, is practically to read the bearing when sighted in the direction of the true meridian. Various modes of finding the meridian are mentioned in the chapter on Astronomical Observations on route surveys; but the most simple and rough methods, not requiring the aid of a Nautical Almanac or of accurate instruments are the two following: 1st. Take the bearings of the sun at sunrise and at sunset, when its lower limb is a semidiameter above the horizon, the bisection of the angle formed will be the direction of the true meridian. 2nd. The direction of the polestar is that of the true meridian, when a plummet shows that a true line passing through it and the star Alioth, ε Ursæ Majoris (the fifth star of the Great Bear) is truly vertical. Having thus either plumbed down and picketed the true meridian from any spot, or determined the correct azimuth of any spire or mark from it, the marks should be made permanent for future purposes of verification.

A specimen of Compass-Survey is given in Plate IV. and Field-record No. III.

Section 6. Sextant Surveys.

The adjustments of the surveying sextant have been explained in the paragraph on Reflecting Instruments (page 40). In the abstract the instrument would appear most perfect for the purposes of the surveyor, being extremely portable and giving angular readings to 20 seconds, as well as being free from all the liability to error that is so inconvenient in magnetic instruments. In practice, however, it is not so much used as would other-

wise be the case, chiefly for the following reasons. In the first place, as it does not give angles in the exact plane of the horizon, but in the plane of the eye of the observer and the two objects observed; it either becomes necessary to take two altitudes with each angle and to reduce it by calculation to a horizontal angle (the formulæ and examples for this are given at page 59 in the collection of formulæ), or it is necessary to set up a rod strictly vertical, or to hang a plumb-line in the exact direction from the observer of one of the objects, and then handle the instrument in an approximate horizontal plane, and observe a horizontal angle. Besides the inconvenience and tediousness of setting up additional rods or plumb-lines with accuracy for every set of angles, in windy weather it is impracticable to observe on them with precision; while permanent vertical lines, such as edges of buildings, are not always available, and anything approaching to the guesswork involved in dispensing with vertical lines may vitiate the whole of the angular observations. In the second place, the exact superposition of the direct and the reflected image is essential to a correct angular observation; in clear weather, and with well-defined objects not more than 90° apart, this is easy; but when the objects are ill-defined, the weather dull (as is frequent in Great Britain), and when the angle is between 90° and 120°, so that the mirror presents a narrow breadth to the observer's eye, the operation involves considerable doubt and has to be repeated once or twice; this is both troublesome and tedious. These two objections render the instrument inconvenient in triangulation, while the difficulty in observing very obtuse angles with it, and, in fact, the impossibility without a supplementary arc, render it inconvenient in

traverse work. It is hence often relegated by surveyors to the class of appliances for occasional use in special cases apart from the above. The special uses are, that an occasional horizontal angle observed with it may be useful as a check in compass triangulation, that an angle of altitude observed with it may be occasionally wanted to determine, or to aid in determining, the height of a building or other such object; and that it is useful in setting out or verifying a horizontal angle up to about 90° with precision in works of construction. Surveys of small extent can, however, under favourable circumstances be accurately made with the sextant.

1st. *By triangulation.* Flat country is most suitable to surveying with the sextant, as the station poles used as survey marks then more frequently afford sufficient length to serve as vertical lines, and aid in obtaining the horizontal angles correctly. A base line is measured (as for compass survey before-mentioned), and a network of triangles conveniently arranged by choosing suitable positions for stations. The best triangle for survey purposes in triangulation is equilateral, but in practice the angle should not exceed 100° or be less than 30°. The three angles of every triangle should be observed, as their sum being 180° affords a check on the angular observations. The larger triangles should be as comprehensive as circumstances admit. Under ordinary circumstances, as, for instance, with a base line of about 1000 feet, and an adopted scale of representation less than $\frac{1}{5000}$th, three sets or series of comprehensive triangles can be surveyed, after starting from the base, without accumulating serious error; but so much depends on the scale of delineation adopted, and the adventitious circumstances affecting the accuracy of the angular

observations, that no precise rule can be laid down. In plotting the work, the whole of the sides of all the triangles may be calculated and then laid down to scale, or the base line may alone be laid down to scale, and the angles plotted with a protractor; the former mode is better, the latter is more rapid; and both methods together afford verification. The principles of triangulation hereafter explained, under the head of Theodolite Survey, are applied in a rough manner in sextant survey of this class, which is generally limited to a square mile of country; the details or filling in of the triangles may be most rapidly carried out by distances and offsets and compass bearings in the way previously mentioned in the paragraph on Compass Survey.

2nd. *By traversing.* In traversing with the sextant flat country is as advantageous as it is in triangulation. The principle of traversing has been already explained at page 72, and its application in chain survey and compass survey at pages 93 and 104. The chief difficulty in traversing with the sextant consists in the observation of any very obtuse angles. These may severally be divided into two acute angles by using an available lateral point; but a better device is to employ advanced points on the produced lines of traverse, and thus to confine angular observation to acute angles only. For instance, if *AB*, *BC*, *CD*, be three lines of traverse, the inward angles *B*, *C*, *D* being obtuse, but less than 180°, let *ABb* be the starting direction and *AB* the first measured distance, *b* being the position of a pole set up further on in the same direction. On arrival at *B*, after setting up a pole at *c* further on, the acute angle *bBc* is observed, and its supplement the inward angle *ABC*

recorded; after measuring *BC*, the acute angle *cCd* is observed, and its supplement the inward angle *BCD* recorded; and so on throughout the traverse, except whenever the inward angle happens to be acute, in which case it can be observed and recorded as such. This process is suited to tolerably open country, and along rather straight roads, but not along very winding lines of route with hedges or obstacles on both sides; it may be much facilitated by any conspicuous object such as a spire in the general direction forward, as the acute angles may then be observed on it and the advanced poles *b*, *c*, *d*, etc. dispensed with; the direction of that object from the first station may then be used as the meridian of reference instead of the starting direction *AB*. It is very necessary to note in the record whenever the actual inward angle is more than 180°, in which case it is not the supplement of the observed acute angle, but is equal to the sum of it with 180°. Checks on lateral objects are more necessary in such traverses than in compass traversing, where every forward bearing is verified by a back bearing, as a corresponding verification is not always possible in such sextant traverses; for example, the angle *bBC* may be observed at *B*, but the angle *bCB* may not be possible of observation, and even if it should be, it does not alone afford any verification; hence the necessity for well-defined lateral objects.

Reduction to a single meridian. — The principles involved in reduction have been explained under compass traversing; the distances, inward angles, and the bearing of one traverse line, being observed, the rest is reduced as in the following example which is illustrated in figure 18.

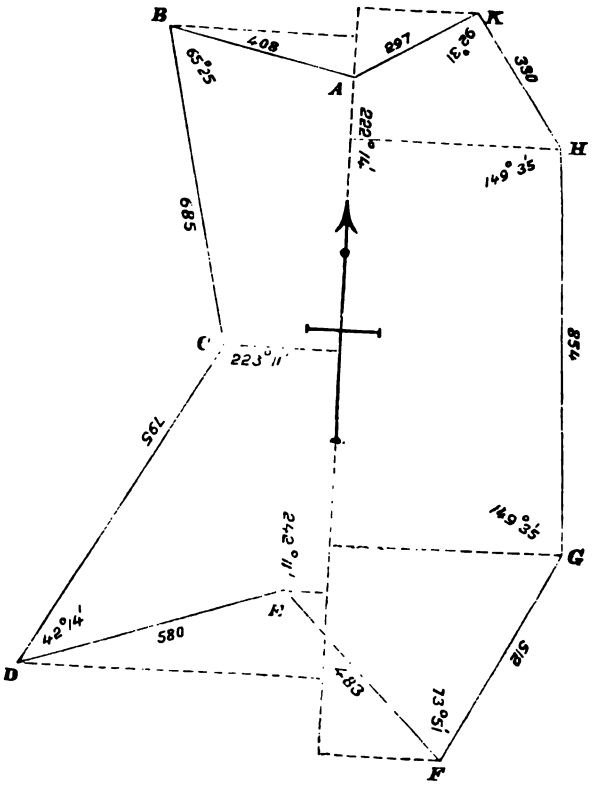


FIGURE 18. SEXTANT TRAVERSE.

The details in proximity to the route traversed are obtained by the method of distances and offsets, aided occasionally by compass bearings.

Reduction of a Sextant Traverse to a single Meridian.

Station	Observed: Inward Angles	Observed: Outward Angles	Magnetic Bearing	Reduced Bearing	Measured Distances	Consecutive Co-ordinates: N	S	E	W	Reduced Co-ordinates	
A	— —	— —	281° 40	78° 20′ NW.	408	84	—	—	399	0	0
B	65° 25′	— —	167 5	12 55 SE.	685	—	666	153	—	84 N.	399 W.
C	223 11	136 49	210 16	30 16 SW.	795	—	687	—	401	582 S.	246 W.
D	42 14	— —	72 30	72 30 NE.	580	177	—	553	—	1269 S.	647 W.
E	242 11	117 49	134 41	45 19 SE.	483	—	340	343	—	1092 S.	94 W.
F	73 5	— —	27 46	27 46 NE.	506	453	—	235	—	1432 S.	249 E.
G	149 35	— —	357 21	2 39 NW.	854	853	—	—	42	979 S.	484 E.
H	149 35	— —	326 56	33 4 NW.	340	277	—	—	186	126 S.	442 E.
K	92 30	— —	239 26	59 26 SW.	297	—	151	—	256	151 N.	256 E.
A	222 14	137 46	281 40	78 20 NW.		—	—	—	—	0	0
$n=9$	1260°0					1844	1844	1284	1284		

Section 7. THEODOLITE SURVEYS.

The adjustments and corrections in angular observation that may be required in using the theodolite are explained in Chapter I.

As surveys in which the theodolite is used may be of any extent, large or small, and may be either for general or special purposes, they can only conveniently be classified under the heads of

Triangulation surveys, and
Traverse surveys,

and combinations of the two; purely traverse surveys being more generally confined to special and engineering purposes, on works of communication.

Triangulation surveys, as ordinarily conducted for topographical purposes of large or of moderate extent, are based on the same principles, operations, and modes of procedure as are now, or were originally adopted in the Great Trigonometrical surveys of large countries. These latter are beyond the scope of this work, but the operations employed in them will be referred to, not for exact imitation, but for the guidance they afford in carrying out corresponding operations with less precision on the same principles. The principal points in which such topographical surveys differ in operation from Great Trigonometrical surveys are the following.

1st. Astronomical observations of latitude and longitude are very frequently dispensed with.

2nd. The spheroidal form of the earth's surface is either entirely neglected, all triangles being treated as plane, or in extreme cases the very simple method of Legendre is adopted for treating large triangles as plane after applying a correction to the angles observed.

3rd. The country is completely covered by a network of triangulation, instead of divided into parallelograms by meridional and latitudinal series of triangles.

4th. The angular observations are made with small portable theodolites, having plates with graduated arcs of 4, 5, or 6 inches, and in exceptional cases 8 inches in diameter, which read to 20, 10, or 5 seconds, instead of with immense theodolites reading to hundredths of a second.

5th. The measurement of base lines is carried out with the simple surveyor's chain, and perhaps repeated a few times without using the troughs, trestles, straining weights, screw-marking-points and compensating bars adopted in the Great Trigonometrical survey.

6th. The whole scale of operation and the amount of accuracy required are smaller, the appliances more portable, the work more rapid, and the number of men employed less.

The preliminary considerations before starting the actual surveying are:

1st. The extent of country to be surveyed and its nature: whether hilly, flat or cramped, or intersected by rivers, covered with buildings, wood, forest, or marsh, or any obstacles that may impede work in certain parts of it. 2nd. The scale on which the survey is to be delineated or represented; and the limiting error in measurement that becomes inappreciable on the final plan. 3rd. The amount of detail required: such as whether every field and hedgerow is to be shown on the plan, or whether every building, or only every block of buildings, or every frontage, or limit of holdings is to be shown. 4th. Any information that may bear on the special objects for which the survey may be made.

5th. The amount of time available, and the number of assistants, men, instruments, and appliances.

Selection of Base Lines and Principal Stations.—These matters having been considered, the first operation is to examine the tract of land, country or town, with the view of choosing suitable positions for measuring a base line, and, if necessary, also bases of verification, and of arranging the distribution of the principal station-points over the whole tract. All such positions may then be roughly marked on the ground, and recorded with distinguishing letters or numbers in a rough sketch in the field-book.

The conditions most favourable for a base line are an open flat extent of country, unimpeded by obstacles of any sort or long grass or vegetation; the ground moderately soft and capable of holding the iron pins or skewers used with measuring chains, or at least capable of being marked without chiselling at each chain length.

The length required for a base line must be suited to the triangulation dependent on it; if the survey is extensive the sides of the main triangles are as long as possible in accordance with the powers of the theodolite-telescopes used, and as the form of triangle most suited to survey purposes is equilateral, the length of base line is then more governed by the two practical circumstances of the conditions of the ground and the powers of the instruments than by other considerations.

The extremities of the base line should, when marked, be clearly visible from the country around, and hence an elevated plateau is an advantageous position for a base line of an extensive survey; but it is not by any means indispensable that the two extremities should be visible from each other; if circumstances are

favorable in other respects, intermediate points in the base line may be ranged with extreme care to make up for the defect.

Should it happen that the ground does not offer facilities for the direct measurement of a sufficiently long base, as may happen when other conditions are favorable, the greater portion of the base should be measured, and the length of an additional piece carefully obtained by calculation by any of the methods of prolonging bases hereafter described in the paragraph on measurement of base lines.

The choice of station-points for the main triangulation, or principal network of triangles to be spread over the tract of country will necessarily be in accordance with the size of the triangles determined as most suitable to the conditions and extent of the tract, or the maximum and minimum limits laid down for the sides of these triangles; the position of the points should be so arranged that the signal flag at each one may be visible from the neighbouring points of observation, and thus admit of the observation of all three angles of every triangle; rather elevated points on open ground are hence to be preferred. If a spire or tower is made one of the main triangulation points, and happens to answer the purposes of a signal point only without affording facilities for setting up an instrument and taking angles from it, a satellite station of observation at a very short distance from the former must be used, and the angles observed there reduced by calculation to the point observed. (See formula in general collection.) This, however, should be avoided whenever possible. Two satellite stations or two inaccessible points are inadmissible in any one triangle. The triangles adopted in main trian-

gulation should not have any angles less than 30° or greater than 120°, and are preferable when nearly equilateral.

The demarcation of the survey points has been already treated on in section vi., Chapter I., pp. 67-69; but in this special branch, the marking of points for extensive main triangulation, it is necessary to mention that much loss of time may ensue from inability to find points badly marked, or from signals being removed or altered in position by the action of the wind; also, that however desirable it may be to have such triangulation as large and comprehensive as practicable, it is a considerable disadvantage to have to wait for very clear weather in order to discern very distant signal poles. In the Great Trigonometrical survey of a country it is worth while to wait months for an opportunity of observing on a very distant point, but in ordinary operations that is not the case, hence these practical disadvantages require attention.

Next as regards very distant stations: the elevated signal point to be seen from a long distance has to be elevated in accordance with available means, and the corresponding point of observation on the ground or position of the instrument must be plumbed down; but at the same time, to afford the necessary amount of convenience, the height of the signal point, which is generally the point where the lower edge or sometimes the upper edge of the bunting joins the pole, should never be less than one foot per mile of the extreme distance of observation above the level of the lowest observing station in the immediate neighbourhood; it is, of course, better to elevate it more. When the signal pole is not fixed to branches of a tree or to other ready struc-

tures, a framed support or tripod becomes necessary ; the base of this should for security's sake be equal to half the actual height.

The single set of triangles constituting the main triangulation require some verification beyond the observations merely sufficient for laying them down, or plotting them ; this is generally effected by extra angular observations which thus afford independent calculations through another set of triangles, but when these fail or are insufficient, two or even three bases of verification may be necessary. The position of these bases should be such as to afford a check on the whole system of triangles, and hence be near the extremities of the tract surveyed ; in other respects their positions should have most of the advantages mentioned as necessary for original base lines.

Measurement of a base.—A tolerably level site having been selected, the base line is measured with a correct chain, either of 100 feet, or of 66 feet, as may be suitable ; careful comparison of the chain with a standard both before and after measurement is necessary, and the operation should be repeated several times to obtain a mean. (The mode of chaining is described in the section on Chain Surveying, page 84). In hot countries or under special circumstances, it may also be necessary to note the temperatures on each occasion, and to reduce the results to some standard temperature, as 62° Fahrenheit. The extremities of the base should have been previously carefully picketed, or chisel-marked on blocks of stone imbedded in the ground ; intermediate points not more than ten chains apart having been marked with pickets ranged in with the theodolite or portable transit instrument to ensure straight chaining. As the

base line is nearly level, or consists of a small number of pieces of gradual inclination, the chaining may be conducted on the actual ground (not by stepping) any small unevenness being removed or avoided by using bricks or planks laid down; a correct section of the base line may then be obtained by levelling, and the horizontal reductions for these pieces applied to obtain a true horizontal reduction. Lastly, as the plan of the survey is to be a horizontal projection at an elevation of mean sea level, the corrected base must be diminished on this account, or reduced to mean sea level, by the following formula.

Let L = the corrected base already obtained,
l = the base when reduced to mean sea level,
h = the elevation of the base above M.S.L. in feet,
R = the radius of the earth in feet,

$$\text{then } l = L\frac{R}{R+h} = L\left(1 - \frac{h}{R} + \frac{h^2}{R^2} - \&c.\right)$$

$$= L - \frac{Lh}{R} \text{ nearly.}$$

And this reduced value should be used in plotting the plan of the main triangulation.

In small surveys all these corrections are not necessary, the amount of exactitude has then to be suited to the conditions of the case, the scale of the plan, and other matters; but the effect of error in the measured distance should always be taken into consideration as regards results at the extreme end of the triangulation depending on it. It is evident that the errors in the calculated sides of the first triangle will be in proportion to the lengths of these sides, and that these errors will be continually magnified in proceeding through a whole

series of dependent triangles, until they become inadmissible from their enormity. Certain limits of error must therefore be fixed as suitable to the case.

Modes of extending a base line without measurement.—It sometimes happens that the ground is not suited to the measurement of the whole of a base line; in this case a portion only is measured and the rest calculated in accordance with one or other of the two following modes.

1st. When the measured base AB can be conveniently prolonged in both directions towards E and F.

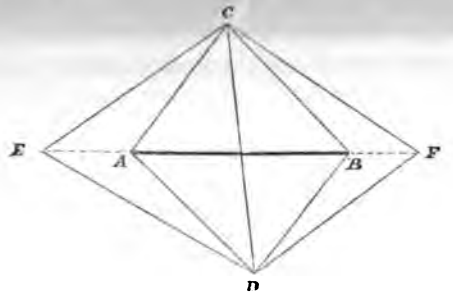


FIGURE 19. PROLONGING A BASE LINE.

Select two temporary station points C and D, so that the resulting triangles ACB, ADB may be well conditioned, and if possible nearly equilateral; observe all the angles of these two triangles and calculate the side CD through each triangle, thus verifying the result; then choose two points E and F in the prolongation of AB, so that the triangles CDE, CDF may be well-conditioned; observe all the angles in these two triangles, and calculate EF twice through separate triangles.

2nd. When the prolongation can be conveniently effected in only one direction, as towards F; a corre-

sponding method can be adopted, which differs only in being one-sided. Choosing points *C* and *D* rather more towards *F*, and observing all the angles, compute *BC* and *BD*; then choosing *F*, so that *CDF* may be well conditioned, and observing all the angles, compute *BF* both in the triangle *BCF* and in *BDF*, thus verifying the result.

In either method the solutions are ordinary cases of plane trigonometry; for which formulæ are given in the general collection, Chapter I., page 47.

These and corresponding methods may be adopted for enlargement of bases by successive steps, but they are not strictly justifiable beyond one operation, as they necessarily introduce the accumulated error due to the successive triangles, and under such circumstances the method can hardly be termed one of prolonging a base, but rather one of enlarging or disposing of the triangles in its vicinity.

When the measurement of the base line of any extensive survey is made, its azimuth should be determined by angular observation with the theodolite on a true meridian; in smaller surveys the magnetic azimuth shown by a good compass may be used for the purpose of obtaining the true azimuth of the base. For the simpler modes of finding the meridian, see Compass Surveys, pages 109; for the more exact modes, see astronomical observations in Route Surveys.

If the latitude and longitude is required of any single point in the survey, it may be obtained by direct astronomical observation (as explained under Route Surveys, in Part IV.) or from a neighbouring observatory which should be connected with the survey by extending the latter for this purpose; the latitude, longi-

tude, and azimuth of any other point in the survey may then be calculated (see formulæ and examples in Chapter I., at pages 54 and 64).

If any two points near the extremities of a large tract of country, whether surveyed or not, are occasionally visible from each other, so that under good arrangements the azimuths of these points from each other may be observed or obtained; then if these points are nearly in the same meridian, and their latitudes are several times carefully observed, their distance apart may be obtained by the formula given in the collection, which is also exemplified in Chapter I.

Distance in feet $= 101{\cdot}36 \times$ secant azimuth $\times (L - L')$.

Main triangulation.—The base line of the main triangulation having been measured and reduced, the observation of all the horizontal angles of each triangle used, as well as of other available angles that may be useful for verification, is proceeded with. Each angle being read with the two or the three verniers of the theodolite and recorded in order, a first check is afforded against error; the vertical angles also are sometimes recorded, as they assist in finding the position of the signal point in dull weather at any subsequent time; and if two observers act in accordance and take them simultaneously, these vertical angles may be utilised in subsequent levelling operations as reciprocal vertical angles; in other respects they are of little or no use, and hence graduated vertical arcs in surveyors' theodolites are often dispensed with. The second check on the accuracy of the horizontal angles is obtained by the summation of the three angles observed in each triangle, the sum of which should in plane triangles be 180°, and in triangles sensibly spherical be $= 180 + E$, where E is

the spherical excess, calculated from the formula given in the general collection in Chapter I., page 49.

Should the sum of the three mean angles show any error, one-third of this error should be applied in the correction of each of the three angles, if the number of observations of each of the angles has been equal; but if unequal, the error must be apportioned according to Gauss' rule or formula, given and exemplified in Chapter I., at pages 49 and 57. It is hence obviously better to adhere to a certain fixed number of observations of each angle, but as some of the readings may be so evidently faulty as to require absolute rejection, an unequal number of accepted observations may sometimes have to be dealt with.

The third check on the accuracy of the horizontal angles is obtained by grouping any number of contiguous triangles into an irregular polygon, and summing the whole of the inward angles along its contour; if the whole area of the polygon can be treated as sensibly plane, without taking the spheroidicity of the earth into consideration, the sum of these angles should $=(N-2)\times 180^\circ$ where $N=$the number of sides of the polygon. If the spheroidal form of the earth is to be considered, as in the case of a large group of extensive triangles, the group should be so formed as to make up one approximate large triangle, the area of which can be calculated, and consequently also the spherical excess E (see formula, page 49, Chapter I.); the sum of the inward angles in this case should be equal to $(N-2)\times 180^\circ+E$.

The horizontal angles observed having thus been reduced to corrected means and verified, they are used in the computation of the sides of the triangles to be

plotted on the plan; the computation starting with the triangles subtending the base line, in each of which three angles and one side are given to find the other two sides, and continuing on through the whole series of triangulation up to the bases of verification, the lengths of which are thus calculated and compared with the reduced measurements. Any very small errors may be neglected; but if they happen to be moderately small, they are generally thrown in or adjusted in the plotting at the portions or points of survey that show traces of weakness; still more important error may be distributed in the plotting by adjustment to an exaggerated or a diminished scale of the whole of the triangulation, so that the error may nowhere in the survey exceed a fixed limit.

The trigonometrical calculations are effected for plane triangles with the formulæ given at page 47, Chapter I., and for spherical triangles in a corresponding way after applying the reduction of Legendre, or reducing each angle by one-third of the spherical excess calculated for each such triangle, as mentioned at page 49, and exemplified at pages 57, in Chapter I. All reductions on account of satellite or supplementary stations of observation must be made previous to these calculations, so that cleared results alone have to be dealt with. Such reductions are made with the formula given on page 52, and exemplified at page 60, Chapter I.

A certain amount of verification of certain sides of triangles may be sometimes obtained by computing them through independent sets of triangles.

The main triangulation being then plotted finally, and ready to receive the secondary triangulation and filling in of detail, the azimuths of all the lines of the

main triangulation may be calculated from that determined for the base line, and recorded for use and reference in that part of the work. The azimuths of any required lines between any two stations of main triangulation can also be calculated, and any latitudes or longitudes required may also be computed. The formulæ for this purpose are given at page 54, and exemplified at page 65, Chapter I.

Secondary Triangulation.—When the main triangulation is so extensive, and the sides so long that the detailed filling in of the survey cannot be directly applied to it without appreciable error, a series of secondary or smaller triangles becomes necessary. This subsidiary network of triangulation is dependent on the former; the limit allowed to the length of sides admissible in it is dependent on the mode and the accuracy with which the detail survey is to be made, and the necessities of the latter. The principles adopted in the secondary triangulation are those of the main triangulation; the survey points used are chosen on the same principles of convenience of inter-observation, many of them being ranged in as intermediate points on the survey lines of the main triangulation, and their distances apart measured to serve as sets of secondary base lines for the subsidiary network; the remaining points are selected in the interior of the main triangles so as to form a series of well-conditioned secondary triangles, and to afford convenience and reduce labour in filling in the detail.

In small surveys secondary triangulation may often be entirely dispensed with.

Filling in detail.—The whole of the detail of the survey—roads, hedges, buildings, streams, etc., is fitted on to the foregoing triangulation; in small surveys it is

more generally effected by direct chainage, sometimes by chained triangulation, and sometimes merely by chained distances and offsets, on the principles already explained in the section on chain surveys. It is, however, preferable to adopt telemetrical observation for a great part of such work on account of its rapidity, and compass bearings may be used to save labour in the less important details.

In large surveys the filling in may be effected either by chain or telemeter work, or by sextant survey, when accuracy is necessary; but whenever the detail may be coarsely done, as for plans or maps on a small scale which are incapable of showing minor detail, this part of the work may be done by compass or by plane-table. The modes of conducting all survey work of these descriptions have already been treated under their respective heads in the foregoing sections of this chapter.

As it is important in ordinary theodolite surveys to avoid loss of time, the triangulation and the detailed survey should proceed nearly simultaneously, and in this case, as the fitting of the plotting of the latter to that of the former requires accuracy, it is necessary that all points of triangulation should have their position relative to surrounding objects carefully measured in a way that admits of no doubt of their exact location. If, however, the triangulation is finished before the detail survey commences, the survey points of the former are allowed to remain visible for the purposes of the latter, which then closes on them.

The detailed survey generally gives a certain amount of written information as to names of places, streets, mansions, rivers, and so forth; information as to imaginary boundary lines of parishes, districts, counties, and




Theodolite triangulation 1859.

provinces must be obtained from the local authorities and entered on the final plan.

In maps or plans on a small scale it is usual to represent roads and railways by double lines and thick lines of any convenient exaggerated breadth, which bears some definite ratio to the actual breadths shown in the original plotting of the details.

The principles of theodolite triangulation are illustrated in the reduced plan, Plate V., where they are applied to a district about four miles long and two miles wide, or nearly nine square miles; the base line is about a mile in length. No secondary set of triangles was used, traversing along all the roads and closing on the main stations having been carried out as a preferable alternative in the secondary part of the survey; the smaller detail was put in with offsets and compass bearings.

Traverse Surveys.—The method of traversing has already been explained in Section I of this chapter, and its application in surveying of other sorts in the succeeding sections. Short portions of traverse work may be carried out with the theodolite wherever the filling in of detail with precision happens to be necessary in parts of triangulation surveys; but when the greater part of any survey is conducted on the principle of traversing rather than of triangulation, its object of operation is confined to a long strip of country of comparatively small breadth, seldom exceeding a quarter of a mile. Surveys of this description are rarely made for other purposes than those of engineering, and more especially the intended location of long lines of communication, roads, railways, or canals, or of conduits and pipes of water supply or drainage; they may hence be generally treated as

engineering surveys, conducted within certain limits of deviation suitable to the proposed works.

The main lines of traverse falling generally within these fixed limits of deviation, the traverse stations are fixed either within or close to these limits, as the survey work proceeds, in accordance with general convenience, and the visibility of neighbouring stations, and lateral objects of observation; the number of stations in a traverse should, apart from these considerations, be as few as possible, as this diminishes the number of angular observations, and hence also the amount of error.

In large traverse surveys of this description there is seldom any opportunity for closing the polygon of main traverse by returning to its starting point, without necessitating a double amount of work, a serious consideration in extended surveys; and sometimes, also, there is no opportunity of closing on points of known position used in former surveys; hence the need of verifying the work fully by sufficient angular observation on lateral objects, spires, towers, buildings, etc. At each traverse station both the inward and the outward angle are severally observed and recorded, and their sum 360° affords a check against inaccuracy of reading. The true bearing of the starting line of traverse is generally obtained with the help of the compass, although it is better to fix a true meridian and observe from it throughout the survey; the bearings of all the remaining lines of traverse will then not require successive computation. The main traverse may either be plotted by assuming a direction for any starting line, scaling distances, and protracting the inward angles, or by scaling distances and protracting the bearings. Both of these methods, however, being liable to undiscoverable errors or mistakes, the mode of

Form for Reduction of a Theodolite Traverse to a single Meridian.

Stations	Distances	Inward Angles	Theod. limb-bearing	Reduced Bearing	Successive Co-ordinates N	S	E	W	Independent Co-ordinates Abscissæ	Ordinates
A	408	— —	281° 40′	78° 20′ NW.	82·5	—	—	399·5	—	—
B	685	65° 25′	167 05	12 55 SE.	—	665·7	152·7	—	82·5 N.	399·5 W.
C	795	223 11	210 16	30 16 SW.	—	686·6	—	400·6	583·2 S.	246·8 W.
D	580	42 14	72 30	72 30 NE.	177·4	—	553·2	—	1269·8 S.	647·4 W.
E	483	242 11	134 41	45 19 SE.	—	339·5	343·4	—	1092·4 S.	94·2 W.
F	1012	73 05	27 46	27 46 NE.	895·5	—	471·5	—	1431·9 S.	249·2 E.
G	830	119 09	326 55	33 05 NW.	687·4	—	—	465·1	536·4 S.	720·7 E.
H	297	92 31	239 26	59 26 SW.	—	151·0	—	255·6	151·0 N.	255·6 E.
A	—	222 14	281 40	78 20 NW.	—	—	—	—	0	0
Sum	.	1080°	.	.	.	.	1842·8	—	1520·8	—
							—	1842·8	—	1520·8

reducing the whole to a single meridian and plotting them by rectangular co-ordinates is to be recommended for accuracy and certainty.

The accompanying form for the reduction of a short piece of theodolite-traverse to a series of rectangular co ordinates may be used as an example.

The reduction is thus computed. Starting with two sets of data, the distances and the checked inward angles, and one given bearing.

1st. To compute the theodolite limb-bearings throughout,

Let P be the given bearing from any preceding station,

a be the inward angle at the actual station,

F the required limb-bearing to the following station,

Then $F = P + a \pm 180°$; using the positive sign when $P + a \angle 180°$, and the negative sign when $P + a \rangle 180°$.

2nd. To obtain the reduced bearings, or angles formed with the meridian either from North or South.

When the theodolite limb-bearing F lies between 0° and 90°, the corresponding reduced bearing is F to the NE.; when

between 90° and 180°, it is $180° - F$ to the SE.; when

between 180° and 270°, it is $F - 180°$ to the SW.; and when

between 270° and 360°, it is $360° - F$ to the NW.

3rd. To obtain the successive co-ordinates for each point from the preceding point.

The abscissa along the meridian = distance × cosin reduced bearing.

The ordinate to the E. or W. = distance × sin reduced bearing.

These quantities may be obtained direct from traverse tables or with the help of tables of sines and cosines as may be preferred.

4th. The final independent co-ordinates for any point, taken with reference to the starting point of the traverse, are obtained by summing up the successive co-ordinates up to that point, treating them as positive or negative in accordance with their signs of direction.

The formula for checking the sum of the inward angles of any closed traverse is $(N-2) \times 180°$, as explained in the collection of formulæ, at p. 48, Chapter I.

By adopting the above method of reduction, which affords a check on itself, the plotting can be afterwards effected by co-ordinates with the minimum of error.

Secondary triangles.—The above main traverse-work is of course the mere skeleton of the survey; and as it frequently happens that these surveys are very narrow, being often confined to a mere road, and that the detail required consists merely in frontages of property and minor detail close to the road; in such a case the detail is surveyed simultaneously with the traverse by the method of distances and offsets. When the limits of deviation are of pronounced width, and the detail within them cannot be very satisfactorily managed by chainings and offsets, aided by compass bearings, recourse must be had to a series of secondary triangles, which may be conveniently arranged by measurement to intermediate points along the main lines of traverse; at these secondary points, angles are taken to lateral points near the intended limits of deviation on each side, and check

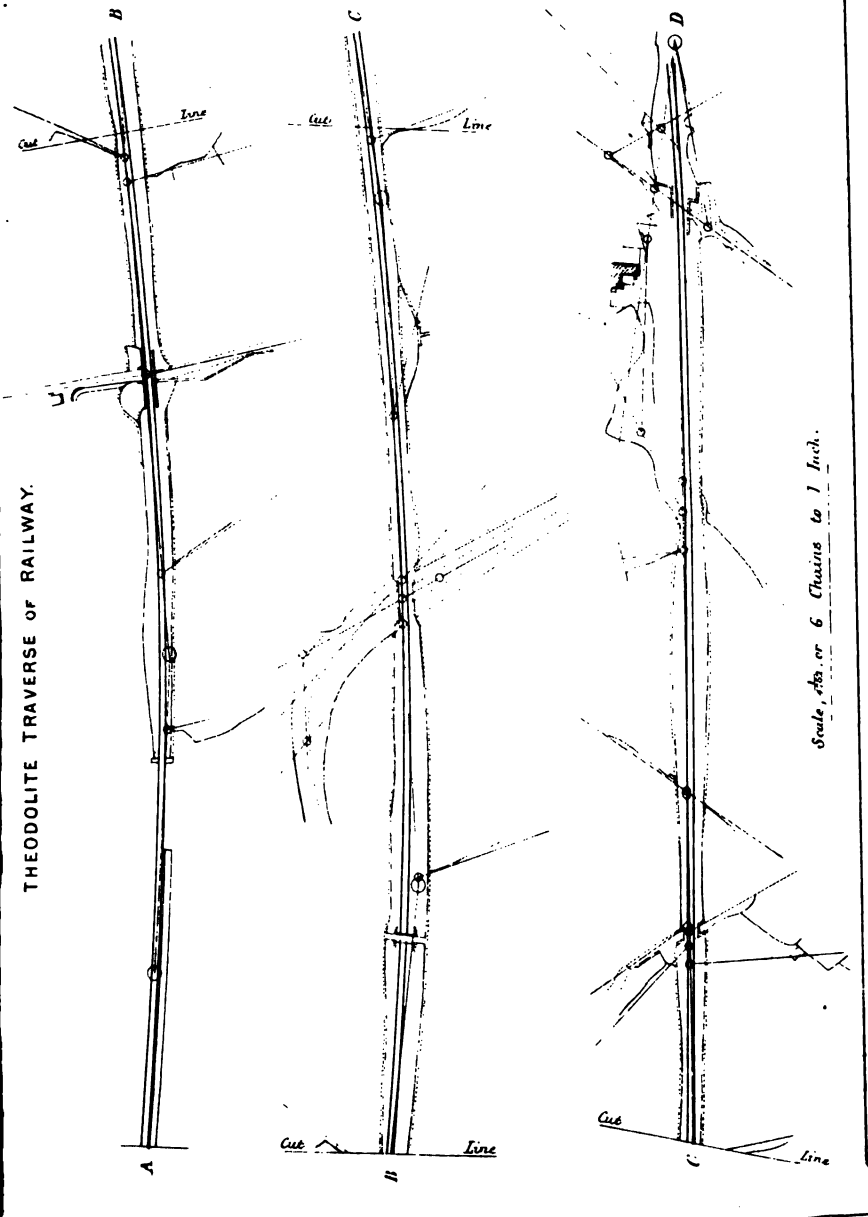
angles are also observed at the lateral points. For this purpose the box sextant is useful.

The filling in, or detail.—This is applied either to the main traverse-work or to the secondary triangulation before mentioned, or perhaps to a secondary set of traverse-lines. The amount of such work in traverse surveys is comparatively small, and generally occurs in detached pieces at places where the construction of some special work of art or building is contemplated. It is, however, generally required with much precision, hence the compass-survey work and plane-table-work suited to subsequent reduction for small-scale plans is not often fitted for this object; and the more accurate results of chain-survey or good telemeter-survey assisted by occasional compass bearings are better for these purposes.

Plate VI., with the corresponding detailed field-book, No. IV., is an illustration of applying the method of traversing with the theodolite to an engineering survey of a portion of railway about one mile in length.

The various examples of field-books actually recorded in the field, and the illustrative field-sketches or reduced plans given in the Plates, are intended to serve for the guidance of those newly entering on the subject.

THEODOLITE TRAVERSE OF RAILWAY.



LONDON, CROSBY LOCKWOOD & CO^R 7, STATIONERS HALL COURT, LUDGATE HILL.



PART II.

LEVELLING.

Section I. INSTRUMENTS AND APPLIANCES.

LEVELLING is that branch of Geodesy which is limited to operations for determining and measuring the relative heights and depths, or elevations of places, points, and objects on and comparatively near the earth's surface.

Absolute elevation of points or places is estimated in linear units (in England, in English feet) above or below mean sea-level; relative elevation is estimated in the same way with reference to any convenient datum level, such as that of Trinity high-water mark, the zero of the Nilometer, the plinth of the nearest town-hall or church, the rail-level at the middle of the platform of the nearest railway-station, or any arbitrary visible or imaginary point. A bench-mark is a subordinate datum-point, marked on some permanent common object, such as a kerbstone, milestone, plinth, stone step, lower hook of gatepost, etc., whose elevation above some datum level has been carefully determined and is known; bench-marks made on surfaces or objects not liable to wear or to change, are useful for reference and saving the relevelling of long distances.

Any set of relative levels of a series of points may be recorded in either of the following ways.

1. In a book, with accompanying descriptions of the points; as for example, in a list of bench marks, or in a level-book.

2. On a plan or map, the exact points being marked and the levels recorded close to them.

3. On a plan, by a series of contour lines, or lines of equal elevation, whose levels are recorded; so that the level of any intermediate point may be approximately determined by interpolation.

4. On a section, or series of sections, taken in various directions, showing the relative heights to scale of any points in that section. A plan is frequently necessary as a guide to show the direction or course of the sections, which may either be straight or circuitous in any direction, while again the section may comprehend levels of two or more sets of objects, and these necessitate reference to more than one plan.

A section generally not only shows by scale the position of various detached points in a vertical plane with reference to some convenient datum level or datum line, but also the position of continuous objects or surfaces, such as ground surfaces, water levels, lines of pipes, road levels, rail levels, edges of strata, mine adits, galleries, shafts, tunnels, bed levels, formation levels, etc., according to the special purposes for which it is intended.

The actual datum level need not occur in the section—any horizontal line of known level may be used for convenience in scaling the relative heights; these latter are frequently also recorded in figures.

Sections being generally short in comparison with a great circle of the globe, horizontal levels and surfaces are represented by straight horizontal lines, and curvature is neglected; the heights and depths indicated on

sections being also small in comparison both with the lengths of the sections and with the radius of the sphere, natural sections, in which the same scale is used both for horizontal distances and vertical heights, are less often used than exaggerated sections, in which the vertical heights are on an exaggerated scale, and differences of height more easily distinguished.

As a general rule, it is most convenient to exaggerate sections 10 times, that is, with a horizontal scale of $\frac{1}{4800}$th, a vertical scale of $\frac{1}{480}$th could be conveniently used to indicate variation of surface; special circumstances may, however, require any amount of exaggeration. For Parliamentary sections of proposed public works, two vertical sets of scales are fixed as the smallest allowed: the one is $\frac{1}{1200}$th. or 100 feet to an inch, in combination with a horizontal scale of not less than 20 chains to the inch, for longitudinal sections; the other, $\frac{1}{480}$th, or 40 feet to an inch, combined with a horizontal scale of 5 chains to the inch, for cross sections showing alterations; but they may for convenience be used in combination with any horizontal scale used in the plan accompanying them. Engineers' working sections are drawn to larger vertical scales; and in natural sections taken at the sites of intended large works in masonry, iron, and timber, the scales used are still larger.

Instruments and Appliances used in Levelling.—Apart from the instruments and appliances used in preliminary surveying, and in the measurement of distances required for levelling (which have been already referred to in Chapter I.), the following are more specially used in the operations of levelling, or the measurement or determination of relative heights and depths.

1st. For direct measurement of height or depth: the surveyor's chain and tape before mentioned, with plummet attached; sounding-cords or lead-lines for depths exceeding 100 feet; permanent and temporary tide-gauges and sounding-rods; graduated level-staves, and sliding vane-staves of various patterns.

2. Angular instruments: the sextant, clinometer, and theodolite.

3. Boning staves.

4. The water level.

5. Spirit-levelling instruments of various patterns.

6. Reflecting levels of various patterns.

7. The barometer.

8. The thermometer, or hypsometer.

Appliances for direct measurement should in the first place be correct to standard; failing that, all such various appliances as may be used in any one set of levelling operations should be correct by inter-comparison, so that one percentage of error alone may have to be applied; or at the worst, all discrepancies should be known.

The special appliances intended for soundings and subaqueous depths should be invariably kept for those purposes; for however convenient it may occasionally be to the surveyor to use his chain or the level-staff as a sounding-chain or sounding-rod, it must be remembered that he may thus rust the former or distort the latter.

In direct measurement of lofty heights, several chains may be used, firmly bound together, and the level-staff in addition; care should then be taken by those managing the upper end that it may not slip away, as its weight, directed by accidental bias, might cause severe injury to those below. Cords and tapes

being liable to stretch and shrink, require frequent verification. Wooden level-staves and vane-staves are liable to warping and distortion. Telescopic level-staves, though convenient for transport, are liable to stick, and thus become useless for a time ; they are also liable to slide gradually : this, when it occurs in the middle of a series of observations, and is unperceived by the staff-holder, may vitiate an unknown number of them. To guard against this contingency, the author has sometimes had a large-headed screw applied to the catch when the staff is extended, which retains it in position, and can be unscrewed on closing the staff. In very accurate levelling operations, telescopic staves should not be used : 10-foot staves, of equal breadth and thickness, with an arrangement for fixing one above another when required, are preferable, as errors are reduced, and the readings and graduations are more distinctly visible to a greater distance. Papered level-staves are seldom very correct, as the paper stretches on application to the staff ; painted staves are to be preferred generally. But as surveyors like to use graduations and marks to which they are accustomed, and frequently keep their own level-papers and apply them themselves, the former thus possess advantages in this respect.

As to patterns for graduation of level-staves, they are exceedingly various. The following three patterns, for use in short, intermediate, or long sights respectively, were devised by the author in 1861, a few being then made for the Government of India. The angular figures are intended to enable the graduated bar-scale to be done away with, the middle of each figure being the exact point of reading for tenths of a foot, the angularities indicating hundredths.

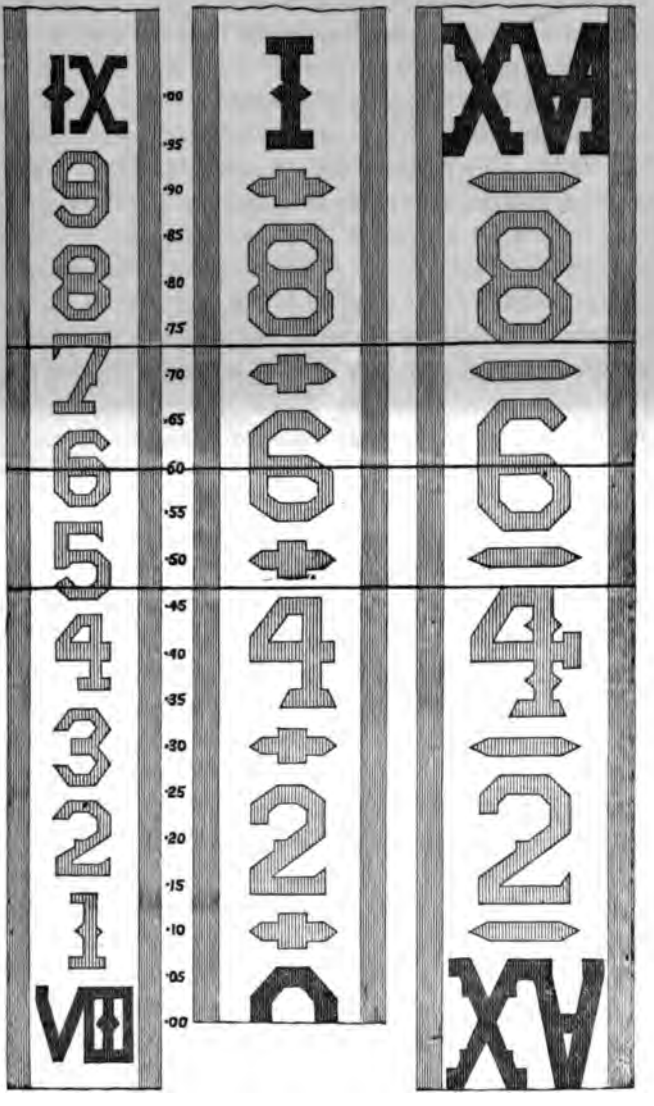


FIGURE 20. PATTERNS OF LEVEL STAVES.

These patterns possess some advantages in distinctness and visibility at a distance, but, like all others, they require examination before commencing their use. Level-staves for use in any accurate work should always be provided with a plummet, to indicate any want of verticality.

Vane-staves were in general use before Gravatt introduced graduated staves and the Dumpy level. The sliding vane having to be repeatedly fixed to some exact points, in accordance with directions given from a distance, is a tedious and inexact process; they are now used only for special purposes.

Tide gauges, whether temporary posts or permanent pillars, with or without self-registering mechanism, generally require stays or very solid foundations, to prevent departure from a vertical position. For temporary purposes, where the rise and fall is great, it is convenient to have a set of tide-posts in preference to a single one at any important point of reference.

The lead-line or sounding-cord, made of tough hard material, is generally marked to fathoms and read to quarter-fathoms for nautical purposes, or marked to 5 feet and read to feet and tenths for engineering purposes with the aid of a foot-rule or staff; it is loaded with a conical lead weight, which may be armed with tallow in the hollow of its base, to bring up specimens of the material or soil at the point of sounding; a jagged plunger is sometimes also needed for that purpose on firm bottoms. The cord should daily during work be tested by comparison with a correct measure, both before and after use, whenever accuracy of depth is important.

2. *Angular instruments* are comparatively seldom used in levelling, or for obtaining relative heights of

survey stations; first, for the reason that the distances apart of the latter are large in comparison with their heights, and the vertical angle, being therefore small, affords but a very coarse means of estimating the height, as may be seen from a consideration of the following specially arranged small table of tangents;

TABLE OF TANGENTS TO A RADIUS OF UNITY.

Tangent	Angle	Tangent	Angle
0·01	0° 34′ 23″	0·10	5° 42′ 38″
0·02	1 8 54	0·15	8 31 51
0·03	1 43 6	0·20	11 18 36
0·04	2 17 36	0·25	14 2 10
0·05	2 51 45	0·30	16 41 57
0·06	3 26 1	0·35	19 17 24
0·07	4 0 11	0·40	21 48 5
0·08	4 34 26	0·45	24 13 40
0·09	5 8 34	0·50	26 33 54

whence it appears that in distances of 2 miles an inaccuracy of one minute may represent an error of 3 feet—an amount totally inadmissible; secondly, our incapability of accurately determining the effect of refraction, which varies so much with the state of the atmosphere, prevents us from obtaining low reduced angles sufficiently correct for practical purposes. There is only one mode of eliminating the latter of these two sources of inaccuracy, and that is by observing reciprocal vertical angles simultaneously. One observer takes the angle of altitude of the second station from the first at the moment when another observer takes the corresponding angle of depression of the first station from the second. This requires preconcerted arrangement as to time, which may be assisted by sunflash signalling. This practice nearly neutralises the effects of curvature and refraction. The difference of level

required is equal to the product of the distance and the tangent of the mean of the two reciprocal angles. Instrumental index error has great effect in these angles, and should be very frequently noted. Such angles may either be taken with the theodolite or with the sextant or reflecting circle, aided by the artificial horizon, which give the double altitude. The first-mentioned source of inaccuracy can only be reduced by limiting such observations to comparatively short distances, and using instruments having large vertical arcs and sensitive levels. For common purposes of determining heights of buildings, towers, etc., in a rough way, when direct measurement is impossible or inconvenient, vertical angles may be employed; the distance of the point of observation is then generally so chosen as to make the angle nearly 45°, as such angles afford nearly double the accuracy of result through their tangents that can be obtained with small angles less than 5°. If the angle can be set at 45°, and the horizontal distance chosen at will, the height and the distance are then equal, and the distance is measured.

The clinometer is a small instrument, having two arms turning on a joint having a graduated arc, and a small spirit-level in one of them, to enable it to be set horizontal; the other arm, when sighted to any point, gives the vertical angle. It is suited to the purposes of roughly determining inclinations and slopes, or of setting out side-slopes on works of construction.

3. *Boning staves.*—A pair of T-headed wooden staves of equal length, generally about 5 feet; they are used within short distances for setting out a third point from two given points on the same level or on a given inclination, and afford a means of levelling in

the line or direction of the two given points as far as the ground allows; they are extremely useful for obtaining intermediate level points about foundations of buildings and public works.

4. *The water level.*—This is also an appliance for rough levelling within short distances, but requires only one point given. It consists of a horizontal tube about two feet long terminated by two bottle-shaped ends, in which water will stand level and thus afford a horizontal line of sight; the instrument turns on a pivot and is supported on a rough stand; the water is colored for convenience in sighting; in frosty weather coarse spirit may be used for the same purpose. This instrument is generally used for the same purposes as the last; it is also very useful in contour-work for rapidly setting out intermediate points between others determined with a more accurate level instrument.

5. *Levels, or telescopic spirit-levelling instruments.*—The main portion of all levelling operations for surveyors' or engineers' purposes is done with these instruments. The type of instrument most in favor now is the Gravatt level, which has a telescope of large clear field with a long spirit-level attached, and is used with graduated level-staves, on which observations are made at moderately long distances; the lines of sight being invariably horizontal. This instrument is so common that any description of it, or of the mode of handling it, is unnecessary; it is easily set up, and worked by thumb-screws until the telescope is shown to be level in any position by the indication of the bubble of the spirit-level; the sights on the distant level-staff are taken with a horizontal wire in the diaphragm of the telescope, the reading recorded, nad the bubble observed to

see whether it has remained steady in the middle; if it has moved, as may be the case in windy weather, or on soft ground, the operation must be repeated.

The adjustments of this instrument are only two in number. As the position of the horizontal crosswire is fixed, and the telescope tube cannot be rolled round, hence the line of collimation cannot be adjusted; the two possible adjustments are for level error, and for the attachment of the level to the vertical axis of the instrument.

1st. For level error.—Whether the line of collimation is slightly incorrect or not is of comparatively little moment; but the level must invariably be set parallel to the line of collimation, whether the latter is in the exact axis of the telescope or not: the setting is effected by moving the capstan-headed screws of attachment until it is so. To test error on this account, set up two equal and equally graduated level-staves about 500 feet apart, and have them held truly vertical with plumb-lines; set up the level halfway between them and take sights and readings on each of them, taking also special care that the bubble is in the centre of its run in each case, even if it requires resetting; the difference of these readings will then be correct. Now set up the level beyond either staff and take a second pair of sights and readings; if the resulting difference is not exactly equal to the former, the capstan-headed screws require moving to be set right, when the correct difference will be shown; the operation should then be repeated until the adjustment is perfect.

This test is more conveniently effected if a sheet of water is available, and the level-staves can then be held or supported on pegs flush to the water level, and the

readings should then be identical in any case; if not, as before, first correct half the error by the plate screws and then the remainder by the capstan-headed screw.

2nd. The error from unadjusted attachment to the vertical axis.—The former adjustment being carried out, the instrument should reverse well, that is, when levelled the bubble should remain steady in the centre of its run, however the telescope may be turned. If it does not do so, set the telescope in such a position as to show the utmost amount of error in position of the bubble; correct half the error by the plate screws, and then correct the remaining half by the special screws that affect the attachment.

In testing the correctness of levels by sighting on pairs of staves, the latter should be held in the same amount of light, that is the direction from one to the other should be at right angles to the direction of the sun.

Levels of other patterns.—Troughton's level is a variation on Gravatt's; the principal difference being that the level is permanently fixed to the telescope, and cannot be adjusted, but the horizontal crosswire can be moved by screws until the line of collimation is set parallel to the level; this adjustment therefore in this instrument takes the place of that for level error just described, and can be tested in the same way.

The Y level, on the contrary, is an older pattern and is now generally used of small size, having a small telescope and short spirit-level; it admits of every sort of adjustment, and frequently requires it. In this instrument the level hangs below the telescope, which rests on Y's or forks, one of which is moveable by a large screw. The adjustment for level error can be tested by lifting

the telescope out of its forks and reversing it end for end; the adjustment of attachment to the vertical axis is easily effected by a movement of the large screw attached to one fork; and the adjustment of the cross-wires can be effected by screws, while the coincidence of the line of collimation with the axis of the telescope can be tested by rolling the telescope round in its forks between two observations on a distant object; as previously explained for the Y theodolite.

The larger levels made on this pattern were sufficiently good for the times when people seemed indifferent whether their drains ran uphill or downhill, but they are now used in small sizes only for rough work, such as foundations of buildings. The inherent defect of the pattern being the shortness of its spirit-level, similar large instruments would be heavy.

An improvement on Gravatt's level is sometimes used, which has a moveable inner telescope-tube and admits of testing and adjustment of the line of collimation.

The theodolite can be used, with its vertical arc clamped, in the same way as a level; but it is heavy. Some Austrian universal instruments are very light theodolites intended also for use as levels in this manner; they mostly suffer from the defect of having the telescopes too small for levelling operations of an extended nature; but their lightness of make and portability present an advantage that might well be partially imitated by English instrument-makers.

There is also a type of level distinct from the above, that admits of taking sights on inclinations; in some, the telescope turns in a vertical plane about its middle, to which an index is attached at right angles reading

on a graduated vertical arc; in others the telescope turns in a vertical arc on a hinge near one extremity, the index at the other extremity reading on a vertical arc of larger radius. They are useful in moderately hilly country or for rather rapid inclinations, but necessarily lack the steadiness and simplicity of instruments of the former type, and are not so well suited for work in level country.

There is also a combination of the level with a reflecting circle invented by Professor Piazzi Smyth, that has some special advantages.

6. *Reflecting Levels.*—These are generally very small instruments without telescopes, intended for rough use in short distances, and generally used in very steep country for interpolating between contour lines, and setting out gradients of hill-roads and paths.

The simplest form of such instruments consists of a half-silvered small mirror attached to a ring and weighted below to keep it vertical in windy weather; it is used either suspended on the finger or on a light staff of known length; the horizontal edge of the silvering acts as the horizontal wire of a diaphragm for sighting on a graduated level-staff, or on a vane staff, the reflection of half the eye being seen in the mirror when the sight is correctly taken.

Another form is the tube reflecting-level; this consists of a small brass tube having a small spirit-level attached, and having a small mirror covering half of one end of the tube; on taking a horizontal sight on a level-staff the round bubble of the level is reflected in the mirror, or appears to pass the horizontal edge of the silvering. The attachment of the level to the tube admits of adjustment by screws.

Each of these instruments should be tested before use, especially the latter, which has not the advantage that the former possesses, of affording a steady sight. The principle adopted in the tube reflecting-level has been applied in larger telescopic levels, and is more satisfactory in instruments with stands; the size of the bubble in a spirit-level may be increased or diminished as required from air in a reserve chamber at one end of its tube specially constructed for the purpose; this is a great advantage in hot countries where the bubble's length may contract to one-half in an hour.

7. *The Barometer* is used for taking flying levels, more especially in hilly country; a pair of tolerably large aneroids, reading clearly to hundredths of an inch, are best for the purpose. The correction for temperature can be either calculated in accordance with the indications of a thermometer, and by comparison with a standard mercurial barometer, or the instruments may be self-compensating for change of temperature; the former method is preferable unless the self-compensation has been thoroughly verified by the observer. The testing and verification should be made both before and after any series of observations, and the correction for temperature suited to each instrument recorded for future reference. Two barometers are necessary for practical work in determining heights; after comparison one is kept at a fixed spot throughout the day and its variations recorded every quarter of an hour during the time that the other is in use at neighbouring points where its readings are recorded. Another method is to take simultaneous observations with two aneroids along a series of stations within signal of each other; thus obtaining a series of relative elevations.

The difference of barometric reading due to increase of height is nearly 0·01 inch less for every 10 feet of increased elevation above mean sea-level, at a permanent standard temperature of 55° Fahrenheit. The following scale is calculated according to the formula

$$h = 55000 \frac{30'' - b}{30'' + b},$$

where b is the barometrical reading at the upper station in inches, and h is the elevation in feet; the values are arranged to admit of interpolation.

b	h	b	h	b	h
31·00	—900	28·00	1900	26·51	3400
30·00	Zero	27·93	2000	26·41	3500
29·03	900	27·49	2400	26·03	3900
28·93	1000	27·39	2500	25·93	4000
28·51	1400	27·00	2900		
28·41	1500	26·90	3000		

For any other permanent temperature (t) at any two stations, where the barometric readings are B and b, the Schuklburgh scale for thermometric variation of the height of a column of air, gives the formula,

$$h = 27470 \left(1 + \frac{t - 55°}{440}\right) \times \log \frac{B}{b}$$

in which, $\log \frac{B}{b} = 2 \left(\frac{B-b}{B+b}\right)$ very nearly.

But when the temperature of the air is different at the two stations,

if B is the barometrical reading at the lower station,

b is the barometrical reading at the upper station,

T the temperature at the lower, t at the upper, the ordinary formula is

$$h = 60360 \times \log \frac{B}{b} \times \left\{1 + 0{\cdot}001014 (T + t - 64°)\right\}$$

But this formula will not by any means apply correctly to *any* aneroid barometer; the correction for each instrument should be determined by experiment and comparison with a good standard.

Elevations under 5000 feet can be determined with aneroids within 10 or even 5 feet, if pairs of instruments be used simultaneously: with instruments used independently the errors may be as large as 30 feet.

8. *The Hypsometer* is a very sensitive thermometer for determining the temperature of pure water when boiling, and thence calculating the elevation of the place above that where the same hypsometer would indicate a boiling point of 212° Fahrenheit; the temperature falls one degree Fahrenheit for about 540 feet of increased elevation.

If D is the depression of temperature of the boiling point in degrees Fahrenheit, h the elevation in feet,

$$h = 517\ D + D^2$$

This instrument is suited to determining elevations above sea-level of spots in mountainous countries in travellers' explorations.

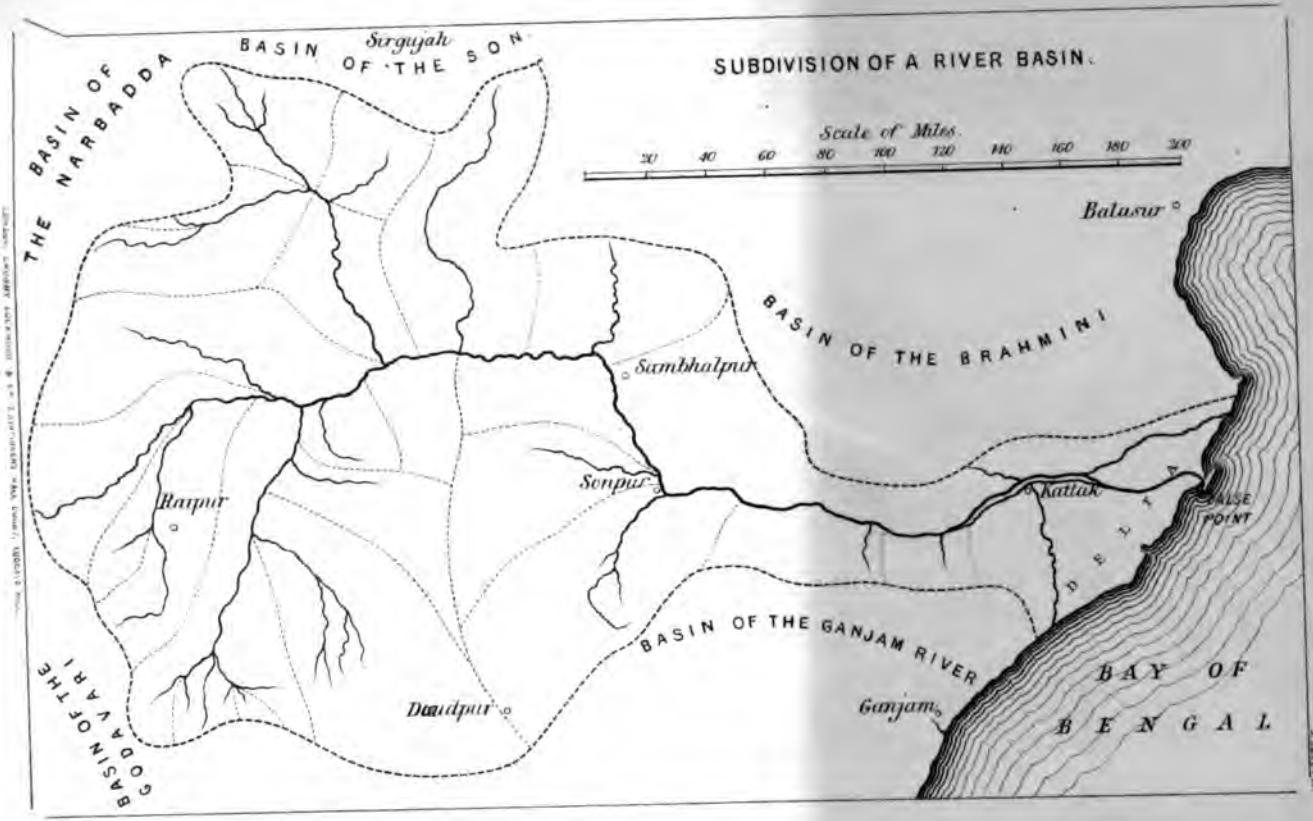
A selection from some hypsometrical observations taken in Thibet will be found in field record No. VII.; they apply to the stations of the route-survey of field record No. VI. shown in reduced plan Plate XI.

Formulæ and Calculations.

The calculations in connection with levelling are generally of a very simple nature; they mostly consist in the addition of lengthy columns of level-readings by various methods that verify the results; examples of the two more common modes of doing this, the Rise-and-Fall, and the Collimation-sight method, are given in the

Forms of Record, or patterns of level-book, following. The formulæ required are very few, and are hence given with the account of the instrument and method to which they more specially apply.

Demarcation of level-points. (See Demarcation of Survey-points, Chapter I., pages 67 to 69).—Temporary points may be marked with whites; more permanent level points by single square pegs, to distinguish them from round survey pegs; and specially important level points by two square pegs near each other, so driven that their heads are level. Such pegs should be nearly flush with the ground. Flat-headed iron pins may be used for the same purpose. Bench-marks are generally chiselled on some permanent stone slab, pillar, or wall; the usual form of mark for them is  . The upper surfaces of portions of permanent ironwork, rails, flanges of girders, parts of gates, are also made to serve the same purpose, some cross-mark being generally dinted with a chisel to mark the point; in every case a sketch and full description of the position of the bench-mark, both with regard to surrounding objects and the parts of the structure on which it is made, should be recorded. More time has perhaps been wasted on account of want of sufficient precaution in this respect than from any other sort of neglect in operations of this class. It may also be noted that giving insufficient information about bench-marks, or marking them insufficiently or doubtfully for others to work from, has always been a fruitful source of error.


SUBDIVISION OF A RIVER BASIN.
Scale of Miles.
20 40 60 80 100 120 140 160 180 200
BASIN OF THE NARBADDA
BASIN OF THE SON
Surgujah
Batasur
BASIN OF THE BRAHMINI
Sambhalpur
Sonpur
Raipur
Katlak
DELTA
FALSE POINT
BASIN OF THE GANJAM RIVER
BAY OF BENGAL
Ganjam
Doudpur
BASIN OF THE GODAVARI

Section 2. CLASSIFICATION OF LEVELLING OPERATIONS.

The operations of levelling may, like surveying operations, be classified according to the principal instrument chiefly used; such an arrangement would not however be very practical, as the greater portion of levelling operations conducted by Englishmen is now generally carried out with levelling instruments of the Gravatt type; it is more convenient for this as well as other reasons to treat the subject with reference to the various intended objects and the scope of operation; and to classify the modes of operation in accordance with the nature of the ground over which the work proceeds.

The more usual objects are:

1. To obtain levels or relative heights of the principal natural features of an extent of country.

2. To obtain sufficient levels over a portion of country for producing a perfectly contoured plan of it, from which the level of any point may be approximately deduced.

3. To obtain a simple chain of distant bench-marks along a line of country for subsequent reference.

4. To take a series of detached or flying levels with a small amount of labour which may serve as a guide in preliminary designs of works of communication.

5. To take such levels in any long strip of country, within previously determined lateral limits, as may sufficiently serve for the purposes of detailed designs of works of communication.

1. *The natural features of a country* are made up of its distinctive varieties of surface, which are most strongly

pronounced and indicated at various points occurring in its series of primary and subsidiary ridges and valleys, or watershed and watercourse lines. This double or combined system of lines constitutes a method of naturally subdividing the surface of any portion of the earth, however large or small. Commencing with the largest continents; these may be naturally subdivided into a series of extensive catchment areas, or river basins, within each of which all rainfall follows an independent course to the sea. The limits of these catchments are main watershed lines or ridges, sometimes marked by mountain ranges and sometimes hardly distinguishable to the naked eye on account of the flatness of some plateau, but in all cases definite lines. These extensive catchments again can be subdivided into a series of secondary catchments, whose limits consist of portions of the main watershed lines and secondary ridges branching from the former. This process of successive subdivision of catchment areas may be continued until the resulting portions become as small as English combes, and possibly again until the parts become microscopic, the limiting lines being always watersheds, main or subsidiary, or both in various orders, primary, secondary, tertiary, and so forth. Commencing afresh with the large rivers that discharge into the sea from any continent; the main courses of these are the main watercourse lines of the continent; their tributaries are the secondary watercourse lines branching off from the former, the affluents of the tributaries form the tertiary watercourse lines, and the subdivision may be carried on by these ramifications until the last watercourse lines are merely excessively small rills in which water may only occasionally run in minute quantity. The sub-

division of catchment areas is illustrated in Plate VII. The points of divergence of the watershed lines, the points of convergence of the watercourse lines, the points of marked change of inclination occurring both along the watershed and watercourse lines, such as peaks, passes, and hollows, and the levels of table-lands, plains, and lakes, are thus the points that most distinctively indicate the natural features of the country. The principal or most marked among these are the points whose levels are observed in order to carry out the object first mentioned. The position of these points is determined by surveying operations, although some additional measurements are sometimes needed in conjunction with the levelling.

2. *Contours*, or contour lines, are lines of equal relative height on the earth's surface; the edge of a lake is an instance of a natural contour line, and the edges of imaginary lakes having various successive depths of water of equal decrement represent the artificial contour lines conventionally used in plans to indicate detail in form of surface. The difference of elevation between any two contours on a plan is a constant quantity, generally some round number in feet, fixed in accordance with the declivities of the country and the scale of the plan. Contour lines are almost invariably at right angles both to the principal watershed lines and to the principal watercourse lines wherever they exactly happen to cross them, the exceptions being where they become coincident with them, as at very sharply-defined ridges or hollows of equal height or equal depth. See Plate VIII.

3. *Lines of distant bench-marks.*—These are generally undertaken by the national survey department of the country, and executed with superior instruments and great

precision for purposes of public utility and reference. In very extended open country the course followed is fixed by convenience and is frequently along main valleys, or in a series of lines connecting the principal towns. The results are a series of correctly marked bench-marks, and a list of them showing their position and height. The experience gained from such operations also forms a guide to ordinary levelling and a means of estimating causes and sources of error; a subject that will be treated in a subsequent section.

4. *Flying levels.*—These are taken at any detached points chosen in connection with some special object, more generally some proposed work of communication, and serve as a basis for preliminary designs. The binding points of such designs are therefore those whose levels are specially required; they are generally practically useful points at or near towns, at various crossing points of rivers, saddles of ridges, points of abrupt change in the general rise or fall of the country, and proposed points of junction with existing lines of communication. Such levels are generally rapidly and rather roughly determined; they may either be recorded on plans or in the form of rough sections on a very small scale.

5. *Complete levels for final designs.*—A series of detailed levels sufficiently complete for purposes of final design for works of communication should consist firstly in a fully corrected set of levels for the points previously determined for the preliminary design; secondly, in an amplification of the same by levels of intermediate and lateral points; and thirdly, in sets of contour levels over such parts of the country as are most specially affected by the intended works or are most likely to affect the

proposed construction and design by their form of surface. Canals and works involving the collection, storage, and distribution of water are more specially affected by inaccuracy of level. The greater portion of such levels are most generally recorded in the form of longitudinal sections, with a series of short or cross sections; the remainder being recorded as contours on portions of the plan.

Under this, as well as the foregoing four heads, the purposes of the actual design or intended object can alone serve as a guide to the extent of work and amount of detail required in the series of levels. The scope of such operations may vary from a few hundred feet to a few hundred miles, and the amount of detail required may also vary enormously.

The nature of the country affects levelling operations to a great degree; the use of the Gravatt level being confined principally to level and moderately undulating localities, the Eckhold principle and instruments of the inclined or A-level type being suited to inclined ground, and the portable reflecting-level being necessary in steep and precipitous country. These three last considerations, as well as the five previously given, and the variety of extent and detail, constitute the practical distinctions in levelling operations of various kinds.

Section 3. LEVELLING OPERATIONS.

Levelling in Moderately Even Localities.

The principal part of all works of engineering, manufacture, agriculture, and trade being carried on in the plains or level portions of any country, the greater portion of levelling operations necessarily also happens to be carried out in similar localities in order to meet

the wants of those residing there. Telescopic levelling instruments and graduated level-staves of the Gravatt type are the appliances most commonly used under such circumstances; the 14 or 16-inch level—that is, an instrument with a 16-inch telescope—being the size adopted for ordinary purposes, and the 18-inch level being used wherever difference of level becomes more important and greater accuracy is required, as for canals in very flat country. The former instrument generally should have a telescope with a magnifying power of about 16 to 18 times, the latter about 18 to 20 times. In either case the level should be as long as the telescope admits of, and should be sensitive, that is of rather flat curvature. It is, however, better in all cases to have a telescope of higher magnifying power—for the former 20 times, for the latter 25 times—for this causes only one drawback, that very short sights cannot be taken with it, while the advantages of being able to take long sights and get over ground faster, and that of using the instrument with additional wires as a telemeter, are relatively important.

It is most usual that surveying operations precede those of levelling, so that the points at which levels are required are shown on some plan, and may be also marked on the ground by whites or pegs ready for the leveller; practically however it often happens that this is only done for a certain number of points, and that additional measurements have to be chained for determining the position of many more; under such circumstances observation of distance with the telescope-wires effects a great saving of time, and within certain distances is quite as correct as chaining. (See paragraphs on Telemetry, page 16.)

The simple operation of levelling from a known point to another whose level is required may, if the distance is short, be obtained from one pair of sights taken with the instrument at one spot. Thus if the known level of a point A be 40·00 feet, and the unknown point B be at a distance of 350 feet from it, the instrument may be set up at any convenient spot from which A and B are visible, the level-staff being held up successively at A and B, and readings on them being obtained 5·25 and 6·75 feet respectively, the required level of the point B is hence 38·50 feet. It is however better in this as in every case to use equidistant sights, that is, that the instrument should be set up at a point equidistant from A and B, or nearly so, but not necessarily in the line AB and at 175 feet from each; as greater accuracy is thus obtained, the distorting effects of curvature and refraction in the two sights being thus equalised and neutralised in the resulting difference.

The circumstances and limits under which a difference of level may be thus taken by one pair of sights are, that the greater distance sighted shall not exceed the powers of the telescope for clearly reading the graduations under the actual atmospheric condition, that the fall or rise in the sum of the two distances sighted shall not be so great that the line of sight may pass below the zero of the one staff or above the top of the other; and lastly, under any circumstances in ordinary work it is rarely advisable to take any sight to a distance more than 650 feet, and hence the two points A and B should not be more than 1300 feet apart. This limit is so arranged that the effect of curvature and refraction may be neglected in levelling operations.

First, to consider the effect of curvature. The earth being a spheroid, which for these purposes may be considered to be a sphere, any truly horizontal line of considerable length is a circular arc described from a centre which is the centre of the earth; the line of sight directed on to a level-staff through a levelled telescope is (if the effect of refraction be neglected) a straight line tangential to this arc at the eye-piece which hits off a graduation on the level-staff above the actual point of equal height; the difference of height is the correction to be deducted from any reading on account of the curvature of the earth, and is a third proportional to the diameter of the earth and the distance between the instrument and the staff, or if taken in sufficiently close numbers,

$$\text{Correction in feet} = (\text{distance in feet})^2 \div 42\,000\,000;$$

hence to need a correction amounting to $-0{\cdot}01$ on this account the distance for the sight would be about 650 feet. In this case the cause of error is invariable, the correction can at any time be calculated and subtracted from any reading, and if nearly equidistant sights are adhered to, it may be entirely neglected.

Next, to consider the effect of atmospheric refraction. The amount of error due to such refraction varies with the condition of the air, and though it would be misleading to apply a mean value which might possibly never be justly due to actual conditions more than once or twice in a lifetime, yet the maximum value, which is about half of the error due to curvature but of opposite sign, can be taken into account and guarded against. The extreme correction on this score hence would be $+0{\cdot}005$ in a sight of 650 feet, and this is just an amount appreciable in ordinary reading.

Lastly, reflex refractive action. When the visual ray of the observer passes close to the surface of the earth in its direction to the object, it is in long sights liable to a certain amount of distortion, the amount of correction for which cannot be estimated or calculated according to any law or formula, but may either be positive or negative in sign. It has been noticed by the author more especially in levelling in South America and India, and occasionally in very hot and bright weather in England and Wales; and it is evidently distinct from the simple refraction before mentioned as due to mere quality of the air. From its variation with the nature of the surface, from dry parched sand up to very damp vegetation of heavily irrigated standing crops and rank lagoon growth, as well as with the temperature and hygrometrical condition of the air, it appears that the cause is a dynamic change, a hygrometrical movement (in distinction to a hygrometrical condition) which causes a reflex refractive action on the visual ray that comes within range of this movement. An increased elevation of the line of sight by a foot, and sometimes only half-a-foot, may be sufficient to free the ray from this distortion. Further experiment on an extended scale will probably result eventually in the determination of some correct law and formula by which this class of error may be rigidly defined; in the interim its existence is here indicated by way of warning, and its mean value as estimated by the author lies between +0·0025 and −0·0025, even at comparatively short distances. (No mention of this phenomenon has been found recorded.) In England it is generally almost inappreciable in amount during cool weather; and probably also in any country when working over dry naked soil.

These two latter elements of uncertainty, added to the first, of alteration, combine to form the limit to levelling sights suited to all ordinary possible conditions, that has already been given. There are, also, other sources of error which when they combine to produce a cumulative result may seriously affect a single pair of sights, but as they generally do not, they will be treated subsequently with reference to extended series of sights, in a section devoted to that subject.

Levelling to obtain solely the relative height of a distant point is effected by taking a continuous series of pairs of sights from the given up to the distant point, each pair of sights being taken in the manner and under the limits already explained. The reduced level of the distant point then = the reduced level of the starting point + sum of all the back sights − sum of all the fore sights.

Levelling to obtain a correct section of a continuous surface is generally similar to the last operation, but varies from it in that the intermediate points and the changing points through which the levelling proceeds must be correctly chosen, and also have their levels determined so that all variation of slope may be truly shown on the section. In the attached figure (21) A is the given starting point, C, D, F are changing points, and B and E are intermediate points, while G is the last point of the section. The sights to A, B, and C are taken from one position of the instrument, and the level of A being given, those of B and C are thus obtained; the level-staff being kept at C the instrument is moved to its second position, whence sights are taken to C and D; the staff at C having been

turned round to face the instrument, this pair of readings gives the level of D; from the third position

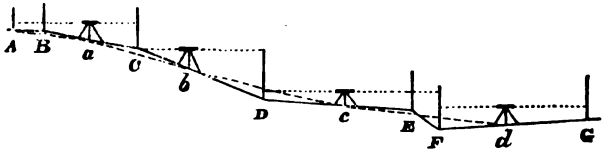


FIGURE 21. METHOD OF LEVELLING A SECTION.

of the instrument sights are taken on D, E, and F; and from the fourth position sights on F and G, from which the levels of all these points are deduced by addition and subtraction. The back sights are the readings on the starting and changing points A, C, D, F, taken in the direction of the starting point; the fore sights are the readings on the changing points, and the final point C, D, F, G taken in the direction of the final point or onward direction; and it is evident that an error in any one of these readings will affect the reduced level of any subsequent point obtained through them; for this reason the level-staff should be held truly vertical with plumb line, or swung gently backwards and forwards by the holder to get a minimum reading at changing points; while any error in observing on an intermediate point B or E will merely affect the reduced level of that point and no other, hence less care is necessary in intermediate readings. The resulting section will not only depend on the actual ground surface, but also on the positions chosen for the level-staves; for if the staves had in this case been held at A, a, b, c, d, G instead of at $ABCDEFG$, the dotted line shown in the figure would in that case be the resulting section; and as this would not truly represent the most marked changes of

inclination, it would be a false section, however correctly the actual readings might have been taken and added together, and would be useless from not showing the true character of the surface.

In levelling on extensive lines of section, a bench-mark is generally made at about every furlong, or at every quarter of a mile, so that any accidental error in the operation may not involve re-levelling the whole, but only recommencement from the bench-mark preceding the error.

Cross sections, or short sections at right angles to any series of longitudinal section, are levelled in a similar manner, the starting point being more generally in the middle of the cross section. When these are very short, the sights necessary for them may often be taken nearly simultaneously with those for the longitudinal section without removal of the instrument, in which case all such become intermediate sights.

Two examples showing the forms for recording and reducing level-readings are attached to this chapter.

Of the two methods, the Collimation method of reducing levels is to be preferred, as it saves one column of figures, and is more simple in working out as the distinction between rises and falls is not considered. Under either method, readings of intermediate sights may either be entered in a separate column or among the foresights, in which case they must be distinguished by a circle round them so as not to interfere with the addition of the columns.

The Rise-and-Fall method is that most commonly adopted by English surveyors, who are apt to adhere to older systems without taking the trouble to investigate new methods.

In either case the original record of staff-readings should never be altered afterwards, however much it may be necessary to re-write the figures used in reduction.

The foregoing description of the mode of levelling a section along a ground surface applies to ordinary work; the levels of exceptional points out of the range of the level-staves are, however, sometimes wanted, in which case the level of some point in the same vertical line must be obtained by this process, and the remaining difference of level to the required point obtained by direct measurement with plumbed chain, extra staves, or sounding-cord. If a required point is only slightly out of the range of the level-staff when held on the ground, as, for example, the under side of a girder or the keystone of a railway arch, the staff may be reversed and held with its lower end to the keystone during the reading; the reversed reading should then be specially marked in the record. If, owing to considerable fall in the ground, a line of collimation should just overshoot the top of the level-staff, the staff may be held up in the hands during the reading, while a second man measures the height of the bottom from the ground with a tape or another staff: the additional height should then be recorded, as well as the reading. This device may be used at intermediate, but not at changing points. Another method of meeting the same difficulty within a small limit is to incline the telescope a little downwards till the staff is visible, and to make use of the bubble-reading of the level to calculate a correction to be added to the staff-reading. A subtense table for the graduations of dislevelment of the instrument used is very useful in such cases.

In ordinary levelling in England no special further precautions or corrections are necessary beyond those already mentioned; but in levelling for long distances in tropical countries when a higher amount of accuracy is required, and in which 18-inch levels become necessary, it becomes necessary to consider and investigate all possible sources of error. These will be treated in another section.

The two sections in figures 22 and 23 are plotted from the two records accompanying.

FORM OF RECORD. RISE-AND-FALL METHOD.

Back Sights	Intermediate	Fore Sights	Rise	Fall	Reduced Level	Distances	Total Distances	Remarks
3·51					350·87			B.M. on lower gate-hook
5·30		5·15		1·64	349·23			
5·51		4·52	0·78		350·01			
4·61		4·76	0·74		350·75			
2·70		6·35		1·74	349·01	Not required		
3·98		6·00		3·30	345·71			
6·84		7·80		3·82	341·89			
		2·16	4·68		346·57			Zero peg of branch-line 15½′ from outer rail main line
Totals and diffs.			6·20	10·50	4·30			
						chains		
2·02	1·92				346·57	0·00	0·00	Zero peg. Ground 1·92 below it.
	3·20			1·18	345·39	1·00	1·00	
4·88		3·60		0·40	344·99	1·00	2·00	
7·05		5·65		0·77	344·22	1·00	3·00	
11·54		13·17		6·12	338·10	1·00	4·00	
	11·65			0·11	337·99	1·00	5·00	On ground surface to the east of formation
6·69		9·89	1·76		339·75	1·00	6·00	
	9·02			2·33	337·42	1·00	7·00	
	10·40			1·38	336·04	1·00	8·00	
10·08		9·34	1·06		337·10	1·00	9·00	
	10·57			0·49	336·61	1·00	10·00	
		14·03		3·46	333·15			At 10·47 ch. on slab of stile scratched cross + b.m.
Totals and diffs.			2·82	16·24	13·42			

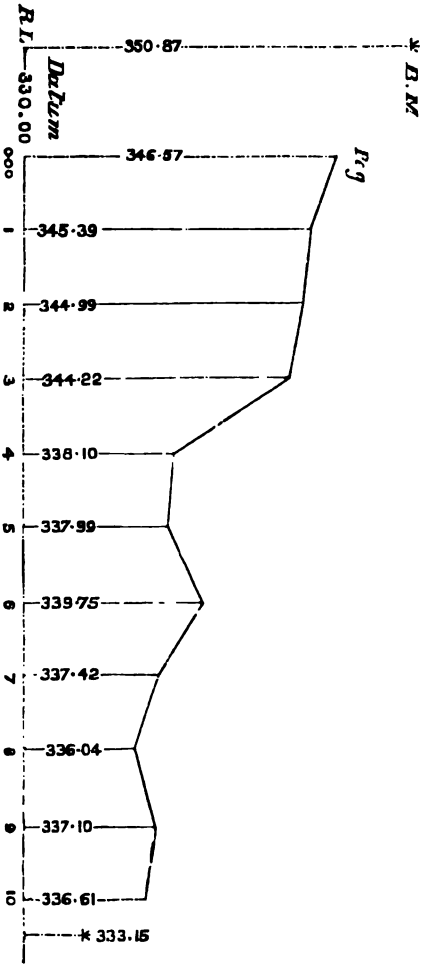


FIGURE 22. SECTION NO. 1.

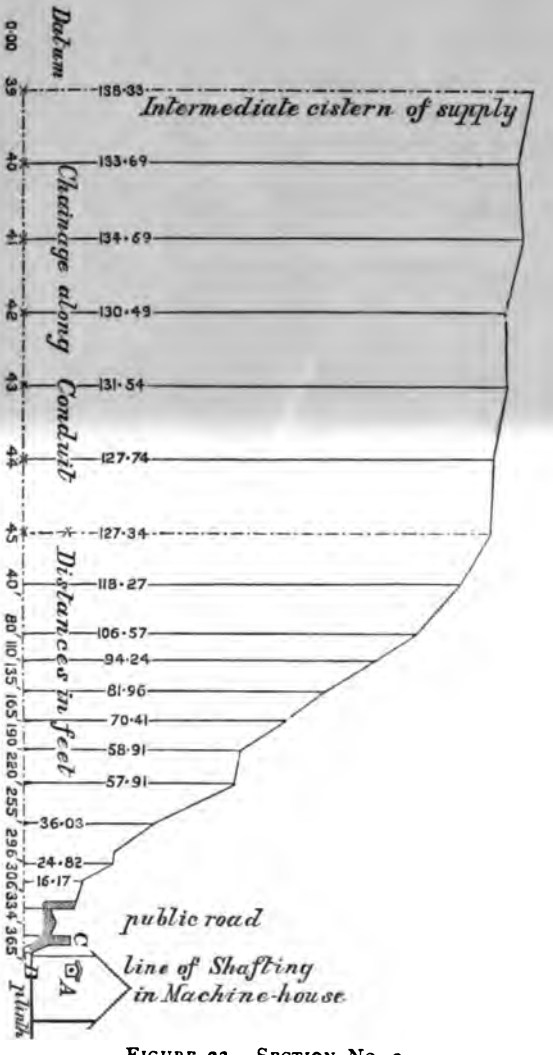


FIGURE 23. SECTION NO. 2.

FORM OF RECORD. COLLIMATION METHOD.

Back Sights	Inter-mediate	Fore Sights	Colli-mation Levels	Reduced Levels	Dis-tances	Total Dis-tance	Remarks
0·36			138·69	138·33		39·00	Peg on site of proposed dam at a cistern
	5·00			133·69	1·00	40·00	Up a valley
	4·00		1·	134·69	1·00	41·00	
6·70		8·20	137·19	130·49	1·00	42·00	At a corner
	5·65			131·54	1·00	43·00	In a bend
	9·45			127·74	1·00	44·00	
1·93		9·85	129·27	127·34	1·00	45·00	By pole at end of gradient, opposite machine-house
1·48		11·00	119·75	118·27	40′	40′	
1·55		12·18	108·12	106·57	40′	80′	
1·64		13·88	95·88	94·24	30′	110′	
1·10		13·92	83·06	81·96	25′	135′	
1·65		16·65	72·06	70·41	30′	165′	
2·00		13·15	60·91	58·91	25′	190′	
1·22		13·00	49·13	57·91	30′	220′	
1·85		13·10	37·88	36·03	35′	255′	
3·70		13·06	28·52	24·82	35′	290′	On a flat bench by a path
5·90		12·35	22·07	16·17	16′	306′	Near edge of lower bench, by wall
	13·30			8·77	28′	334′	On the road
0		9·00		13·07	26′	360′	C, on wall, north of road
		10·00		3·07	5′	365′	B, plinth of machine-house
31·08		166·34					
Diffs.		135·26		135·26			

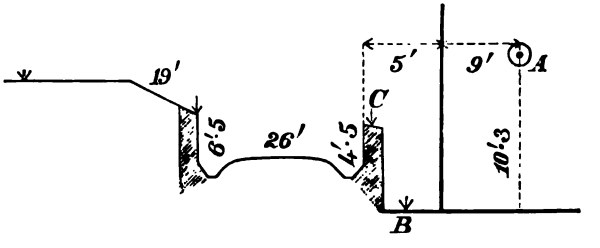


FIGURE 23A. SKETCH AT MACHINE-HOUSE IN LINE OF SECTION.

Levelling on Inclined Ground.

The levelling instrument of the Gravatt type, with horizontal sights on graduated staves, may be used on moderate inclines in the manner already described with a very fair rapidity; but when the incline becomes rapid, that mode of operation has to be conducted with short sights and a telescope of low power: the number of sights per mile is enormously increased, and the work becomes laborious and tedious. Under such circumstances, therefore, it becomes necessary to depart from the system of horizontal sights, and to adopt inclined sights or reciprocal vertical angles.

1st. When the levels of comparatively distant detached points are alone required, such as of survey-stations visible from each other. The use of aneroid barometers, giving results within 10 feet or at the best within 5 feet of the truth, is more suited to very steep country, and is inapplicable to accurate levelling in this case. The method of reciprocal vertical angles may, however, be conveniently adopted by two observers with accurate well-graduated instruments—reflecting circles, sextants, or theodolites—with or without the aid of mercurial horizons. Bright weather is necessary for this work. Preconcerted times and signals should be arranged, watches compared, and sunflash signals made, to ensure that the observations may be perfectly simultaneous. The reciprocal angles of elevation and depression should be observed at least three times to obtain a good mean, after rejecting faulty observations; index error should be carefully and frequently noted. The reduced horizontal distance between the two stations being known from previous survey operations, the ele-

vation is equal to that distance multiplied by the tangent of the resulting mean of the six corrected angles.

2nd. When a longitudinal section, or series of levels, is required between two such stations situated at some rather long distance apart, which may sometimes be not visible from each other, levelling on graduated staves with inclined sights may be necessary. In such operations the levelling may be executed either with the theodolite, the omnimeter of Eckhold, or with a levelling instrument specially constructed for this purpose. The use of these instruments has been already explained at pages 18 and 149. It is better, as far as possible, to keep to one fixed angle of inclination throughout a set of sights between stations, and to adhere to round numbers in distances, as the reduction of sights and levels can be more easily and rapidly calculated. If the distances used in calculation have been measured on the incline, the correction for declivity = inclined distance × sine of the angle used; but if horizontal distances are used, the correction for declivity = horizontal distance × tang. of the angle used. Apart from this correction, the level-readings are reduced in the same way as ordinarily with horizontal sights. Index error should be carefully and frequently noted during such work. In some instruments, the graduation of the arc of the instrument is set to the ratio of the inclination, and the ratio given can then be used instead of the angle in calculating the correction, provided that there is not any index error.

Levelling in Steep Country.

1st. The levels of detached points may be obtained with aneroid barometers, as much accuracy can be

generally dispensed with in hilly or mountainous country. The use of the aneroid has already been explained at pages 151 and 152, where the formulæ for reducing the elevations are given.

2nd. When a longitudinal section, or the levels of a series of points at short distances, are required. The levelling may be executed with the instruments before referred to, that are adapted to inclined sighting on graduated staves, in all cases in which the steepness of the country admits of it. When the hill-sides are so steep that the instrument is removed with difficulty, and the work becomes very laborious and slow, it is better to use a small reflecting level. This has been described at pages 150 and 151. Either a vane-staff or a graduated

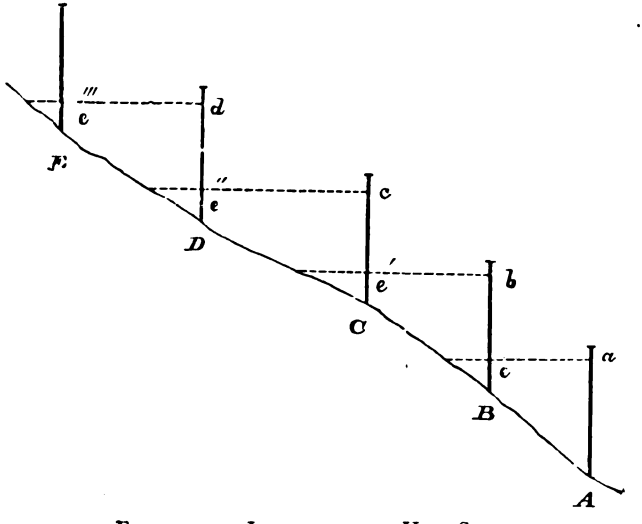


FIGURE 24. LEVELLING ON HILL-SIDES.

staff may be used, and a supporting-rod for hanging the reflecting-level at a constant height is generally neces-

LONDON, CROSBY LOCKWOOD & Co'S STATIONERS HALL COURT LUDGATE HILL

sary. The distances, which are necessarily short, are measured horizontally with a good tape or chain, and in order to proceed with rapidity, each reading on the staff should be taken nearly at its extreme length and if possible near the top of the staff. The rise or fall at each sight, according as the work proceeds uphill or downhill, is equal to the staff-reading less the height of the level, measured on the supporting-rod. (See figure 24). In this work, going uphill is the better plan in practice. Let A be the starting-point of known or assumed level, where the graduated staff is held: the observer proceeds uphill in the required direction, and sets his supporting-rod at B, hangs the reflecting-level on it, and takes a reading on the staff at A; the staff-holder then brings the staff to B, and the observer goes on to C to take a reading on the staff at B, and so on. The records are thus kept:

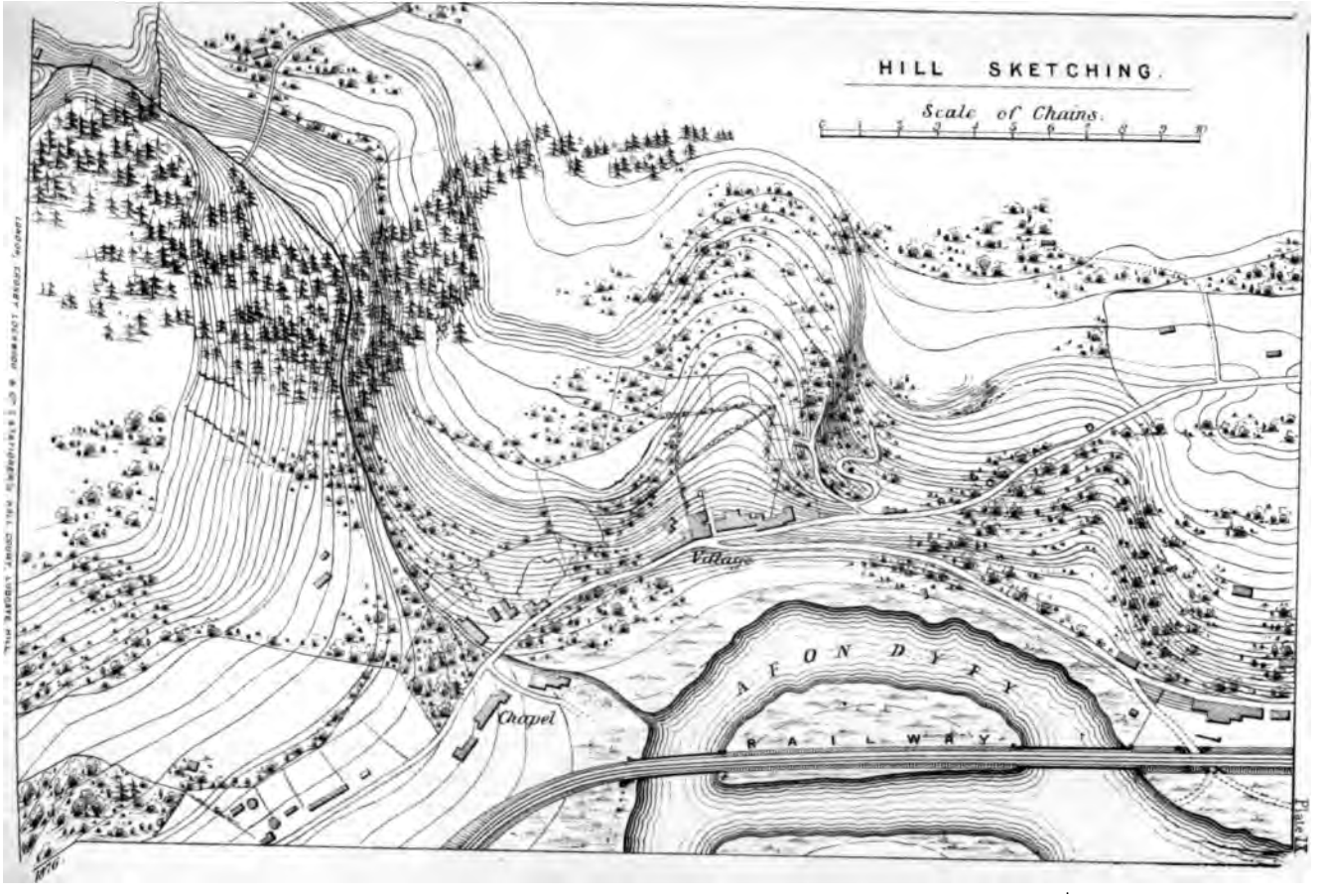
	Sights	Distances	
A.	13·30	30·	Permanent height of level on rod 3·5 feet
B.	12·50	35·	4 sights × 3·5 = 14·00 feet
C.	12·75	28·	51·80 − 14·00 = 37·80 feet. = Total rise in 145 feet
D.	13·25	52·	
	51·80	145	

When hill-sides are excessively precipitous, and often exceed an inclination of 30°, the reflecting-level may be superseded by a mason's level, the horizontal distances being measured with a 10-foot staff held to level, and the vertical heights with a graduated level-staff; but when the fall is nearer 30°, the former method is more rapid, and preferable. It is generally more convenient to adhere to straight lines, and not to vary the direction, in levelling on steep hill-sides.

Contouring.—The difference of level between contour lines to be shown on a plan is made dependent on the scale of the plan, the steepness of the ground, and the object specially under consideration; it may be either one foot or fifty, or any convenient number of feet between the two. The principal contours, which occur at every fourth or fifth contour-line, are surveyed and plotted with special accuracy; the remainder are interpolated in a rougher way. Thus, if the principal contours have a difference of level of 100 feet, the intermediate contours may be taken at differences of 25 or 20 feet; or if the former are taken at every 8 feet of elevation, the latter may be taken at every 2 feet or at each foot.

The principal contours are based on a series of sections taken on the principal watercourse and watershed lines of the country, according to which the position of points of any required set of altitudes, such as 100, 200, 300, 400, etc. feet, can be plotted on the plan and marked on the ground; such points are then extremely distant points on the principal contours. Each principal contour is then separately set-out between those points with the levelling instrument, and the points thus set-out are surveyed (generally with compass and chain or telemeter) and plotted on the plan as well as marked on the ground.

Intermediate contour lines are interpolated in various ways: generally intermediate level-points are set-out and surveyed on every watershed and watercourse line, between which these contours can be either set-out and surveyed, or accurately sketched. In the former case the interpolation of contours should in flat country be done with the water-level, and in steep country with the reflecting-level, whenever rapidity is an im-


HILL SKETCHING.
Scale of Chains.
Village
Chapel
AFON DYFY
RAILWAY



portant object: in the latter case the sketching may be aided by measurements.

As all such work must be done on the spot, and generally also in pieces, tracings of the original survey-plan must be used, and the resulting work transferred back on to the original. The finished work being of value in accordance with its accuracy, the latter must be vouched for by inscribing in figures on the plan the level of every point that has been actually observed. See Plate VIII.

Hill-sketching.—Such work very much resembles the interpolation of contours just described; the difference being that the intermediate contours are very numerous, and are simply sketched in without the aid of level or measurement, while the principal contours are very few, far between, and roughly determined.

Hill Reconnaissances.—In this work even principal contours are dispensed with; a few detached levels of principal points, peaks and passes, etc. are given, and the general form of the ground is shown on the plan by hill-shading, or a close-hatching of broken contour-lines sketched on the spot—the utility of this work is confined to representing the extent of the hilly ground, and giving some notion of the comparative steepness of the hill-sides. It is suited to rough surveys in unexplored countries, and to military purposes.

Section 4. SOURCES OF ERROR IN LEVELLING OPERATIONS.

The foregoing explanation is intended to apply to ordinary levelling in the ordinary practice of the surveyor, most of which is carried out with the 14-inch or 16-inch Gravatt level. When a higher amount of accu-

racy is necessary, as for instance on canals having a fall of only four inches per mile, it becomes necessary to use an 18-inch level, and besides to guard against all possible sources of error—at least such is the author's opinion, forced on him by experience in 1869; others may prefer to persevere in their ordinary practice, and make up for deficiencies by solemn asseverations of wonderful results.

In order to guard against all possible sources of error, these must be investigated, and always kept in view.

During a careful examination of this subject, it occurred to the author that the levelling operations of the Indian Trigonometrical Survey might, as well as those of others, throw considerable light on this subject; he consequently extracted the following notes from their reports, as well as the account of the instruments used by them and the errors they acknowledge, and followed up the same by a list of errors to be guarded against in very accurate canal-levelling, and their corresponding remedies. The whole was published in the Appendix to the author's Curve-Book, dated Calcutta, March 25, 1870, a copy of which was forwarded to the Library of the Institution of Civil Engineers in London. The matter referred to is here reprinted :—

In adding this to the Curve-Book, it is necessary to mention that it is from no love of intense exactitude, nor with any hope that it can be obtained, or wish that it should be attempted ; but it is important, as in the case of angular observations and their errors treated in the first chapter, that all sources of error should be known, and hence also the means of avoiding them to a certain extent understood ; and that a guide to the

ordinary limits of error, both in angular and in levelling observations, should be forthcoming to the engineer or the surveyor, whenever he thinks fit to refer to it.

Neither in angular observations nor in levelling does the railway engineer or even the canal-engineer use very exact instruments: the more exact instruments of both these classes are the altitude and azimuth instrument of the astronomer, and large theodolites of the Great Trigonometrical Surveyor, and the levelling instruments and appliances used in determining heights by the Great Trigonometrical Survey Department. From the former the principles of correction for angular error have been taken and applied to the surveying engineer's wants; from the latter, and their system, we can judge in what respect and to what amount the ordinary results of levelling of the engineer are deficient.

General sources of Error thus indicated.

1st. When a series of levelling operations are united in a circuit, or when the same ground is levelled by successive parties, *the lengths of the staves are sometimes found to vary*, sometimes as much as ·05 of a foot in a 10-foot staff; thus eventually causing discrepancy of height. The discrepancies of height thus due to differences of unit should be corrected, the corrections being made proportional to the height actually measured by the staves whose units are incorrect; such discrepancies being treated distinct from those due to cumulative or accidental error, which again should have their corrections distributed over each station of the line of levels in proportion to its distance from the station

of junction with the absolutely correct or accepted starting level.

2nd. *In continuing to level, along a long line that is really level, the further end constantly appears from the observation to be the lower end*, and the amount of this depression appears to increase with the distance, the result of one's work showing the distant point to be always lower than it actually is. The difference arising from this cause is never considerable, but is always in the same direction, and in the same series of operations is greater in proportion as the distance levelled is greater. The causes of this error are as yet unassignable, as they take place in an instrument shaded from the sun, and on hard ground as well as on soft, where sinking might take place in the interval between observation of the back and fore staves. There are probably several causes of different sorts which are minute and variable, and again sometimes accumulate rather than cancel. The only way to eliminate this error to a certain extent is to level the whole line twice over in opposite directions, and accept the mean of both results.

The following amounts of error due to this cause are recorded, and are useful to show in what distance this error is serious when the instruments and appliances are of a very perfect description.

IN STRAIGHT LINES.

Miles	Inches	Place	By whom
11	−1·07	Bristol to Portishead	by Mr. Bunt, C.E.
16	−1·94	Bridgewater to Quantocks' Head	do.
29	−7·6	Bridgewater to Portishead	do.
40	−4·11	Bridgewater to Uxmouth	do.
12·85	− ·19	Nar to Kadar	Indian G. T. S.

IN CIRCUITS ROUND TRIANGLES AND QUADRILATERALS.

Miles	Inches	Place	By whom
29·26	+ ·06	Mugo, Bala, Shahjamal.	I. G. T. S.
30·06	− ·53	Mugo, Lodri, Hujan.	do
33·09	+ ·78	Mugo, Fattī, Hujan.	do.
33·17	−1·58	Hazara, Mugo, Bala.	do.
31·64	− ·40	Shahjamàl, Mugo, Fattī.	do.
36·88	+ ·71	Jeto, Hazara, Gunia.	do.
38·17	−2·42	Kadar, Nar, Jeto.	do.
38·99	−1·16	Kadar, Nar, Kothiàlà.	do.
46·40	− ·07	Nar, Gunia, Jeto, Kadar.	do.
47·12	− ·97	Jeto, Bala, Hazara, Gunia.	do.
36·48	− ·56	Average result of the 10 circuits.	do.

It is probable that the greater error in the levels by Mr. Bunt on behalf of the British Association were due to his using only one staff, and that a vane-staff.

3rd. *The dislevelling effect of the sun's action, which tends to raise the end of the telescope towards itself, and to depress the opposite end.* The amount of this was proved by the surveyors of the Indian Great Trigonometrical Survey by adding together algebraically the respective level-corrections of each instrument for a whole season, when the negative corrections were found to predominate in every section worked from south to north, and the positive corrections in the opposite sections. Their amounts, when referred to a common origin, were as follows at the end of 310 miles:—

No. 2 Level, No. 4 „	Interchanged between two of the observers	1·51 feet. 1·60 feet.
No. 3 „	used by the same observer	3·55 feet.

All three instruments were always carefully shaded from the sun by large umbrellas while set up for observation, and by blankets over their boxes while being carried. Being all of the same pattern, they were pro-

bably equally affected by the sun's rays: the apparent excess of corrections for No. 3 level being due to the observer, who had a habit of not relevelling his instrument during observations; his results therefore give the fairest measure of the sun's influence. They give an average dislevelment of 0·92 second of arc per station, if supposed to act only during the hottest part of the day; this, though small in amount, has a serious cumulative effect.

This is now cancelled in the practice of the G. T. S., at least as far as possible, by watching the corrections, adding them together algebraically as the work proceeds, and tilting the instrument by its foot-screws slightly upwards to the north away from the sun, whenever necessary to counteract the tendency to droop in that direction; otherwise, any error in the value of the run of the level would affect the final results by the same fraction of the accumulated corrections that it is of the run.

In ordinary levelling operations, the action of the sun is more liable to produce error than in those of the G. T. Survey, where the bubble-reading is noted and recorded immediately after the staff has been observed; because, in ordinary cases, the bubble is first adjusted and the staves read afterwards, leaving an interval for the sun to act on the instrument: the amount of error resulting will depend on the length of time of this interval, and with the direction of line of operations. Change of weather and time would probably cancel or affect this error, but there are no means of entirely removing it; and no method of working in a circuit or otherwise would indicate the amount of error. As long as the cause remains constant, these errors re-enter with-

out attracting notice to an equal extent in the up and down lines; thus the opposite extremes of a circuit, which closes without apparent error, may be yet considerably erroneous.

4th. *Atmospheric influence.*—Different results may be obtained from levelling at different times of the day, especially when the rays of light graze the ground in passing from the staff to the observer; this occurs in cloudless weather, when the sun shines brightly, but never in the cloudy. The following result of experiments on two following days, January 10 and 11, 1859, at Hatedara, in Sind, by surveyors of the G. T. Survey, shows a tendency to a diurnal law of variation.

A.M.	Differences from mean in feet	P.M.	Differences from mean in feet
h. m.		*h. m.*	
7 12	+·0023	5 6	+·0055
7 35	+·0019	4 45	+·0010
8 35	−·0002	4 27	+·0006
9 3	−·0022	3 6	−·0011
10 4	−·0013	1 49	−·0027
11 10	−·0033	12 19	−·0030

Such atmospheric influences would tend to cancel each other in a long line of operations, excepting under the following circumstances:—First, when operations are carried on more before than after noon, they are more under a sinking than under a rising refraction; hence the first pair of staff-readings will have a tendency to be more refracted than the second, thus introducing cumulative error, unless the precaution is taken to alternate the order of observations. Secondly, when operations are carried over a line of country which slopes uniformly in one direction, when the rays of light from the upper staff to the observer are usually nearer the ground than those from the down staff, and are there-

fore more subject to extremes of refraction. In India there are fewer working hours before than after the ground has been heated by the sun in the practice of the G. T. Survey; consequently, the rays from the up staff must have an excess of negative refraction, compared with those from the down staff, and the result on an ascent would be to make it too small. The amount of this error will vary with the season, and is evidently beyond control.

With regard to the extremes of refraction, the following has been observed by the G. T. S. in the plains of the Punjab. A station or object seen from another station ten miles off, appears at a minimum height between 1 P.M. and 3 P.M.; it then rises gradually, and afterwards rapidly as the sun sinks and the dew falls, to its maximum at night; on the following morning this occurs in the reverse order, the extreme height through which it appears to rise and fall in 24 hours being 100 to 150 feet. A few feet of difference in the absolute height of the observed station has a considerable effect on these phenomena, which vary with the heights at which the rays from the object pass above the ground level, also with the atmospheric moisture, the soil, and the heat of the sun's rays.

The following experiment was also made:—A pair of signals was placed $10\frac{3}{4}$ miles off, the one signal being $16\frac{1}{2}$ feet above the other vertically, and truly subtending a vertical angle of 60 seconds; during different times of the day, the subtenses varied as follows:—

at	2h.	33m.	P.M.	74 seconds.
	2	43	,,	70 ,,
	4	16	,,	58 ,,
	4	33	,,	49 ,,
	4	43	,,	42 ,,

From this it appears that it is absolutely necessary to take vertical angles reciprocally and simultaneously if possible, and that they should be taken at the period of minimum refraction.

5th. *The settlement of the instrument on its axis.*—This was shown to the observers on the G. T. Survey by examination of the recorded bubble-readings of operations made, which indicated that the index error of the level had a tendency to alter always in the same direction during the observations at each station. Thus in one instance taken at random out of many, the index error in the second pair of observations at forty-seven consecutive stations was found, when compared with its value for the first year of observations, to have decreased 39 times, increased 5 times, and remained unchanged 3 times, the algebraical mean decrease being 1·2 seconds of arc. This indicates a constant deviation in the adjustment of the level to the axis of rotation during the settlement of the instrument on its axis, on being taken out of its box and set up on the stand, which has to be done at every station, when a large heavy level is employed. It is of no importance by itself, save as indicating that a similar alteration may take place simultaneously in the adjustment of the usual axis of the telescope to the level. Were the forward staff to be invariably read after the back staff, as is the usual custom, the error mentioned, 1·2 second per station, would amount to 1½ feet per 100 miles. To cancel this, therefore, the back staff and the fore staff should be alternately read first in a series of levels.

6th. *Personal errors.*—Among practised observers, those in connection with the manipulation of the instrument are mostly not great; those in connection with the

staff-reading are again likely to cancel themselves, except perhaps in working in a meridional direction, when, one staff being invariably more illuminated than the other, these errors become cumulative.

The most probable source of personal error is in reading the bubble. The observer generally gets a side view of the bubble refracted obliquely through the thickness of the glass tube, which is not so clear as the view from above. The rim round the bubble caused by the adhesion of the liquid to the sides of the tube becomes so prominent, that its extremities may be read, instead of the actual ends of the bubble. When light falls obliquely on the instrument, the outer edge of the rim, towards the light, is more clearly defined than the inner, while at the opposite end of the bubble, the inner edge of the rim is most clearly defined; hence there is a tendency to bring the bubble too much towards the light, and to give the readings an erroneous bias by an amount equal to half the breadth of the rim, which might have a considerable effect on a long line of levels.

7th. *General remarks.*—Generally, to prevent bias or accumulation of error, changes of observers, changes of instrument, change of weather, change in the putting up of the instrument at each station—at one time putting one side foremost, the next time the other—and change in the manner of carrying the instrument from one station to another, all have a decided effect.

The Instruments, Appliances, and Methods employed by the Great Trigonometrical Surveyors in Levelling.

The levels were standard levels, by Messrs. Troughton and Simms, of 20 or 21 inch focal length, and powers

averaging 42, of a very superior description ; their levels were fitted with finely graduated scales for reading the positions of the ends of the bubble, which had always to be recorded. The run of the level was determined by a series of observations on the vertical circle of a large theodolite or astronomical instrument: from the mean values run tables, or subtense tables, showing the correction to the level of a station for different degrees of dislevelment at different distances, were prepared, and applied to every observation in the field. The value of one division on the scale of the instrument being known, as for instance 1·709 seconds of an arc, the following was the rule for correcting dislevelment:—Consider the back end level readings to be minus, and forward end to be plus: find their difference, and enter it with sign of whichever is greatest. Half the algebraic sum of the differences is the quantity for which a correction is to be taken from Subtense Tables; the correction to have the same sign as the half sum.—There was therefore no need to level the instrument accurately at every station, and time was thus saved in manipulation, while a trained recorder accompanied the surveyor in the field to make the necessary computation.

The staves were 10 feet long, and were occasionally compared with a portable 10-feet iron bar: one face had its divisions marked in black on a white ground, being marked from 0 to 10 feet tenths and hundredths; the other side had a black face, and had white divisions marked from 5·55 to 15·55; the difference of reading on the same staff being always 5·55. On such staves there is no bias to repeat an error of reading. The staves were read to the third place of decimals of a foot; and whenever the difference between the two values obtained exceeded ·006

of a foot, after the correction for dislevelment had been applied, the observation was repeated. The staves were supplied with plummets let into their sides and visible through glass doors, and with four guy ropes attached to a swivel at the top in order to keep them in a vertical position. The staves were always placed on wooden pegs, in which convex brass brads are driven, allowing the staff to rotate freely, and serving as marks for the second and third set of observers to work on. The level was protected by an umbrella when placed in position; and by a blanket when being carried in a dooly.

To eliminate errors of adjustment, effect of curvature and of constant refraction, the instrument was invariably, even in hilly ground, placed midway between the staves, the distances varying from three Gunter's chains on bad ground in the middle of a hot day, to ten or twelve Gunter's chains on clear mornings or evenings.

On levelling a long line, it was always divided into sections of about five miles, adjacent sections being levelled in opposite directions. Forward staves were alternately observed first or last, the back staves being observed first at stations of an even number, and *vice versâ*. An error of ·006 of a foot required the observations to be repeated. A day's work consisted of about six hours in the field, and was made to close on some masonry object, or on two firmly planted pegs for the last two stations which were checked on recommencing work. The permanent bench-marks, about ten miles apart, were blocks of masonry flush with the ground, and hidden with earth, three mounds being erected around. As to absolute resulting errors, they were as follows :—

Miles	Feet.
310	0·98 Maru Pir to D. Ghazikhan.
301	0·942 Maru Pir to Karadie.
360	0·01 Dera Ghazikhan to near Attok.
440	0·527 Mittankot to Ambala.

Errors in ordinary Levelling.

The six sources of error that have been shown to exist by the levellers of the Great Trigonometrical Survey, must also exist in ordinary levelling as practised by engineers and their assistants In addition to these, there must be others which their arrangements occasion; among which are the following:—

I. *Those from using less perfect levelling instruments.* The level ordinarily used by railway engineers is a 14 or a 16-inch, and that by canal engineers an 18-inch Gravatt level. These, from being lighter, are more affected by the wind, and are very liable to damage in transport. An ordinary Gravatt level requires its permanent adjustments examined, and if necessary corrected, about once a week during ordinary work, as well as immediately after it has been transported in any rough manner at any time. These adjustments, and their tests, have been already explained at page 147, but even if the adjustments be perfect, the imperfections inherent in the instrument must be taken into consideration.

In using the telescope near its extreme power, all error due to slight defects in the lenses will be increased, and if combined with very slight parallax, become serious. Any looseness of attachment, as represented by a turn of a screw, has greater angular effect in a small instrument and causes greater error. The same may be said as regards other such instrument-derangement. Also

many such instruments are unprovided with screws for directing the telescope exactly towards the midwidth of the staff, and this may also conduce to error.

II. *Those resulting from the staves*, their patterns, length and graduation, and their manipulation. Level-staves have been made of many patterns, some of which are confusing, and cause uncertainty about the hundredths; the pattern adopted by the author, shown in figure 20, avoids all such graduations, while a wider staff of very similar pattern gives also the feet at every fifth of a foot, thus avoiding errors of feet. Papered staves may be very incorrect in parts from the stretching of the paper during its application, hence painted staves are preferred for all accurate work.

It is not necessary to number each side of the staff; if a second reading is wanted as a check, reverse the staff, and read it so. Telescopic staves are very liable to slide or stick; and perhaps simple ten-foot or eight-foot staves, that admit of being joined for lengthening, are best. As to their not being held upright, but inclining laterally, the observer can see that while standing by his instrument, and correct it by motions with his hand; but as to its inclining backwards or forwards, that he cannot see. To assist the man, a single plumb-bob should hang down the back of the staff to nearly the middle, or opposite the man's eye. In default of this, he should, while being observed, swing his staff slowly a little both backwards and forwards from the perpendicular, so as to show two maxima and one minimum reading, the latter of which will be correct. The cumulative error from want of verticality of the staves may be immense, especially when the staves are often read very high up.

III. *Those due to the want of pegs and convex brass*

brads.—On inclined ground, or on wet or rough ground, when the staff-man turns round the fore-staff at a changing point, he may sink it an inch or two; more on the slope of a ditch. A leveller cannot watch this at every step, and such errors, when accumulated, are very large indeed. Twelve-inch pegs, about an inch in diameter (in India of bamboo), are very convenient, and are sufficient to prevent serious error. On ground where pegs cannot be driven in, flat pieces of tile are perhaps better than nothing, though small convex iron discs would be better still. Without some such appliances, the observer is entirely at the mercy of his men as regards errors due both to lateral and vertical movements, from carelessness in turning the staves.

IV. *Personal errors*.—This class of errors, before described, has more effect on ordinary than on more perfect levelling. There, observers, recorders, and staff-men are all well-trained men, employed in that special way continuously; an engineer, again, has to do it at varying and uncertain times—sometimes with an instrument strange to him, and often with strange chainmen that have never been employed as such before, and do not think it worth while either to learn their work or to attend scrupulously to what is ordered: under which circumstances, not only is the greater part of the time spent in watching and correcting the chainmen, but besides strictly personal errors, mistakes become more numerous. While some mistakes may be due to carelessness, or to over-rapidity, most are certainly due to involuntary inattention caused by worry, annoyances, accidental occurrences, and interruptions; all of which are more frequent in ordinary levelling than in that of the I. G. T. Survey.

Summary of sources of Error.

1. Unsteadiness of the stand, and dislevelment by wind or accidental motion.

2. Looseness of any attachment of the instrument; or of any screw not firmly set.

3. Settlement of the stand in the soil, or settlement of any part of the instrument.

4. Dislevelment by the sun, from unequal expansion during reading the sight.

5. Want of instrumental adjustment:
   - A. Focus error, or parallax.
   - B. Collimation error.
   - C. Level error.

6. Want of correction for curvature of the earth.

7. Result of atmospheric effect on the visual ray:
   - A. Refraction under the varying conditions.
   - B. Reflex refractive action.
   - C. Apparent depression of the distant point.

8. In connection with the level-staves:
   - A. Unequal, or unequally graduated staves.
   - B. Want of verticality.
   - C. Lateral and vertical motion at changing points.

9. Inexactitude of datum or bench-mark used.

10. Personal errors:
    - A. In instrumental manipulation.
    - B. In bubble-reading.
    - C. In staff-reading.
    - D. From visual defect.
    - E. Simple mistakes from misconception.

Corresponding Remedies.

1. A heavy firm stand, permanently attached to the lower part of the instrument.

2. A well-made instrument; a thorough knowledge of it, its peculiarities and defects; care in removing and transporting it; repeated careful examination from time to time.

3. To invariably examine the bubble after reading on the staff. The bridge-reflector fails to be of much use for this purpose, as the length of the bubble is always varying. Piazzi Smyth's reflected bubble would guard against this.

4. Cover up the instrument when not in use. Set up the instrument alternately one way to the sun and the other.

5. Keep all in good adjustment. Have an arrangement for focussing the eye-lens as well as the object-lens.

6. Allow for curvature.

7. Note barometer and thermometer, and use a refraction table.

8. Use plummets; use discs or tiles; compare staves thoroughly; and employ good men as staff-holders.

9. Use three or four bench-marks for reference.

10. These are mostly unavoidable, although some may be reduced on noticing the cause of error.

General Remarks.

The system of equidistant sights should be adhered to as much as possible in long lines of levels: it eliminates errors due to curvature and constant refrac-

tion, also any collimation error, and, if the bubble is brought into the centre of the run at each sight, the level error also. The error due to the earth's curvature and mean refraction is almost inappreciable in hundredths of a foot in sights less than 650 feet: it is hence better not to exceed this distance in ordinary work. See page 162.

It is absolutely necessary always to attend to parallax, that is, the object viewed should be in focus, and not appear to move when the eye is moved from side to side; observations should also be always made with the middle of the horizontal wire, which by the way is not always exactly horizontal. It is not strictly necessary to bring the bubble to the centre of the run; instead of this, the position of the bubble-ends may be read, and the correction applied from a small subtense table made for the instrument and its scale.

As measurements are sometimes on sloping ground, a small table for reduction of distances to their true horizontal value should be inserted in the level-book. As to level-books, their patterns are endless: the collimation form of record, explained on page 171 has several advantages; the results are so placed that they catch the eye, independently of the side-work, and the reduced levels can be easily kept up while going along in the field. It is best to have a level-book that will go in the pocket, $7'' \times 4''$, and to have all its pages divided into little squares by ruling in pencil colour, pale grey; if these divisions are 100 to a foot, it makes it useful for sketches to scale; it also serves for tabular memoranda. These squares, any number of which can be marked off into a column-breadth, allow any number of columns to be made to suit any form.

No important levelling should be done when the wind is very high, nor when the air is so hot that the divisions on the staves seem to dance, and the bubble is so short that its motion is a comparatively useless indication; nor in very severe cold, when the fingers are stiff, benumbed, and incapable. A waterproof cloth should always be at hand to throw over the level in case of rain, which ruins an instrument; dust should be daily wiped off gently; screws occasionally washed in vinegar, well dried, and wiped with a clean oiled rag; staves should be wiped dry after being much wetted, laid horizontally to prevent warping, and occasionally compared with a standard length. Bench-marks should be made at distances of one or two furlongs on permanent objects; they can be painted (a bottle of red paint and a brush is easily carried) at the time, and chisel-marked afterwards. Permanent setting-out pegs should be two feet long and two inches square, distinct from survey pegs, which may be round. On recommencing work, always check over the last two pegs. Be careful that the pegs are always flush with the ground, especially in India, where simple-minded natives will cook their dinners with the marks of many miles' work, if they can find them.

As to the amount of accuracy with which levelling operations can be done with an ordinary set of instruments and appliances, and with ordinary chainmen, it is impossible to speak exactly; many and wild accounts of accurate levelling for long distances—100 and even 200 miles without any error, or next to none—have been frequently heard, though not often believed. Excessive good fortune may have occasionally produced such things in connection with good management, but not often; nor is the substantiation of this point so im-

portant as at first appears. The best plan is to determine beforehand the maximum amount of error that will not affect, to any important degree, the object for which the levelling is done, and then working with ordinary care and precaution to make the error less than that for certain. Regularity of result or evenness of error is perhaps of more importance than being occasionally wonderfully correct; this is generally obtained by steady systematic persons, who thoroughly study the particular instrument they use and its defects, who keep to this one instrument, and one set of appliances, and have a set of chainmen whose habits are known, and on whom they can depend as far as small matters are concerned.

As to the amount of work that can be done by one observer in one day. This is a point over which endless difficulties have been made, and which will never be settled; levelling being of different sorts and for different purposes, and involving generally many things besides getting over the ground to ascertain the level of a single distant point. But when this latter is the sole object, a wonderful speed may be attained by an observer having instruments and appliances that he has often used, having excellent chainmen that are willing to run hard all day with the staves, etc., a long stretch of unbroken flat ground, and a powerful telescope. Results obtained this way cannot, however, be taken as a basis of amount for ordinary work; and hurrying observers is not an advisable proceeding under any circumstances.

PART III.

SETTING-OUT.

Section I. SETTING-OUT ON THE STRAIGHT.

SETTING-OUT, or the location of intended works or of any predetermined points, is that branch of geodetical operations which is the converse of surveying and levelling in their exact sense, which consists in discovering the position of a series of actually-existing points, and through them forming a correct representation of existing features. Setting-out points as regards level differs little from levelling, as it is so much dependent on existing points that the operations are nearly the same in each case; the principal difference being that the exact points levelled must be marked by a peg or other means that will indicate correctly the position; this branch of the subject therefore requires but little further notice. Setting-out points, as regards position corresponding to design on plan demands that their relative positions shall be correctly marked on the ground. The greater portion of such ranging or setting out of engineering works is in straight lines, of long extent as centre-lines, and in transverse straight lines at right angles to these; most of the remainder is in straight lines at various angles and on circular curves having various radii.

Setting-out on the straight.—The setter-out, being in

all cases an adept at the use of surveying instruments, and competent to execute surveys and make trigonometrical calculations, requires but little additional experience, thought, and care to execute his duties in this branch; he simply applies the principles with which he is well acquainted in the most ready manner, with an amount of accuracy well suited to the requirements of the circumstances; sometimes the operations require little or no reflection, sometimes what is called usual practice—or the stereotyped habits of others—may be advantageously followed and sometimes advantageously improved upon. A few special tables for various purposes are occasionally useful; and with these, his instruments, and a few chainmen, the surveyor can undertake the duties of setting-out works.

Setting-out short distances on the straight can be effected by simply ranging three or four straight poles in the straight line, and taking the necessary measurements with a chain: right angles may be set out with the ordinary cross-staff, and one or two check measurements used to verify the resulting marks or pegs, which should be firmly driven in. The date of setting-out detached works, bridges, culverts, &c., should be invariably recorded at once, as there is a weakness among foremen and contractors' agents for removing such marks, and either declaring that none ever existed or eventually making fresh ones for themselves and declaring them to be the original marks: the former plan to justify delay, the latter to absolve themselves from blunders. Ranging in straight lines with poles may be done for several miles by very exact persons with hardly any error; but the attempt is hardly to be recommended generally. The permanent marking of extensive works,

such as those of centre-lines of canals, railways, or roads, is generally effected with a theodolite; although any good telescope with wires will range points in one direction, yet for ranging both ways a theodolite is required that will reverse well, and a transit theodolite is generally to be preferred, as the correctness of ranging would be entirely spoilt by a slight bend at the point of observation. When a Y theodolite is used for such purposes it is not sufficient to trust to the graduation at 180°, the telescope should be taken out of its bearings and reversed in them, and a sight taken on the same object by way of test. In ranging very long straight lines, such as occasionally occur on canals or railways in flat countries, which may be even fifty miles in length, a portable transit-instrument should be used in preference to a transit-theodolite; the ranging of very distant points should always be done with it, while intermediate or smaller distances can be ranged with the theodolite. The setting up of the transit-instrument, and its corrections and adjustments, are treated in the chapter on Astronomical Observations on Route Surveys.

Obstacles, such as pieces of marsh-land, immense trees, etc., sometimes occur in lines of long alignment, which impede the vision and obstruct measurement; the operations and devices used in passing such obstacles are very similar to those adopted in ordinary chain survey.

Points of given level.—Sometimes it is customary to have only one set of ranged pegs or marks both for position and level, the level points being marked on these; but in many cases it is necessary to have a second set of pegs to denote level. Level-marks are required at every change of gradient, at crossings, and

at all masonry works or special constructions; permanent bench-marks should be set at about every quarter of a mile over extensive works; a description of their positions should be recorded; and no error of more than an inch or two per mile allowed to remain permanently.

The permanent stakes used on centre lines of works of communication are generally placed at distances of 100 feet apart throughout the whole line from the commencement; in towns or crowded localities they are interpolated by pegs at every 50 feet apart, and on sharp curves at 25 feet apart. On some works, and in most English railway-practice, where Gunter's chains and furlongs are preferred, the distances of the stakes are taken at similar convenient distances expressed in those units. Such stakes are of uniform pattern, square in section, 3 feet long, painted and capped and numbered. In open country their distances apart may be increased to 400 feet, or to 5 Gunter's chains. The permanent level-stakes, if required, should be round, of a slightly different length, and otherwise painted so as to be easily distinguished.

When the land is tolerably flat, the staking and setting-out on the straight is comparatively simple and rapid; but on embankments, in cuttings, tunnels, or places where there is water, difficulties present themselves. The true position of a stake in cutting can only be arrived at when the cutting is completed, and any temporary stake would be liable to removal as the work proceeds. Hence the usual mode is to set out two lateral marks clear of the cutting on each side of the centre line and at right angles to it; the point in a cord strained between the two marks can then be plumbed

down to obtain the true position of the permanent stake at any time; while as regards level, if any two level-marks set to the proper gradient are given at each end of the cutting, a foreman can use boning-staves on them to obtain the level at any required point, and thus regulate the depth of excavation.

For high embankments lateral points are set out in the same way, being marked by tall poles with cross-pieces nailed to them, showing the intended final level; the permanent stake is set after the embankment is finished and has had a little time to settle.

In setting-out tunnels the centre line is generally first set-out on the ground surface; various points in it are afterwards plumbed down through the shafts to the required level, and the centre line set out afresh from the points below ground. As the shafts are rarely more than 10 feet in diameter this operation requires extreme nicety. The method adopted on the Metropolitan Railway was as follows. At each shaft the above-ground centre line was marked with great precision by two pegs each 25 feet distant from the centre of the shaft; in each peg a nail or eyelet was driven and sighted in with a transit theodolite; a cord strained between these nails then showed the exact position of the centre line over the shaft mouth. Two points were then marked on the cord, their distance apart being about six inches less than the width of the shaft, and these two points were plumbed down to the bottom of the shaft, where two corresponding permanent iron marks were set; thus giving the direction of the centre line below-ground. This operation was repeated several times to eliminate errors in the plumbing down, which at all times, and especially in windy weather, is tedious and doubtful in

result. The unsteadiness of the plummet may be mitigated by hanging it in water. The centre line is afterwards extended both ways in the tunnel with a transit-theodolite. Theodolites having hollow trunnions through which a light may be thrown on the cross-wires are very convenient in tunnel work.

Temporary marks for tunnel work are generally iron-staples in the leading and following frames, which are set to their correct position with the theodolite at each shift of frame; the temporary marks in the advanced heading are made on the timbers, and are sighted in the same way with the help of candles. Permanent marks in tunnels are generally made at every length or portion constructed at a time; they are generally iron spikes driven in between the bricks of the invert, and sometimes nails driven in wooden bricks introduced for this purpose. Permanent level-marks in tunnels are spikes driven at the level of the skewback; there should also be a datum staple near the mouth and at the bottom of each shaft.

At the adit entrances of tunnels the proper alignment for working is obtained by sights taken in the reverse direction; should a deep valley intervene and prevent convenient sighting, some mark may be set on the next rising ground or hill to serve the purpose of getting the direction at any time.

The same method may also be adopted for fixing points over deep water, such as the extremities or corners of a jetty; two points in the correct alignment can be fixed on shore, and the position of the required point, when wanted for setting up staging or other purposes, is indicated by them, and also if need be by a cross alignment given by two other points.

At sites of bridges, aqueducts, and such works, marks and stakes are liable to frequent removal during the progress of work; lateral marks are hence planted clear of the work in the alignment of the faces of abutments and piers or at any convenient distances, so that the positions of the corners of the abutments and piers may be easily obtained from them at any time. In checking the points set out in this rectangular mode, the diagonals may be measured and compared; a hypothenuse-table is sometimes useful for these purposes.

Side-widths of roads, railways, and canals.—These are set out on each side of every permanent stake at right angles to the centre-line. Cross-sections of the ground surface having been previously taken at these points and plotted to a natural scale, the central heights or depths of formation level are marked off on them, and a horizontal line drawn to represent the formation level in each section. A cardboard templet, having the proper formation breadth and side slopes, can then be applied to each section, and the earthwork section ruled in accordingly; the side-widths can then be obtained by scale from these sections, recorded in the Working Section-Book, and set-out on the ground.

This mode is sufficiently accurate for most purposes, but if it be preferred to calculate the side-widths, it may be done with the help of the four following formulæ which include all cases that can occur.

Each side-width is composed of the half of the formation breadth added to an extra portion, due to the slopes of the ground and of the intended side-slopes; the section being reversed in position for cuttings.

Let h=the central height or depth of earthwork at any given section or half-section,

$\frac{b}{2}$ = the half breadth at formation,

f to 1 = the intended side-slope.

g to 1 = the natural lateral inclination of the ground.

Then in the first case in figure 25

$$\text{the side-width } AB = \frac{b}{2} + fh.$$

In this case the ground is horizontal. A reversed figure would suit the corresponding case in cutting.

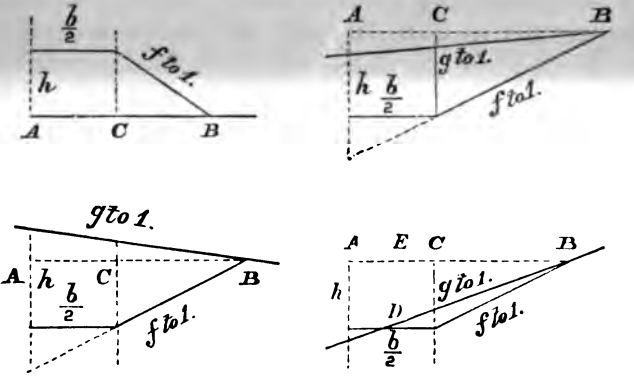


FIGURE 25. SIDE-WIDTHS.

In the second case in figure 25

$$\text{the side-width} = \frac{b}{2} + \frac{fg}{g-f}\left(h + \frac{b}{2g}\right).$$

In this case the ground slopes away from the formation level. A reversed figure would do for a corresponding case in embankment.

In the third case in figure 25

$$\text{the side-width} = \frac{b}{2} + \frac{fg}{g-f}\left(h - \frac{b}{2g}\right).$$

In this case the ground slopes towards the formation levels as it recedes from the centre line. A reversed figure would do for a corresponding embankment.

In the fourth case in figure 25

$$\text{the side-width} = \frac{b}{2} + \frac{fg}{g-f}\left(\frac{b}{2g} - h\right).$$

In this case the ground line intersects the formation level at D, and $ED = h$. The reverse of this figure would be a corresponding case, more in embankment and less in cutting.

The land-breadths, which include the side-widths and additional land for hedge or ditch, are marked on the plan and set on the ground accordingly when required. The area of land taken up may be obtained from the plan by planimeter measurement.

The Working Section-Book.—In former practice it was customary to enter in the Working Section-Book a mass of information and data which is now more often entered on folded tracings of working plans and sections for use in the field. The information now given in Working Section-Books, when used, is limited to some of the data mentioned in the headings of the following form; six of these columns are generally on one leaf, and the seventh on the opposite leaf of the book.

Distance from the Beginning of the Section	Formation-level	Levels of Bench Marks, Stakes, or other Marks	Depths of Cutting and Heights of Embankment	Lateral Slope from the Centre Line outwards		Side Widths on each Side		Remarks Gradients, Curves, Spans, Headways, Foundations, Springing, etc. etc. Descriptions of Bench-marks
				Right	Left	Right	Left	

Setting-out hill-roads and mountain-passes.—Setting-

out work of this description differs from the ordinary setting-out of roads and railways in that no preconceived design or plan can be exactly adhered to on account of the difficulties of the ground; in such cases the surveying and setting-out proceed nearly simultaneously with but little assistance from plans, hence the elements of design enter largely into work of this sort. It is necessary first to go over the ground to determine the binding-points and limits of deviation within which the road can be made, the saddle-backs, bridge-sites, and alternative routes that may be necessary on account of land-slips or bad ground; also the general route, whether it may best be carried up the flank of one or more large ravines, or up and around a spur or series of spurs; in this work some judgment must be exercised. These matters having been roughly determined, the setting-out can proceed at once; the portions in which ordinary spirit-levelling is practicable are set-out in the usual manner, but the steeper portions where moving a heavy instrument is almost impracticable can be set-out with the reflecting level. The application of this instrument to levelling in steep country has been explained at page 174. With this instrument, a level- or a vane-staff, a strong 50-feet cord or chain, plenty of pegs and one or two mallets, the pegging-out of 50 feet or 25 feet distances to any predetermined incline may be rapidly done on bare ground; but if the land is particularly woody and difficult, men with axes and billhooks must be employed to clear the way as the work proceeds. As an example let 1 in 20 be the gradient to be set-out; this gives a rise or fall of $2\frac{1}{2}$ feet in 50, and if the support for the reflecting level be 5 feet, the reading on the level staff for every 50 feet should be $7\frac{1}{2}$ feet; if a vane-staff

be used the vane should be set to this height; then with the aid of the 50-feet chain, points can be marked or pegged on the ground which will follow the required gradient. This pegging should be connected by cords by a laboring party immediately following, and used as a centre line for a footpath 3 feet wide to be made at once for the convenience of men and animals, as well as for enabling the engineer to go over the whole of this work with a levelling instrument before the bridges and other works are commenced.

On hill-roads the calculation and setting-out the side-widths is roughly done on the principles already explained, page 204. The final breadth of formation of a hill-road is generally 12, 18, 21, or 24 feet, exclusive of the drain on the inner side in side cutting.

As to gradients, 1 in 20 is the steepest ascent for wheeled vehicles that can be continued for many miles without causing animals extreme fatigue, it is also the steepest declivity for a horse to canter down for a long distance; a gradient of 1 in 16 should never be continued in exceptional places for more than a quarter of a mile, nor one of 1 in 8 for more than 50 feet; this being the limiting maximum for wheeled vehicles on roads in good order.

For mountain-passes or paths to allow of the passage of beasts of burden only, the gradient generally adopted is 1 in 10, or 1 in 8, and the maximum continuous gradient admissible is 1 in 4, although 1 in 3 is sometimes used for a very short length.

In work of this class extended vertical cliffs often present serious impediment to carrying on the path; if they also happen to be so hard that blasting them away to the required width becomes too expensive, the only

alternative is to adopt a gallery supported on iron cantilevers sunk in the cliff. The setting-out in such work is by pegs at every 25 feet in the mode already described for hill-roads; in the general management much is left to the discretion and skill of the setter-out, who should be conversant with the subject, as well as capable of setting-out pegs.

Section 2. Determining a Curve.

Before proceeding to describe practical methods of ranging on the curve, the determination of the curve to be adopted, and the functions and formulæ employed by the setter-out will be given and explained by very short and simple operations.

A curve being, for the purposes under consideration, a convenient method for changing the direction of a route, and a substitute for an angle, the first thing on which it will depend will be the angle formed between the two directions, or angle of intersection as it is termed, as the curve will be tangential to them; the second, the radius of the curve, rendering it sharper or flatter; these being fixed, the curve is determined—and if shown on a plan, the two quantities, designated by the symbols α and R, are noted on it also for convenience of reference.

The former of these two is the result of the previous determination of the centre line of the route with regard to general intention and local circumstances; the second is only approximately so, and generally admits of alteration within certain limits: it should generally be a round number, some multiple or sub-multiple of a hundred being most convenient, as Tables are constructed to suit such numbers in facilitating or saving calculation.

From these two all remaining functions of the curve may be calculated by means of formulæ and Trigonometrical Tables; those, however, that are more practically of use may be easily obtained with the help of the attached Tables by very short and simple operations; while the two pages of curve-formulæ at the end of this section, explained by the figure attached to them, include all such data necessary for the use of the setter-out.

In determining the curve, however, after obtaining the angle of intersection (a) and the radius (R), the two quantities required are the curve-tangent (T), which is the trigonometrical tangent of half the angle, and the total length of the curve (L); values of the former are given in a Table at the end of this section, and values of the latter in another table following it, which is also specially used for purposes of ranging.

The angle of intersection of the tangents is generally observed on the ground with a theodolite, and noted on the plan; in the absence of any instrument the angle may be obtained through calculation in the following way. Let BAC be the angle of intersection whose value is required, measure any two equal distances AB, AC, along the tangents and measure also BC.

Then since $BC^2 = BA^2 + AC^2 - 2BA.AC \cos BAC$

and $BA = AC$

$$\therefore BC^2 = 2\,AB^2\,(1 - \cos BAC)$$

$$= 4\,AB^2 \sin^2 \frac{BAC}{2}$$

$$\therefore \sin \frac{BAC}{2} = \frac{BC}{2\,AB}$$

hence the half-angle of intersection may be found by

means of a trigonometrical table of sines, and by doubling this, the whole angle (a) BAC.

Having therefore obtained the angle of intersection (a) and assumed arbitrarily a radius (R), the distances of the tangent points, or beginning and ending of the curve from the point of intersection, must be obtained; these distances are called the curve-tangents, and are given in the Table of curve-tangents at the end of this section, for any angle of intersection and for a radius of 1000: for any other radius they can therefore be obtained by simple proportion.

For example,

If $a = 102° 40'·4$ and $R = 600$ feet;
for rad. 1000 and $a = 102° 40'$ $T = 800·196$
the difference for $1' = ·2389$,
and the difference for $·4'$ $= \underline{\quad ·0955}$

$\therefore$ for rad. 1000, $a = 102° 40'·4$ $T = 800·291$
and for radius of 600

$$T = 800·291 \times \frac{600}{1000} = 480·175 \text{ feet.}$$

It is advisable not only to mark off these tangent-points on a plan, but also to mark them permanently and accurately on the ground, as from these the setting-out or ranging the curve can be done at any time afterwards, either with or without an instrument. The total length of curve should be also recorded on the plan: this may be easily obtained by means of the Table of curve-lengths.

For example.— Let the angle of intersection
$a = 128°45'·1$, and the radius $= 790$ feet.

Then the angle of deflection for the whole curve (γ), or tangential angle to the other tangent point, will $= \frac{1}{2}$ $(180° - 128°45'{\cdot}1) = 25°37'{\cdot}45$.

For this angle we get from the Table the curve for a radius 1000,

25°	872·665
37	21·526
·45	·262
25°37′·45	894·453

and the required curve-length

$$(L) = \frac{894{\cdot}453 \times 790}{1000} = 706{\cdot}618 \text{ feet.}$$

In this way the four necessary quantities a, R, T, L, are determined and recorded independently of the subsequent ranging of the curve on the ground.

Systems of nomenclature.—The practice in England formerly was to designate curves by their radii in Gunter's chains of 66 feet each up to 80 chains, and beyond that in miles; this system retains but few advantages, the Gunter's chain being of use chiefly to the land or parish surveyor for calculating acreage rapidly; but since railways have been constructed to a very great extent with such radii, and branches and junctions have to be made into such lines, the old system cannot very well be ignored, and the tables here given are suited to it as well as to the more modern way.

At present the system in vogue in England is to designate curves by their radii expressed in hundreds of feet, or chains of 100 feet, whatever may be their length, thus avoiding both miles and Gunter's chains. As, for instance, a curve of 500 feet radius, or one of 1000 feet radius, is used in preference to an 8 or 16 chain-curve;

for greater radii, instead of using the mile of 5280 feet, the round numbers, 5000 feet, or 6000 feet would be preferred.

The American system of nomenclature also requires explanation. It is one of great convenience in one respect, and for one system of setting-out, viz., that where an instrument is used, as it assumes for its basis of nomenclature the centre angle subtended by a chain of a hundred feet; thus naming curves, as one-degree curves, two-degree curves, three-degree curves, etc., when the centre angle subtended by a hundred feet on the curve is one degree, two, or three, etc. The mistake adopted in this system consists in naming the centre angle the angle of deflection; this not only creates confusion, but is obviously incorrect. It is most evident that the true angle of deflection is that made by the chord of the curve with its last or previous piece of straight, that is to say, the angular deviation of direction as observed by a person walking along the line; and this *true* angle of deflection or angle at the tangent point is exactly a half the centre angle subtended by the same arc. Again, a little consideration will show any one that it is quite unnecessary to deal with centre angles at all; no practical system of setting-out recommends one to proceed to the centre of the curve-circle.

An improved angular nomenclature would deal only with the *actual* angle of deflection; the curves being named one-degree, two-degree, etc., curves in accordance with the angle subtended at the tangent-point by an arc of 100 feet. The following small Table gives the radii in feet corresponding to curves thus named.

TABLE GIVING ANGLES DUE TO AN ARC OF 100 FEET FOR VARIOUS CURVES.

Angle of Deflection Minutes	Radius Feet	Angle of Deflection Minutes	Radius Feet
30	5730	1° 27·5	1980
32·5	5280	1° 42·5	1650
35·	4950	2° 0′	1432·5
37·5	4620		
40·	4290	2° 7′·5	1320
		2° 52·5	990
45·	3960	3° 0′	955
47·5	3630	4° 0′	716·25
52·5	3360	4° 22·5	600
57·5	2970		
1° 0′	2865	5° 0′	573
		5° 25·	528
1° 5′	2640	8° 30·	330
1° 12′·5	2310	14° 5·	198

This system of nomenclature, though preferable to the former, will doubtless soon become obsolete, as it is based on a single method of ranging curves now not exclusively adopted; and the other method of naming curves according to their radii in feet seems more likely to remain in vogue.

Collection of Formulæ for Curves.

(These angles are for conciseness in degrees and decimals.)

Angle of intersection of tangents $= a$.
Centre angle $(\beta) = 180° - a$.
Angle of deflection (γ) for the whole curve $= 90° - \frac{1}{2}a$.
Angle of deflection (δ) for an arc $(l) = 28°{\cdot}64789 \times \frac{l}{R}$.

Angle of deflection to middle point of curve $=45^\circ-\frac{1}{4}a$.

Length of curve-tangent, $T=R \tan. \gamma$.

Distance of the middle point of curve from point of intersection, $D=R \overline{\sec \gamma - 1}$.

Chord of the whole curve,

$$C=2R \sin \gamma=2\sqrt{2V.R.-V^2}.$$

Distance from middle of chord to middle of curve,

$$V=R \text{ vers. } \gamma=R-\sqrt{\left(R^2-\frac{C^2}{4}\right)}=\frac{L^2}{8R^2} \text{ (approx.)}.$$

Total length of curve, $L=\cdot 0349066 \times R\gamma=ln$.

Number of equal arcs (l) in a curve, $n=\frac{\gamma}{\delta}$.

The length of any arc (l) whose chord is (c)

$l=\frac{1}{3}(8 \times \text{chord of } \frac{1}{2}l-c)$ or $c=l\left[1-\frac{l^2}{24\ R^2}\right]$ approximately.

The ordinate (y) for any abscissa (x) measured from the tangent-point along the tangent $=R-\sqrt{R^2-x^2}$.

A normal (d) taken at any point in a curve, when (t) is the intercepted distance along the tangent,

$$d=\sqrt{R^2+t^2}-R.$$

The ordinate (y') for any abscissa (x') measured from the middle of the whole chord along it

$$y'=\sqrt{R^2-x'^2}-\overline{R-V}$$

$$\text{vers } \frac{\gamma}{2}=1-\sqrt{\left(1-\frac{\text{vers in } \gamma}{2}\right)}.$$

Offsets from the production of a Series of equal chords.

Rectangular offset $= 2y$; y being the first tangential ordinate.

Oblique offset $=\frac{c^2}{R}$ when taken, at a distance along the produced chord equal to the chord itself (c),

Also sin $\delta=\frac{y}{c}$; and the radius $R=\frac{c^2}{2y}$.

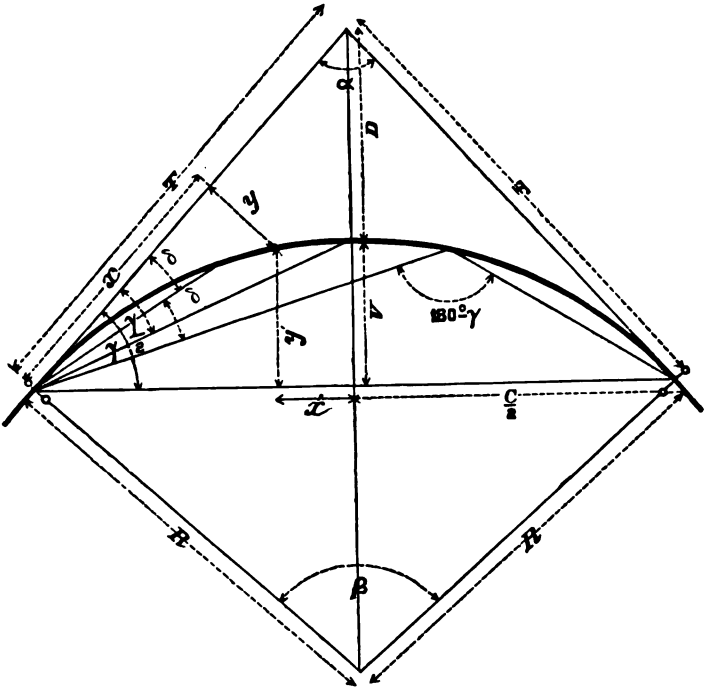


FIGURE 26. SYMBOLS USED IN FORMULÆ FOR CURVES.

TABLE OF CURVE-TANGENTS.

Giving curve-tangents for every angle of intersection, from a straight line, viz., 180°, to 60°, for a radius of 1000.

	50′	40′	30′	20′	10′	0′	Diff. for 1′	
179°	1·454	2·909	4·364	5·818	7·272	8·727	·1455	179°
178	10·181	11·636	13·090	14·545	16·000	17·455	·1455	178
177	18·910	20·365	21·820	23·275	24·730	26·186	·1456	177
176	27·641	29·097	30·553	32·009	33·465	34·921	·1456	176
175	36·377	37·834	39·220	40·747	42·204	43·661	·1457	175
174	45·118	46·576	48·033	49·491	50·949	52·408	·1459	174
173	53·866	55·325	56·784	58·243	59·703	61·163	·1460	173
172	62·623	64·083	65·543	67·004	68·465	69·927	·1462	172
171	71·389	72·851	74·313	75·775	77·238	78·702	·1464	171
170	80·165	81·629	83·094	84·558	86·023	87·488	·1465	170
169	88·954	90·420	91·886	93·354	94·821	96·289	·1468	169
168	97·757	99·225	100·694	102·164	103·634	105·104	·1470	168
167	106·575	108·046	109·518	110·990	112·462	113·935	·1473	167
166	115·409	116·883	118·358	119·833	121·308	122·784	·1476	166
165	124·261	125·738	127·216	128·694	130·173	131·652	·1479	165
164	133·132	134·613	136·094	137·575	139·057	140·540	·1483	164
163	142·024	143·508	144·993	146·478	147·964	149·451	·1487	163
162	150·938	152·426	153·914	155·403	156·893	158·384	·1491	162
161	159·875	161·367	162·860	164·353	165·847	167·342	·1495	161
160	168·838	170·334	171·831	173·329	174·827	176·327	·1500	160

TABLE OF CURVE-TANGENTS (*continued*). $R = 1000$.

	50′	40′	30′	20′	10′	0′	Diff. for 1′	
159°	177·827	179·327	180·829	182 331	183·835	185·339	·1504	159°
158	186·843	188·349	189·855	191·363	192·871	194·380	·1509	158
157	195·890	197·400	198·912	200·424	201·938	203·452	·1514	157
156	204·967	206·483	208·000	209·518	211·036	212·557	·1521	156
155	214·077	215·599	217·121	218·645	220·169	221·695	·1526	155
154	223·221	224·749	226·276	227·806	229·336	230·868	·1532	154
153	232·400	233·934	235·468	237·004	238·541	240·079	·1538	153
152	241·617	243·157	244·698	246·241	247·783	249·328	·1545	152
151	250·873	252·420	253·967	255·517	257·066	258·618	·1552	151
150	260·169	261·723	263·278	264·834	266·390	267·949	·1559	150
149	269·508	271·069	272·631	274·195	275·759	277·325	·1566	149
148	278·891	280·460	282·029	283·600	285·171	286·745	·1574	148
147	288·320	289·896	291·473	293·052	294·632	296·214	·1582	147
146	297·796	299·380	300·965	302·553	304·141	305·731	·1590	146
145	307·322	308·914	310·508	312·104	313·700	315 299	·1599	145
144	316·898	318·500	320·102	321·707	323·312	324·920	·1608	144
143	326·528	328·139	329·750	331·364	332·978	334·595	·1617	143
142	336·213	337·833	339·454	341·077	342·701	344·328	·1627	142
141	345·954	347·584	349 215	350·848	352·482	354·119	·1637	141
140	355·756	357·395	359·036	360 680	362·324	363·970	·1646	140

TABLE OF CURVE-TANGENTS (*continued*). $R = 1000$.

	50′	40′	30′	20′	10′	0′	Diff. for 1′	
139°	365·618	367·268	368·919	370·573	372·227	373·885	·1658	139°
138	375·543	377·204	378·866	380·530	382·196	383·864	·1668	138
137	385·534	387·205	388·878	390·554	392·231	393·911	·1680	137
136	395·591	397·275	398·961	400·647	402·335	404·026	·1691	136
135	405·719	407·414	409·110	410·810	412·510	414·214	·1704	135
134	415·918	417·626	419·335	421·046	422·759	424·475	·1716	134
133	426·192	427·918	429·634	431·358	433·084	434·812	·1728	133
132	436·543	438·276	440·010	441·748	443·487	445·229	·1742	132
131	446·972	448·719	450·467	452·217	453·970	455·726	·1756	131
130	457·484	459·244	461·006	462·771	464·538	466·308	·1770	130
129	468·079	469·854	471·630	473·409	475·191	476·975	·1784	129
128	478·762	480·551	482·342	484·137	485·933	487·732	·1799	128
127	489·534	491·338	493·145	494·955	496·767	498·582	·1815	127
126	500·399	502·219	504·041	505·867	507·694	509·525	·1831	126
125	511·359	513·195	515·034	516·875	518·719	520·567	·1848	125
124	522·417	524·270	526·125	527·984	529·845	531·709	·1864	124
123	533·576	535·446	537 319	529·195	541·074	542·956	·1882	123
122	544·840	546·728	548·619	550·512	552·409	554·310	·1901	122
121	556·212	558·118	560·027	561·939	563·854	565·773	·1919	121
120	567·694	659·619	571·547	573·478	575·412	577·350	·1938	120

TABLE OF CURVE-TANGENTS (*continued*). $R = 1000$.

	50′	40′	30′	Diff. for 1′	20′	10′	0′	Diff. for 1′	′
119°	579·291	581·235	583·183	·1948	585·134	587·088	589·045	·1957	119°
118	591·006	592·970	594·937	·1967	596·908	598·882	600·860	·1978	118
117	602·842	604·826	606 815	·1989	608·806	610·802	612·801	·1999	117
116	614·803	616·809	618·819	·2010	620·832	622·849	624·869	·2020	116
115	626·893	628·921	630·953	·2032	632·988	635·027	637·070	·2043	115
114	639·117	641·167	643·221	·2054	645·280	647·342	649·408	·2066	114
113	651·477	653·551	655·629	·2078	657·711	659·796	661·886	·2090	113
112	663·979	666·077	668·178	·2101	670·284	672·394	674·508	·2114	112
111	676 627	678·749	680·876	·2127	683·006	685·141	687·281	·2140	111
110	689·425	691·572	693·725	·2153	695·881	698·042	700·208	·2166	110
109	702·377	704·551	706·730	·2179	708·913	711·101	713·293	·2192	109
108	715·489	717·691	719·897	·2206	722·107	724·322	726·543	·2221	108
107	728·707	730·996	733·230	·2234	735·469	737·712	739·961	·2249	107
106	742·214	744·472	746·735	·2263	749·003	751·276	753·554	·2288	106
105	755·837	758·124	760·418	·2294	762·716	765·019	767·327	·2308	105
104	769·640	771·959	774·283	·2324	776·612	778·946	781·286	·2340	104
103	783·630	785·980	788·336	·2356	790·697	793 064	795·436	·2372	103
102	797·813	800·196	802·585	·2389	804·979	807·378	809·784	·2406	102
101	812 195	814·612	817·034	·2422	819·462	821·896	824·336	·2440	101
100	826·782	829·234	831·691	·2457	834·154	836·624	839·099	·2475	100

TABLE OF CURVE-TANGENTS (*continued*). $R = 1000$.

	50′	40′	Diff. for 1′	30′	20′	Diff. for 1′	10′	0′	Diff. for 1′	
99°	841·581	844·069	·2488	846·562	849·062	·2500	851·568	854·080	·2512	99°
98	856·599	859·124	·2525	861·655	864·192	·2537	866·736	869·287	·2551	98
97	871·844	874·407	·2563	876·976	879·593	·2577	882·135	884·725	·2590	97
96	887·322	889·924	·2602	892·534	895·151	·2617	897·774	900·404	·2630	96
95	903·041	905·685	·2644	908·334	910·994	·2658	913·659	916·331	·2672	95
94	919·010	921·697	·2687	924·390	927·091	·2701	929·799	932·515	·2716	94
93	935·238	937·969	·2731	940·706	943·451	·2745	946·204	948·965	·2761	93
92	951·732	954·509	·2777	957·292	960·083	·2791	962·882	965·689	·2807	92
91	968·504	971·327	·2823	974·157	976·995	·2838	979·842	982·697	·2855	91
90	985·560	988·432	·2872	991·312	994·200	·2888	997·096	1000·	·2904	90
89	1002·913	1005·835	·2922	1008·765	1011·703	·2938	1014·650	1017·607	·2957	89
88	1020·572	1023·546	·2974	1026·528	1029·520	·2992	1032·521	1035·530	·3009	88
87	1038·550	1041·578	·3028	1044·614	1047·660	·3046	1050·715	1053·780	·3065	87
86	1056·854	1059·938	·3084	1063·031	1066·134	·3103	1069·247	1072·369	·3122	86
85	1075·501	1078·642	·3131	1081·793	1084·955	·3162	1088·127	1091·309	·3182	85
84	1094·501	1097·702	·3201	1100·913	1104·137	·3224	1107·368	1110·612	·3244	84
83	1113·869	1117·131	·3262	1120·405	1123·691	·3286	1126·988	1130·294	·3306	83
82	1133·610	1136·941	·3331	1140·283	1143·633	·3350	1146·994	1150·368	·3374	82
81	1153·752	1157·150	·3398	1160·560	1163·977	·3417	1167·407	1170·850	·3443	81
80	1174·304	1177·770	·3466	1181·249	1184·738	·3489	1188·240	1191·754	·3514	80

TABLE OF CURVE-TANGENTS (*continued*). $R = 1000$.

	55′	50′	Diff. for 1′	45′	40′	Diff. for 1′	35′	30′	Diff. for 1′	
79°	1193·516	1195·281	·3530	1197·048	1198·818	·3540	1200·593	1202·370	·3554	79°
78	1214·896	1216·699	·3606	1218·504	1220·312	·3616	1222·124	1223·938	·3628	78
77	1236·735	1238·576	·3682	1240·421	1242·269	·3696	1244·120	1245·974	·3708	77
76	1259·051	1260·932	·3762	1262·817	1264·706	·3778	1266·598	1268·495	·3794	76
75	1281·862	1283·786	·3848	1285·714	1287·645	·3862	1289·579	1291·518	·3878	75
74	1305·190	1307·158	·3936	1309·129	1311·105	·3952	1313·084	1315·057	·3966	74
73	1329·054	1331·070	·4032	1333·086	1335·109	·4046	1337·133	1339·164	·4062	73
72	1353·480	1355·539	·4118	1357·607	1359·675	·4136	1361·750	1363·827	·4154	72
71	1378·489	1380·600	·4222	1382·716	1384·835	·4238	1386·959	1309·088	·4258	71
70	1404·107	1406·270	·4326	1408·438	1410·610	·4344	1412·787	1414·967	·4360	70
69	1430·361	1432·577	·4438	1434·800	1437·026	·4452	1439·258	1441·495	·4474	69
68	1457·278	1459·552	·4548	1461·831	1464·115	·4568	1466·403	1468·697	·4586	68
67	1484·889	1487·222	·4666	1489·561	1491·904	·4686	1494·252	1496·606	·4708	67
66	1513·222	1515·620	·4796	1518·020	1520·426	·4812	1522·838	1525·253	·4830	66
65	1542·319	1544·779	·4920	1547·245	1549·715	·4940	1552·192	1554·675	·4966	65
64	1572·208	1574·736	·5056	1577·269	1579·808	·5078	1582·353	1584·903	·5100	64
63	1602·927	1605·526	·5198	1608·132	1610·742	·5220	1613·359	1615·982	·5246	63
62	1634·519	1637·192	·5346	1639·871	1642·558	·5374	1645·250	1647·948	·5396	62
61	1667·024	1669·775	·5502	1672·534	1675·298	·5528	1678·075	1680·848	·6546	61
60	1700·490	1703·323	·5666	1706·164	1709·011	·5694	1711·866	1714·728	·6724	60

TABLE OF CURVE-TANGENTS (*continued*). $R = 1000$.

	25′	20′	Diff. for 1′	15′	10′	Diff. for 1′	5′	0′	Diff. for 1′	
79°	1204·149	1205·933	·3568	1207·719	1209·507	·3576	1211·301	1213·097	·3592	79°
78	1225·757	1227·579	·3644	1229·403	1231·232	·3658	1233·063	1234·897	·3968	78
77	1247·832	1249·693	·3722	1251·558	1253·428	·3738	1255·297	1257·172	·3750	77
76	1270·393	1272·297	·3808	1274·202	1276·113	·3822	1278·025	1279·942	·3834	76
75	1293·460	1295·406	·3892	1297·355	1299·307	·3914	1301·265	1303·225	·3920	75
74	1317·054	1319·044	·3890	1321·039	1323·036	·3994	1325·039	1327·045	·4012	74
73	1341·196	1343·233	·4074	1345·275	1347·320	·4090	1349·369	1351·422	·4106	73
72	1365·910	1367·995	·4170	1370·086	1372·180	·4188	1374·279	1376·381	·4204	72
71	1391·220	1393·357	·4274	1395·498	1397·643	·4290	1399·794	1401·948	·4308	71
70	1417·513	1419·343	·4380	1421·537	1423·735	·4396	1425·940	1428·157	·4414	70
69	1443·733	1445·980	·4494	1448·230	1450·485	·4510	1452·745	1455·009	·4528	69
68	1470·995	1473·298	·4606	1475·607	1477·919	·4624	1480·238	1482·561	·4646	68
67	1498·964	1501·328	·4728	1503·697	1506·071	·4748	1508·451	1510·835	·4768	67
66	1527·675	1530·103	·4856	1532·535	1534·973	·4880	1537·416	1539·865	·4898	66
65	1557·162	1559·655	·4986	1562·154	1564·660	·5012	1567·169	1569·686	·5034	65
64	1587·461	1590·023	·5124	1592·593	1595·168	·5150	1597 748	1600·335	·5174	64
63	1618·612	1621·246	·5268	1623·889	1626·536	·5294	1629·191	1631·851	·5320	63
62	1650·655	1653·366	·5422	1656 085	1658·809	·5448	1661·542	1664 279	·5474	62
61	1683 634	1686·426	·5584	1689·225	1692 031	·5612	1694·844	1697·663	·5638	61
60	1717 598	1720 473	·5750	1723·357	1726·247	·5780	1729·146	1732·050	·5808	60

TABLE OF CURVE-LENGTHS OR ARCS CORRESPONDING TO ANY ANGLE OF DEFLECTION, WHEN R = 1000.

Angle (Minutes)	Arc	Angle (Minutes)	Arc	Angle (Degrees)	Arc	Angle (Degrees)	Arc
′		′		°		°	
·05	·029	21	12·217	1	34·907	36	1256·637
·10	·058	22	12·799	2	69·813	37	1291·544
·15	·087	23	13·381	3	104 720	38	1326·450
·20	·116	24	13·963	4	139·626	39	1361·357
·25	·145	25	14·544	5	174·533	40	1396·263
·30	·175	26	15·126	6	209·440	41	1431·170
·35	·204	27	15·708	7	244·346	42	1466·077
·40	·233	28	16·290	8	279·253	43	1500·983
·45	·262	29	16·872	9	314·159	44	1535·890
·50	·291	30	17·493	10	349·066	45	1570·796
·55	·320	31	18·035	11	383·972	46	1605·703
·60	·349	32	18·617	12	418·879	47	1640·610
·65	·378	33	19·199	13	453·786	48	1675·516
·70	·407	34	19·780	14	488·692	49	1710·423
·75	·436	35	20·362	15	523·599	50	1745·329
·80	·465	36	20·944	16	558·505	51	1780·236
·85	·494	37	21·526	17	593·412	52	1815·142
·90	·524	38	22·107	18	628·319	53	1850·049
·95	·553	39	22·689	19	663·225	54	1884·956
′		40	23·271	20	698·132	55	1919·862
1	·582	41	23·852	21	733·038	56	1954·769
2	1·164	42	24·434	22	767·945	57	1989·675
3	1·745	43	25·016	23	802·852	58	2024·582
4	2·327	44	25·598	24	837·758	59	2059·489
5	2·909	45	26·180	25	872·665	60	2094·395
6	3·491	46	26·762	26	907·571	61	2129·302
7	4·072	47	27·343	27	942·478	62	2164·208
8	4·654	48	27·925	28	977·384	63	2199·115
9	5·236	49	28·507	29	1012·291	64	2234·021
10	5·818	50	29 089	30	1047·198	65	2268·928
11	6·399	51	29·670	31	1082·104	66	2303·835
12	6·981	52	30·252	32	1117·011	67	2338·741
13	7·563	53	30·834	33	1151·917	68	2373·647
14	8·145	54	31·416	34	1186·824	69	2408·554
15	8·727	55	31·997	35	1221·731	70	2443·461
16	9·308	56	32·579				
17	9·890	57	33·161				
18	10·472	58	33·743				
19	11·054	59	34·324				
20	11·636	60	34·907				

TABLE OF ORDINATES FOR EQUIDISTANT POINTS ON THE TANGENT. $R = 1000$.

Abscissa	Ordinate	Curve	Abscissa	Ordinate	Curve
5	·0125	5·000	205	21·2381	206·463
10	·0500	10·000	210	22·2986	211·573
15	·1125	15·001	215	23·3860	216·683
20	·2000	20·002	220	24·5002	221·812
25	·3125	25·004	225	25·6412	226·941
30	·4500	30·005	230	26·8093	232·071
35	·6127	35·007	235	28·0047	237·209
40	·8002	40·010	240	29·2270	242·358
45	1·0130	45·015	245	30·4770	247·517
50	1·2505	50·021	250	31·7541	252·675
55	1·5137	55·030	255	33·0589	257·843
60	1·8018	60·040	260	34·3915	263·021
65	2·1148	65·050	265	35·7514	268·209
70	2·4529	70·060	270	37·1396	273·386
75	2·8165	75·075	275	38·5570	278·593
80	3·2051	80·104	280	39·9999	283·790
85	3·6190	85·114	285	41·4725	289·007
90	4·0582	90·124	290	42·9734	294·224
95	4·5227	95·144	295	44·5027	299·450
100	5·0127	100·169	300	46·0609	304·696
105	5·5277	105·199	305	47·6477	309·941
110	6·0677	110·227	310	49·2635	315·197
115	6·6345	115·260	315	50·9084	320·452
120	7·2261	120·292	320	52·5825	325·727
125	7·8432	125·324	325	54·2861	331·012
130	8·4861	130·371	330	56·0191	336·296
135	9·1543	135·416	335	57·7818	341·600
140	9·8485	140·462	340	59·5747	346·923
145	10·5696	145·518	345	61·3973	352·237
150	11·3140	150·574	350	63·2503	357·570
155	12·0852	155·645	355	65·1337	362·912
160	12·8831	160·696	360	67·0477	368·264
165	13·7060	165·753	365	68·9924	373·626
170	14·5559	170·829	370	70·9682	379·008
175	15·4320	175·910	375	72·9751	384·389
180	16·3333	180·981	380	75·0134	389·790
185	17·2614	186·071	385	77·0836	395·210
190	18·2159	191·162	390	79·1851	400·631
195	19·1969	196·259	395	81·3186	406·070
200	20·2049	201·353	400	83·4848	411·510

TABLE OF ORDINATES FOR EQUIDISTANT POINTS ON THE TANGENT (*continued*). $R = 1000$.

Abscissa	Ordinate	Curve	Abscissa	Ordinate	Curve
405	85·6833	416·979	605	203·7745	649·767
410	87·9145	422·447	610	207·5986	656·060
415	90·1787	427·945	615	211·4729	662·382
420	92·4760	433·443	620	215·3982	668·742
425	94·8065	438·960	625	219·3752	675·132
430	97·1711	444·493	630	223·4049	681·551
435	99·5697	450·033	635	227·4878	688·009
440	102·0022	455·599	640	231·6252	694·496
445	104·4694	461·174	645	235·8175	701·031
450	106·9714	466·769	650	240·0658	707·586
455	109·5085	472·373	655	244·3715	714·179
460	112.0810	477·997	660	248·7344	720·820
465	114·6894	483·631	665	253·1567	727·491
470	117·3336	489·285	670	257·6390	734·212
475	120·0142	494·956	675	262·1822	740·960
480	122·7315	500·647	680	266·7879	747·757
485	125·4859	506·359	685	271·4569	754·603
490	128 2776	512·090	690	276·1907	761·492
495	131·1071	517·836	695	280·9903	768·420
500	133·9745	523·599	700	285·8572	775·401
505	136·8807	529·378	705	290·7928	782·421
510	139·8256	535·186	710	295·7983	789·500
515	142·8099	541·004	715	300·8755	769·617
520	145·8337	546·851	720	306·0260	803·802
525	148·8987	552·717	725	311·2511	811·035
530	152·0024	558·593	730	316·5528	818·317
535	155·1479	564·507	735	321·9329	825·667
540	158·3349	570·432	740	327·3931	833·065
545	161·5640	576·389	745	332·9355	840·531
550	164·8354	582·358	750	338·5622	848·065
555	168·1496	588·360	755	344·2752	855·654
560	171·5075	594·362	760	350·0769	863·298
565	174·9091	600·432	765	355·9697	871·036
570	178·3554	606·507	770	361·9561	878·829
575	181·8446	612·562	775	368·0388	886·715
580	185·3835	618·729	780	374·2205	894·665
585	188·9667	624·876	785	380·5043	902·694
590	192·5967	631·043	790	386·8932	910·810
595	196·2744	637·258	795	393·3906	919·023
600	200·	643·949	800	400·	927·293

TABLE OF ORDINATES FOR EQUIDISTANT POINTS ON THE TANGENT (*concluded*). $R = 1000$.

Abscissa	Ordinate	Curve	Abscissa	Ordinate	Curve
805	406·7250	935·671	905	574·5884	1131·381
810	413·5701	944·155	910	585·3918	1143·288
815	420·5391	952·727	915	596·5462	1155·505
820	427·6365	961·411	920	608·0816	1168·061
825	434·8664	970·201	925	620·0329	1181·035
830	442·2367	979·108	930	632·4405	1194·406
835	449·7501	988·128	935	645·3523	1208·258
840	457·4136	997·281	940	658·8255	1222·632
845	465·2336	1006·570	945	672·9297	1237·594
850	473·2173	1015·985	950	687·7501	1253·234
855	481·3710	1025·546	955	703·3942	126 ·659
860	489·7060	1035·271	960	720·	1286·996
865	498·2281	1045·142	965	737·7501	1305·438
870	506·9483	1055·197	970	756·8951	1325·229
875	515·8771	1065·436	975	777·7952	1346·715
880	525·0263	1075·860	980	801·0025	1370·462
885	534·4089	1086·487	985	827·4457	1397·369
890	544·0394	1097·344	990	858·9327	1428·713
895	553·9339	1108·420	995	900·1251	1470·750
900	564·1101	1119·765	1000	1000·	1570·796

Section 3. THE VARIOUS SYSTEMS OF RANGING CURVES.

The curves usually adopted for centre lines of railways and canals are circular, and it is for these only that the foregoing data for determining a curve are applicable. Other curves that have been occasionally used are the parabola, Gravatt's and Froude's curves, and the compound circular curve of the author; but these will be noticed in a subsequent section.

Ranging a curve or a centre line simply consists in setting out a number of pegs along the proposed curved centre line of the works; sometimes they are placed at

the same intervals as the permanent stakes, which may be 50, 66, 100 or 200 feet; but this is comparatively of little use, if the pegs on each curve are started from each tangent point, as the permanent stakes will afterwards require setting out throughout the line, and no saving of labor is effected; it is hence equally usual to range the curve with pegs at any convenient intervals, and to set-out the permanent stakes afterwards by measurement on the alignment thus obtained.

The following are practical methods of ranging simple circular curves of centre lines on canals, railways, and main roads.

1. With one theodolite and a chain.
2. With two theodolites and observers, without a chain.
3. Kröhnke's tangential system, giving equidistant points on the curve.
4. Baker's parabolic system.
5. The author's first polygonal system, or six-point system, giving unequal intervals on the curve.
6. The author's equidistant six-point system, giving equal intervals on the curve.
7. The single chord system.

These methods will here be described separately, illustrated by figures, and accompanied by small tables to facilitate their application in the field.

1st. *The one-theodolite and chain system.*

The method of ranging a curve most usually adopted in setting-out with one theodolite is the following:

Having obtained the necessary preliminary data, set

a theodolite over one of the tangent points, obtain from the following table (on p. 230) the angle of deflection cor

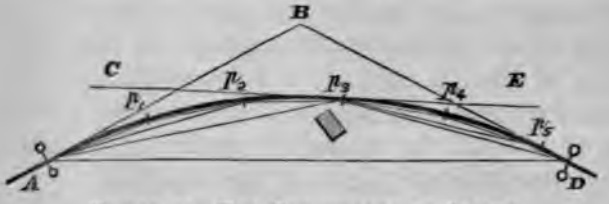


FIGURE 27. THE ONE-THEODOLITE SYSTEM.

responding to an arc or chord of 50 feet to the adopted radius, and set it off from the tangent, fix or hold one end of a 50-feet measure, whether chain or metallic tape, to the tangent point, and keeping it stretched, cause it to be moved around that point as a centre until the other end falls in the direction set off by the instrument, mark that end, thus obtaining one point in the curve. Keeping the instrument at the tangent point, set off successively the double, treble, etc., of the angle originally obtained, by continually adding its value to the last amount, and cause measurements of 50 feet to be made from the last point set-out in each case so as to fall in the corresponding directions set-out: the intersections, or points, may be often set-out in this way throughout the whole of the curve, without moving the instrument. On arriving at the direction of the other tangent point, the total angle of deflection should be observed, and its value compared with half the supplement of the angle of intersection originally obtained; thus affording a check on the work.

As the total length of curve is rarely an exact multiple of 50, there is generally a short piece remaining

after setting out a number of these points, between the last point and the tangent point. To verify this, observe the angle of deflection subtended by this piece; also measure the piece, multiply its value by 1000, and divide it by the radius used, and obtain, corresponding to this result, the subtended angle given in the table on p. 232; these values should be nearly equal.

In case obstacles should prevent all the points from being set-out from the one tangent point, move the instrument to the last point set-out, having the last angle clamped on its upper plate; sight with this on the original tangent point, and set off the angle, thus obtaining a new tangential direction from the production of which angles can be set off for more points as at the commencement.

In the accompanying figure (27), the instrument is supposed to be placed at A, the angles BAp_1, BAp_2, BAp_3, being set off from the tangent BA with the distances or chords, Ap_1; p_1p_2; p_2p_3; measured in the manner described. An obstacle prevents the point p_4 from being seen from A; the instrument is then moved to p_3 having the angle Bap_3 clamped on it, the lower circle being moveable, with which sighting A, the angle Ap_3C is set off, thus obtaining the direction of a new tangent Cp_3E: from which the angles Ep_3p_4; Ep_3p_5; etc., Ep_3D; are laid off in the same way as the others.

This method of setting-out assumes that the chord of an arc is equal to the arc itself, and hence the check before-mentioned will never be exactly right, although sufficiently so for many practical purposes. The error for 50-feet arcs when the radius is greater than 500 feet and less than 2000 feet, may be neglected—when the radius

is less than 500 feet, 25-feet arcs may be set-out with greater correctness: when the radius is greater than 2000 feet, 100-feet arcs may be considered equal to their chords. In obtaining angles of deflection from the tables under these latter circumstances, it is useful to remember that when both the arc and the radius are doubled or halved, the angle is doubled or halved also.

It is exceedingly rare that greater exactness is required than the foregoing method admits of: but in such cases, the value of the chord corresponding to the arc of 50 should be calculated and used instead of a 50-feet chord; for this purpose Rankine's approximate formula (see p. 214) may be sufficiently precise.

TABLE OF ANGLES OF DEFLECTION SUBTENDED BY USUAL CHORDS FOR VARIOUS RADII.

Radius in feet	Angle for Arc of 50′	Radius in feet	Angle for Arc of 50′
	° ′		′
250	5 43·77	2000	42·97
300	4 46·48	2250	38·19
350	4 5·55	2500	34·38
400	3 34·86	2750	31·25
450	3 10·99	3000	28·65
500	2 51·89	3250	26·44
600	2 23·24	3500	24·55
700	2 3·46	3750	22·92
800	1 47·43	4000	21·44
900	1 35·49	4250	20·26
1000	1 25·94	4500	19·10
1100	1 18·13	4750	18·09
1200	1 11·62	5000	17·19
1300	1 6·12	5280	16·28
1400	1 1·39		
1500	57·29		
1600	53·71		
1700	50·55		
1800	47·75		
1900	45·23		

Radius in Gunter's Chains	Angles for Arc of 50′	Angles for Arc of 66′
	° ′	° ′
5	4 20·44	5 43·77
7½	2 53·42	3 49·18
10	2 10·82	2 51·89
12½	1 44·17	2 17·51
15	1 26·81	1 54·59
20	1 5·11	1 25·94
25	52·87	1 8·75
30	43·40	57·29
35	37·20	49·11
40	32·55	42·97
45	28·94	38·19
50	26·04	34·38
55	23·68	31·25
60	21·70	28·65
65	20·03	26·44
70	18·60	24·55
75	17·36	22·92
80	16·28	21·49

2nd. *The two-theodolite system.*

Setting-out a curve with two-theodolites is, when circumstances permit of it, by far the most rapid method of ranging.

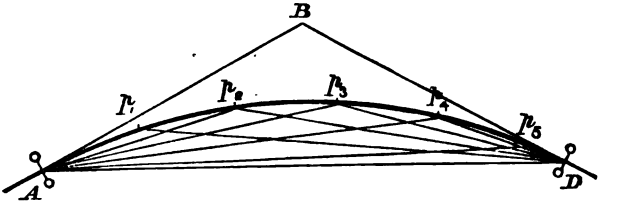


FIGURE 28. THE TWO-THEODOLITE SYSTEM.

The two operators set their instruments on the two tangent points, the one setting off from the one tangent any convenient angles, and their multiples (round numbers are generally chosen), the other simultaneously setting-out angles made with the other tangent equal to the differences between those angles and the total angle of deflection for the whole curve, mentioned as (γ) in the formulæ ; the intersections of all such corresponding pairs of directions are points in the curve.

The small angles should be so chosen with the help of the following table as to subtend arcs, either less than 50 or 100, according to the distances required for points along the curve. To check these points, an instrument is set up at each one of them, and if the angle contained by the directions of the two tangent points amounts exactly to the supplement of the angle of deflection, viz. $180° - \gamma$, that point is correct.

For example—(refer to figure 28).

Let $R = 2000$ feet and $\gamma = 30°$ (BAD) it is required to set-out points not more than 50 feet apart.

The angle corresponding to this arc will be that given for a radius of 1000 in the Table for an arc of

$$\frac{50 \times 1000}{2000} \text{ or } 25 \text{ feet}$$

Thus for	20 feet. . . .	34′·38
And for	5 feet. . . .	8·59
Hence for	25 feet. . . .	42′·97

consequently 40′ subtending less than 50 feet will be the largest convenient angle in round numbers.

The one operator will then set off an angle $BAp_1 = 40'$ simultaneously with the other setting off an angle of $30° - 40' = 29°20'$, viz., BDp_1 from his tangent: the intersection of these two directions is in one point in the curve, p_1; then the first operator will set off the angle $BAp_2 = 1° 20'$ while the other sets off the angle $BDp_2 = 28°40'$; the one adding and the other subtracting 40′ each time until all the points are ranged in.

To check the work they would then go to each of the points $p_1\ p_2\ p_3$ etc., thus set out, and see that each of the angles Ap_1D, Ap_2D, Ap_3D, etc., were equal to $180° - \gamma = 150°$.

The following table, constructed for a radius of 1000, may be used for obtaining either the angle subtended at the tangent-point by any arc or length of curve: for the converse purpose the table of curve-lengths and arcs on p. 223 will give the arc subtended by a given angle, for any radius whatever; as illustrated in the example already given.

TABLE OF ANGLES OF DEFLECTION SUBTENDED BY ANY LENGTH OF ARC WHEN $R = 1000$.

Arc	Angle	Arc	Angle
	′		° ′
·005	·01	10	17·19
·01	·02	20	34·38
·02	·03	30	51·57
·03	·05	40	1 8·76
·04	·07	50	1 25·77
·05	·09	60	1 43·13
·06	·10	70	2 0·32
·07	·12	80	2 17·51
·08	·14	90	2 34·69
·09	·16		
		100	2 51·89
·1	·17	200	5 43·77
·2	·34	300	8 35·66
·3	·52	400	11 27·55
·4	·69	500	14 19·44
·5	·86	600	17 11·32
·6	1·03	700	20 3·21
·7	1·20	800	22 55·10
·8	1·38	900	25 46·99
·9	1·55		
		1000	28 38·87
1	1·72	2000	57 17 75
2	3·45	3000	85 56·62
3	5 16	4000	104 35·50
4	6·88	5000	143 14·40
5	8·59	6000	171 53·24
6	10·31		
7	12·03		
8	13·75		
9	15·47		

3rd. *Kröhnke's tangential system.*

Kröhnke's system consists in setting-out equidistant points on the curve by ordinates from the two tangents.

In doing this the tangent points B, C (figure 29), are taken as the origins of rectangular co-ordinates, the abscissæ being taken along the tangents and measured from them as Be, Be_1, Be_2, Be_3, and Cg, Cg_1, Cg_2, Cg_3, etc., the ordinates at right angles to them as ef, e_1f_1, e_2f_2, etc., gh, g_1h_1, g_2h_2, etc.; thus giving number of points f, f_1, f_2, etc., h_1, h_2, etc.; the values of ordinates and abscissæ corresponding to equidistant points on the curve may be obtained from the following Table. When the two last points set-out on either branch as f_4 and h_4, are sufficiently near each other, the curve is set-out; and

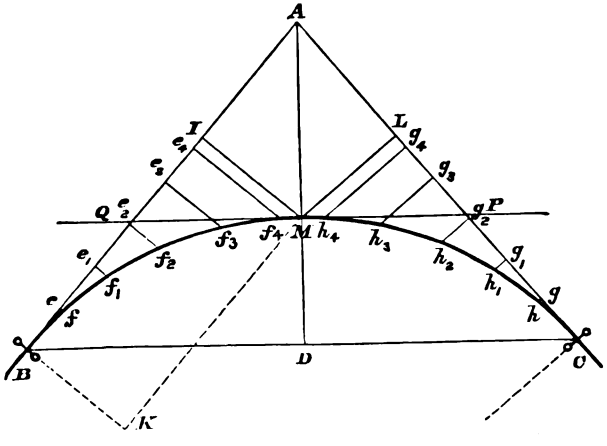


FIGURE 29. KRÖHNKE'S SYSTEM.

if for any special reason the middle point of the curve M is wanted, it may be obtained sufficiently closely by bisecting the distance between the two last points.

This setting-out may be checked by comparing the calculated total length of curve which may be obtained from the preceding Table of curve-lengths, as due to the angle of deflection, or (which ought to have been previously found and registered on the plan) with the sum of the distances between the equidistant points, together with the small additional pieces f_4M, $h\ M$ which must be measured on the ground.

Thus, if L length of curve $= 523{\cdot}7$, and five points have been set-out on either branch at distances of 50′ apart, then $523{\cdot}7 - 500 = 23{\cdot}7$, which ought to be the sum of the additional pieces measured on the ground; and $f_4M = h_4M$ should $= 11{\cdot}85$ feet. When the radius is large and the angle of intersection small, the ordinates near the middle point of the curve become inconveniently long, and if not laid off at very accurate right angles with the abscissa, great error will result. As for instance with a radius of 5000 feet and a central angle of 100°, the last ordinates will be about 1000 feet long, and the serious error from a defective right angle may be imagined.

To avoid these inconveniences it is necessary to make use of an additional tangent at the middle point of the curve, making this a new origin for rectangular co-ordinate, and measuring abscissa both ways from it, the ordinates will be smaller—thus each branch of the curve will be treated as the whole curve was in the first instance. In this case the tangent QMP must be set-out very accurately, the distances BQ and CP must each $= R \tan \frac{\beta}{4}$ and M should be at the middle of QP; as a further check on the position of the point M, it may be set-out by means of abscissa and ordinate BI, IM from the original tangent, where

$$BI = R \sin \frac{\beta}{4}, \text{ and } IM = R \left(1 - \cos \frac{\beta}{2}\right).$$

β being the centre angle or supplement of the angle of intersection (a); this being done, M should fall in the line QP. Should it happen to be more convenient, BK and KM may be measured instead of BI and IM, the distances being identical.

The table herewith given is confined to a radius of 1000; and from this ordinates and abscissæ for equidistant points on curves having other radii may be calculated by either multiplying or dividing the whole of the three corresponding quantities—curve, abscissa, and ordinate—by the ratio that the given radius bears to 1000. In the author's Curve Book, published in 1869, such results were given for sixteen values of radius, and occupied fifteen pages of tabular matter, which were seldom used, and are hence not here reproduced. In Kröhnke's work a comparatively large table, consisting of 47 pages, and containing a great deal of repetition of figure, is given to enable one to set-out curves on this system.

There is a great disadvantage in this system, that of requiring a bisection of the curve, and sometimes even many bisections, when the offsets from the tangent become long; and this necessitates the use of an angular instrument. The presumed advantage of this system, viz., that of setting-out points for permanent stakes that are exactly 50 feet or 100 feet apart on the curve itself is unreal; for as they happen to commence from the tangent point of one curve, they would not suit that purpose on other curves further on, and hence the intention fails.

ORDINATES AND ABSCISSÆ FOR EQUIDISTANT POINTS ON A CURVE, TO A RADIUS OF 1000.

Curve	Abscissa	Ordinate	Curve	Abscissa	Ordinate
10	9·9999	·0500	400	389·4180	78·9390
20	19·9986	·2000	410	398·6092	82·8792
30	29 9954	·4500	420	407·7606	86·9110
40	39 9894	·8000	430	416·8710	91·0340
50	49·9792	1·2498	440	425·9396	95·2482
60	59 9640	1·7994	450	434·9656	99·5528
70	69·9430	2·4488	460	443·9482	103·9474
80	79·9148	3·1980	470	452 8864	108·4316
90	89 8786	4·0470	480	461·7792	113·0050
100	99·8334	4·9958	490	470·6238	117·6672
110	109·7782	6·0440	500	479·4254	122·4176
120	119·7122	7·1916	510	488·1772	127·2556
130	129·6342	8·4382	520	496·8802	132·1808
140	139·5432	9·7840	530	505·5334	137·1928
150	149·4382	11·2288	540	514·1360	142·2912
160	159·3182	12·7726	550	522·6872	147·4754
170	169·1824	14·4152	560	531·1862	152·7448
180	179·0296	16 1564	570	539·6320	158·0990
190	188·8588	17·9960	580	548·0238	163·5374
200	198·6692	19·9336	590	556·3608	169 0594
210	208·4598	21 9690	600	564·6422	174·6644
220	218·2294	24·1022	610	572·8672	180·3520
230	227 9774	26·3332	620	581 0350	186·1216
240	237·7024	28·6618	630	589·1448	191·9726
250	247·4038	31·0876	640	597·1956	197·9044
260	257 0806	33·6102	650	605·1866	203·9164
270	266·7316	36·2294	660	613·1170	210·0080
280	276·3558	38·9448	670	620·9860	216·1786
290	285·9522	41·7562	680	628·7930	222·4274
300	295·5200	44·6634	690	636 5370	228·7540
310	305·0584	47·6662	700	644·2174	235·1578
320	314·5664	50·7644	710	651·8336	241·6380
330	324·0430	53·9576	720	659·3848	248·1942
340	333·4872	57·2454	730	666·8696	254·8256
350	342 8980	60·6374	740	674·2878	261·5314
360	352·2744	64·1034	750	681·6386	268·3110
370	361·6154	67·6726	760	688·9214	275·1638
380	370·9202	71·3356	770	696·1352	282·0892
390	380·1880	75·0908	780	703 2796	289·0864

Curve	(continued) Abscissa	Ordinate	Curve	(continued) Abscissa	Ordinate
790	710·3534	296·1546	920	795·6008	394·1798
800	717·3562	303·2932	930	801·6200	402·1660
810	724·2872	310·5016	940	807·5580	410·2120
820	731·1458	317·7988	950	813·4154	418·3170
830	737·9314	325·1242	960	819·1916	426·4802
840	744·6432	332·5370	970	825 8858	435·7006
850	751·2804	340·0166	980	830·4974	442 9774
860	757·8426	347·5622	990	836·0260	451·3100
870	764·3290	354·1732	1000	841·4710	459·6971
880	770·7390	362·8486	1010	846·8318	468 1392
890	777 0720	370 5878	1020	852·1080	476·6340
900	783·3272	378·3900	1030	857·2988	485·1812
910	789·5040	386·2542	1040	862·4040	493·7798

4th. *Baker's parabolic system.*

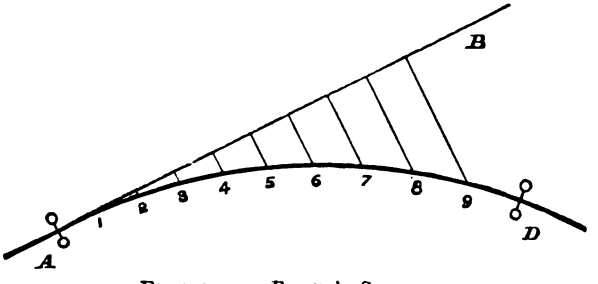


FIGURE 30. BAKER'S SYSTEM.

This method was formerly much used from its possessing the advantages of requiring few figures for use in the field. The abscissæ are equal and are measured along the tangent from one tangent point; at the end of the first chain, whether of 66 or of 100 feet, an ordinate (y) is set out at right angles to the tangent to obtain one point on the curve. The first ordinate (y) is calculated for any abscissæ (x) and any radius R by the formula

$$y = R - \sqrt{R^2 - x^2}$$

(These values corresponding to a radius of 1000 have been given in the table of ordinates at the end of the last section at pp. 223, 225, and values of y or first ordinates are given in the following table for various radii, and certain usual and convenient values of x the first abscissa.) Then the ordinates to be set-out successively have the following values:

At the end of the	1st	chain, ordinate	$=y$
„ „	2nd	„	$=2^2 \times y = 4y$
„ „	3rd	„	$=3^2 \times y = 9y$
etc.	etc.	etc.	
„ „	n^{th}	„	$=n^2 y.$

It must be evident to everyone that such an equation could not be true for a circular curve, being parabolic; we must not, however, entirely throw aside such a convenient method, but notice between what limits we may safely use it for circular curves.

A parabola may, for many practical purposes, be considered to approximate to a circular curve when the versine is not more than one-eighth of the chord. These circumstances in practice rarely occur on sharp curves of roads, railways, or canals, but frequently on flat curves of large radius; and we may hence consider 2000 feet as generally a safe minimum limit for the radius, as it is comparatively rare that a curve of so large a radius goes round a large portion of its circle.

Under the above-mentioned limits, therefore, the parabola, or the tangential system of Baker, may be safely adopted, and treated as a circular curve; beyond these limits if a parabolic curve be preferred to a circular one, as is often the case on the French railways, it becomes necessary to calculate the radius of curvature

at many points throughout it, both for the camber and elevation of the rail, and this involves additional work.

This system seems at first sight to be exceedingly convenient; the radius and the length of chain being determined, the length of the first ordinate is all that is absolutely required, and the rest can be worked out in a moment, during the setting-out. But in practice the ordinates become long, after the first few, and then the slightest inaccuracy in setting-out the right angle of the ordinate vitiates extremely the position of the point on the curve; hence a new tangent must be set-out and a theodolite must be brought to bear on the work. This system, therefore, cannot be considered practically superior to that of Kröhnke, as it possesses the same defect; while its advantage of requiring but few figures for use is counterbalanced by the disadvantage that the curve it sets out is parabolic, and hence is only an approximation to a circular curve suitable to special limited circumstances.

TABLE OF FIRST ORDINATES, AT THE EXTREMITY OF ONE CHAIN MEASURED ON THE TANGENT, FOR VARIOUS RADII.

Radius in Chains	Ordinate in Chains	Radius in Chains	Ordinate in Chains
$7\frac{1}{2}$	0·067	25	0·020
10	0·050	30	0·017
$12\frac{1}{2}$	0·040	35	0·014
15	0·033	40	0·012
20	0·025		

TABLE OF FIRST ORDINATES, AT THE EXTREMITY OF TWO CHAINS MEASURED ON THE TANGENT, FOR VARIOUS RADII.

Radius in Chains	Ordinate in Chains	Radius in Chains	Ordinate in Chains
40	0·050	65	0·031
45	0·044	70	0·028
50	0·040	75	0·027
55	0·036	80	0·025
60	0·033		

TABLE OF FIRST ORDINATES, AT THE EXTREMITY OF 50 FEET, MEASURED ON THE TANGENT.

Radius in feet	Ordinate in feet	Radius in feet	Ordinate in feet	Radius in feet	Ordinate in feet
500	2·505	1000	1·250	1500	0·833
600	2·086	1100	1 136	1600	0·780
700	1·788	1200	1·041	1700	0·734
800	1·564	1300	0·962	1800	0·700
900	1·389	1400	0·893	1900	0·658

TABLE OF FIRST ORDINATES, AT THE EXTREMITY OF 100 FEET, MEASURED ON THE TANGENT.

Radius in feet	Ordinate in feet	Radius in feet	Ordinate in feet	Radius in feet	Ordinate in feet
2000	2·501	3250	1·536	4500	1·112
2250	2·223	3500	1·419	4750	1·052
2500	2·005	3750	1·333	5000	1·000
2750	1·818	4000	1·250	5280	0·947
3000	1·667	4250	1·178		

5th. *The author's first polygonal system.*

The radii of all the curves throughout the line being previously determined, and the tangent points fixed, the setting-out of the curve commences from one tangent point. Measure from this point along the tangent six equal distances, whether chains or fifty-feet distances, according as the radius used is in chains or feet, and from these six points set off at right angles to the tangent the distances corresponding to them given in the table, or copied from it on a slip of paper; the points thus obtained will be six points on the curve.

It now becomes necessary to get a new tangent to the curve touching at the sixth point: then from this new tangent six more points may be set-out with the same numbers in exactly the same way: and this operation may be repeated until the whole of the curve is set-out.

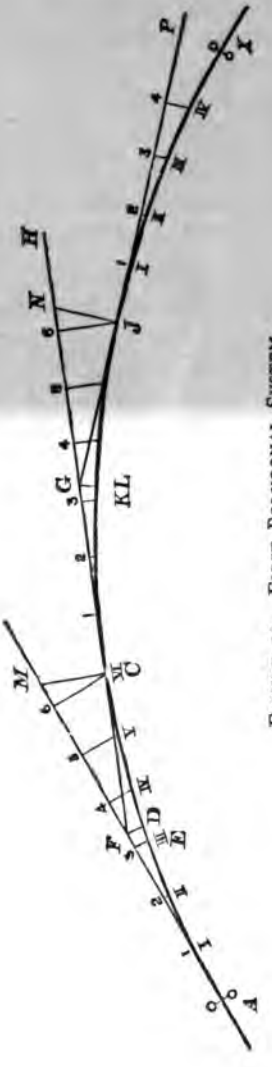


FIGURE 31. FIRST POLYGONAL SYSTEM.

In order to get the direction of the new tangent onwards from the sixth point, measure from the third point in the direction of the fourth the small extra distance *ED*, given in the tabular data: thus obtaining a point on the curve exactly half-way between the starting and the sixth point, or in other words, the middle point of that portion of curve. From this point measure the normal distance *DF*, also given, in such a way that the other end of it may fall in the original tangent. The point in the tangent thus obtained gives a direction taken through the sixth point on the curve onwards, which is that of the new tangent.

This operation may be repeated for every new tangent, and will be found to be practically correct, the theoretical error caused by measuring the small extra distance *ED* as a chord instead of on the curve being insignificant in practice.

The check given in the table is the distance *CM* from the sixth point on the curve to a point on the original tangent, and is measured in the same way as the quantity *DF*. The point thus obtained gives a direction, taken through the twelfth point on the curve onwards, which is the direction of the third tangent used. And this will be found a useful check on the direction of this tangent, obtained by means of the second tangent in the manner previously described.

The tabular quantity (curve), gives the exact length of curve set out for every six points.

The few numbers required for setting-out a curve to any one radius may be copied on a slip of paper from the table, and after a little use, may even be borne in mind without that aid.

The first portion of the table will be found most

applicable for radii in Gunter's chains, up to a mile or 80 chains: the intervals when the radius is more than 40 chains, being double or 2 chain distances: the latter part again is more suitable for radii in hundreds of feet up to 5280 feet or 1 mile, and for intervals of 50 or 200 feet: but the numbers given have a sufficiently wide range for setting-out a curve, either with inches, yards, mètres, or any unit whatever.

For Example.

In setting out a curve to the radius 900 feet by offsets from six points, taken 50 feet apart on successive tangents.

The tabular quantities are

	Abscissa	Ordinate
	50	1·39 feet
	100	5·57 ,,
	150	12·59 ,,
	200	22·50 ,,
	250	35·33 ,,
	300	51·47 ,,
Extra; distance from third point to middle point of the piece of curve . .		2·22 ,,
Normal; distance from this to tangent .		13·15 ,,
Check		54·59 ,,
Curve set-out up to sixth point . .		305·85 ,,

Referring to figure 31, the points marked 1, 2, 3, 4, 5, and 6, are the equidistant points 50 feet apart on the tangent, as given in the set above, in distances from *A*, viz., 50, 100, 150, 200, 250, 300; from these points the rectangular offsets given in the slip as 1·39, 5·57, 12·59, etc., are set off, giving as points on the curve the points marked I., II., III., IV., V., VI.

Now to obtain a new tangent at the point VI., first measure from point III. towards point IV. the short dis-

tance ED, given on the slip just quoted as $M = 2{\cdot}22$ feet, and thus fix the point D. Now put the handle of the chain or tape on the peg at D, and taking the distance $DF = 13{\cdot}15$ feet on the chain, let it rotate freely and range this length so that F may fall exactly in the tangential line AM; this can be easily done by an observer standing at 6, the sixth point on the tangent and looking in the direction of A. The point F being thus obtained the direction $FCGH$ through VI., the sixth point on the curve, is the new tangent required.

From the new tangent six more points can be set-out with the same foregoing figures, and after that a third tangent may be obtained in the same way as the last; and so on to the end of the curve. Each time that six points are set-out the length of curve set-out by them amounts to 305·85 feet, and the distances AD, DC, CL are each 152·92 feet. CM or IN is the quantity given as the check normal = 54·59 feet; it is, when required, laid off from C or $\mathcal{J}$ in the same way that the normal DF was laid off from D; the extremity M being in the tangential direction is marked by a pole; on arrival at $\mathcal{J}$ it will be found that the points G and M are in exactly the same direction, if the curve has been carefully set-out, and this affords a useful check on the work.

The table and the figure are made to suit a six-point system, but the same principle might be applied to four, eight, or any even number of points, not requiring long offsets. The advantage of this system is that angular instruments are entirely dispensed with, a cross-head staff being sufficient for all alignment and setting-out the right angles; the method is so simple that after explaining it once or twice in practice, good chainmen can do it by themselves.

TABLE FOR RADII AND DISTANCES IN CHAINS AND DECIMALS.

Length of Abscissa	R = 7½ Ord.	R = 10 Ord.	R = 12½ Ord.	R = 15 Ord.	R = 20 Ord.	R = 25 Ord.	R = 30 Ord.	R = 35 Ord.	R = 40 Ord.
1	·067	·050	·040	·033	·025	·020	·017	·014	·012
2	·272	·202	·161	·134	·100	·080	·069	·057	·050
3	·627	·461	·365	·303	·226	·181	·150	·129	·113
4	1·156	·835	·657	·543	·404	·322	·268	·229	·201
5	2·509	1·339	1·044	·858	·635	·505	·419	·359	·314
6	3·000	2·000	1·534	1·252	·921	·731	·606	·518	·453
Extra Normal	·391	·171	·099	·066	·036	·022	·015	·011	·008
Check	·884	·542	·402	·323	·234	·185	·153	·130	·114
Curve	5·000	2·500	1·749	1·366	·966	·753	·719	·526	·458
	6·955	6·435	6·258	6·173	6·094	6·058	6·041	6·029	6·023

Length of Abscissa	R = 40 Ord	R = 45 Ord.	R = 50 Ord.	R = 55 Ord.	R = 60 Ord.	R = 65 Ord.	R = 70 Ord.	R = 75 Ord.	R = 80 Ord.
2	·050	·044	·040	·036	·033	·031	·028	·027	·025
4	·201	·178	·160	·146	·137	·123	·114	·106	·100
6	·453	·402	·361	·328	·301	·276	·257	·240	·225
8	·808	·717	·644	·585	·536	·494	·459	·428	·401
10	1·270	1·125	1·010	·917	·839	·774	·718	·669	·627
12	1·842	1·629	1·461	1·325	1·212	1·117	1·036	·966	·905
Extra Normal	·073	·055	·044	·036	·030	·026	·022	·018	·017
Check	·469	·413	·369	·334	·305	·281	·261	·243	·227
Curve	1·931	1·681	1·505	1·358	1·238	1·137	1·052	·979	·916
	12·188	12·147	12·118	12·097	12·081	12·069	12·059	12·055	12·046

TABLE FOR RADII AND DISTANCES IN FEET AND DECIMALS.

Length of Abscissa	R = 200 Ord.	R = 250 Ord.	R = 300 Ord.	R = 350 Ord.	R = 400 Ord.	R = 450 Ord.	R = 500 Ord.
25	1·569	1·253	1·043	·874	·782	·695	·625
50	6·351	5·051	4·195	3·589	3·137	2·786	2·506
75	14·595	11·515	9·526	8·130	7·094	6·293	5·657
100	26·795	20·871	17·157	14·589	12·702	11·252	10·102
125	43·875	33·494	27·282	23·083	20·033	17·709	15·877
150	67·712	50·	40·192	33·772	29·190	25·736	23·030
Extra	7·929	4·263	2·737	1·926	1·431	1·112	·887
Normal	19·415	13·543	10·583	8·761	7·503	6·575	5·859
Check	102·373	62·498	46·410	37·379	31·486	27·296	24·143
Curve	169·613	160·873	157·079	155·019	153·755	152·924	152·348

Length of Abscissa	R = 500 Ord.	R = 600 Ord.	R = 700 Ord.	R = 800 Ord.	R = 900 Ord.	R = 1000 Ord.	R = 1100 Ord.	R = 1200 Ord.
50	2·505	2·086	1·788	1·564	1·389	1·250	1·136	1·041
100	10·100	8·391	7·179	6·975	5·572	5·013	4·554	4·173
150	23·030	19·052	16·260	14·188	12·585	11·314	10·275	9·411
200	41·740	34·314	29·179	25·403	22·503	20·205	18·333	16·781
250	66·985	54·564	46·165	40·065	35·327	31·754	28·784	26·401
300	100·	80·474	67·543	58·290	51·472	46·061	41·721	31·105
Extra	8·525	5·474	3·852	2·862	2·224	1·774	1·453	1·215
Normal	27·082	21·166	17·523	15·006	13·149	11·718	10·586	9·641
Check	124·997	92·820	74·757	62·976	54·592	48·288	43·341	39·341
Curve	321·747	314·159	310·038	307·511	305·848	304·696	303·849	303·190

TABLE FOR RADII AND DISTANCES IN FEET AND DECIMALS.

Length of Abscissa	R = 1300 Ord.	R = 1400 Ord.	R = 1500 Ord.	R = 1600 Ord.	R = 1700 Ord.	R = 1800 Ord.	R = 1900 Ord.	R = 2000 Ord.
50	·962	·893	·833	·780	·734	·700	·658	·625
100	3·848	3·750	3·437	3·428	2·944	2·780	2·642	2·501
150	8·684	8·052	9·019	7·046	6·630	6·260	5·928	5·633
200	15·470	14·359	13·393	12·558	12·979	11·143	10·555	10·025
250	24·264	22·502	20·976	19·651	18·482	17·447	16·519	15·686
300	35·088	32·520	30·307	28·376	28·106	25·171	23·855	22·628
Extra	1·018	·871	·760	·671	·597	·501	·452	·424
Normal	8·862	8·214	7·635	7·142	6·710	6·325	5·997	5·684
Check	36·059	33·292	30·929	28·888	27·107	25·531	24·139	22·888
Curve	302·719	302·334	302·030	301·786	301·582	301·401	301·281	301·148

Length of Abscissa	R = 2000 Ord.	R = 2250 Ord.	R = 2500 Ord.	R = 2750 Ord.	R = 3000 Ord.	R = 3250 Ord.	R = 3500 Ord.
100	2·501	2·223	2·005	1·818	1·667	1·536	1·419
200	10·025	8·904	8·013	7·309	6·874	6·159	5·719
300	22·628	20·089	18·065	16·411	15·638	13·875	12·880
400	40·409	35·841	32·208	29·248	26·786	24·709	22·933
500	63·508	56·259	50·512	45·835	41·952	38·691	35·897
600	92·121	81·475	73·068	66·352	60·614	55·865	51·813
Extra	3·548	2·772	2·219	1·815	1·521	1·303	1·089
Normal	23·436	20·648	18·470	16·715	15·270	14·056	13·030
Check	96·572	84·536	75·260	67·883	61·889	56·839	52·591
Curve	609·392	607·354	605·895	604·854	604·059	603·460	602·980

TABLE FOR RADII AND DISTANCES IN FEET AND DECIMALS.

Length of Abscissa	R = 3750 Ord.	R = 4000 Ord.	R = 4250 Ord.	R = 4500 Ord.	R = 4750 Ord.	R = 5000 Ord.	R = 5280 Ord.
100	1·333	1·250	1·178	1·112	1·052	1·000	·947
200	5·322	5·002	4·707	4·449	4·212	4·010	3·789
300	12·019	11·266	10·599	10·112	9·485	9·009	8·530
400	21·381	20·051	18·865	17·763	16·875	16·026	15·174
500	33·458	31 373	29·512	27·864	26·389	25·064	23·728
600	48·312	45·256	42·563	40·179	38·047	36·131	34·203
Extra	·915	·828	·748	·662	·575	·530	·486
Normal	12·134	11·364	10·655	10·078	9·541	9·059	8·574
Check	48·945	45·776	42·997	40·544	38·354	36·395	34·428
Curve	602·760	602·296	602·017	601·803	601·592	601·460	601·323

6th. *The author's equidistant six-point system*, giving equidistant points on the curve.

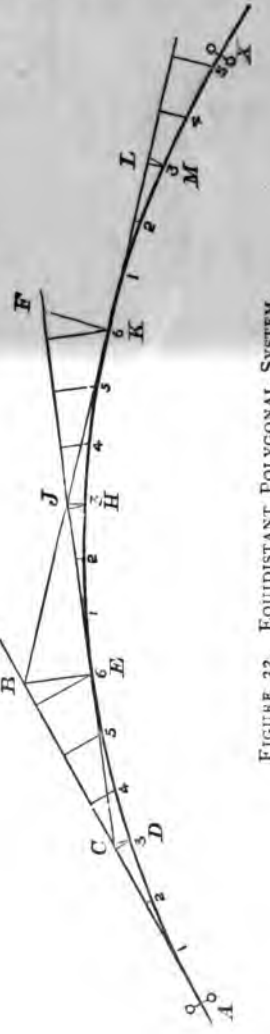


FIGURE 32. EQUIDISTANT POLYGONAL SYSTEM.

This method slightly resembles the last, hence care must be taken not to confound them in practice. In two respects it is preferable, that the points on the curve obtained on this system are equidistant, and are hence more easily verified, and that the new tangents are more easily and rapidly set out; but in another respect it is less convenient, the abscissæ used are not round numbers, and hence are less rapidly set out.

The preliminary data of the curve being previously determined, the setting-out commences from one tangent point.

Produce this tangent, and along it measure and fix the points in it (due to the proper radius) obtained from the following Table, as abscissæ, six in number, and from them set off at right angles to the tangent the corresponding ordinates there given. The ends of these ordinates are the first six points on the curve.

To obtain a new tangent at the sixth point:—Measure from the third point on the curve the normal distance given in the Table in such a way that its extremity shall fall in the original tangent; the direction from this extremity through the sixth point produced onwards is a new tangent. From this sixth point and from this new tangent set-out six more points with the same numbers and in the same way as before: and continue setting-out in this very simple manner fresh tangents and more sets of six points until the final tangent point is reached. These points being equidistant, the calculated total length of curve gives a check on the length of curve thus set-out.

If required, three points may be checked backwards from each of these subsidiary tangent points from the new tangent: and if all the points are found correct

when set off from two tangents, the work is correct to all purposes.

For example. Let it be required to range a curve to a radius of 900 feet; the necessary data from the table following are as follows for 50-feet distances along the curve :—

Abscissa	Ordinate	Curve
49·97	1·39	50
99·79	5·55	100
149·31	12·47	150
198·38	22 13	200
246·80	34·50	250
294·48	49·54	300

Normal at 3rd point 12·65.
Check-normal at 6th point 52·42.

Referring to the figure, the points marked I., II., III., IV., V., VI., are the points set-out on the tangent or the abscissæ in distances from A given in the set just quoted; from these points the rectangular offsets, given as ordinates in the same set, are set-out, giving points in the curve marked 1, 2, 3, 4, 5, 6, which are 50 feet apart. At the third point, marked 3 or D, which is 150 feet from A the normal DC, given as 12·65 feet, is set-out from D, so that the point C falls in the line AB, the original tangent; the direction CEF is then a new tangent. Recommencing at E six more points are set-out in the same way as before with the same set of figures; and a new tangent FKL obtained in the former manner. To check the work, measure from E, the normal $EB = 52·42$ feet, so that B may fall in the line AB; then if the curve has been well set-out, the points F and B will appear in exactly the same direction when viewed from K.

Although this method requires more figures than the former polygonal system, it is more accurate on long

curves, and more convenient in other respects. It will be observed on examining the following table that often the normal and the ordinate at the third point are practically equal; in such cases it becomes unnecessary to set-out any normal at the third point, as the third point on the tangent answers all purposes for sighting in a new tangent. This simplifies the work considerably; but the figures for the normal at the sixth point should, however, always be available for use.

The tables and the figure are arranged to suit a six-point system, but the same principle may be applied to four, eight, or any even number of points, when the offsets used are not long. In this polygonal system no angular measurement is necessary, a cross-head staff is sufficient for the alignment and setting-out right angles; the method is so simple that, after explaining it once or twice in practice, good chainmen can easily apply it by themselves.

RANGING TABLE SUITED TO RADII AND DISTANCES IN CHAINS AND DECIMALS.

Length of Curve	R = 7½		R = 10		R = 12½		R = 15	
	Absc.	Ord.	Absc.	Ord.	Absc.	Ord.	Absc.	Ord.
1	0·997	0·067	0·998	0·050	0·999	0·040	0·999	0·033
2	1·976	0·265	1·987	0·199	1·991	0·160	1·994	0·133
3	2·921	0·592	2·955	0·447	2·971	0·358	2·980	0·299
4	3·813	1·042	3·894	0·789	3·932	0·635	3·953	0·530
5	4·638	1·606	4·794	1·224	4·868	0·987	4·908	0·827
6	5·380	2·275	5·646	1·747	5·772	1·413	5·841	1·184
Normal .	0·643		0·467		0·366		0·305	
Check .	3·264		2·116		1·583		1·286	

FOR RADII AND DISTANCES IN CHAINS AND DECIMALS (*continued*).

Length of Curve	R = 20		R = 25		R = 30		R = 35	
	Absc.	Ord.	Absc.	Ord.	Absc.	Ord.	Absc.	Ord.
1	1·000	0·025	1·000	0·020	1·000	0·017	1·000	0·014
2	1·997	0·100	1·998	0·080	1·999	0·067	1·999	0·057
3	2·988	0·225	2·993	0·180	2·995	0·150	2·996	0·129
4	3·973	0·399	3·983	0·319	3·988	0·266	3·991	0·228
5	4·948	0·622	4·967	0·498	4·977	0·416	4·983	0·357
6	5·910	0·893	5·943	0·717	5·960	0·598	5·971	0·517
Normal .	0·227		0·181		0·151		0·130	
Check .	0·935		0·738		0·610		0·525	

Length of Curve	R = 40		R = 45		R = 50		R = 55	
	Absc.	Ord.	Absc.	Ord.	Absc.	Ord.	Absc.	Ord.
2	1·999	0·050	1·999	0·044	1·999	0·040	2·000	0·036
4	3·993	0·200	3·995	0·178	3·996	0·160	3·996	0·146
6	5·978	0·449	5·982	0·399	5·986	0·360	5·988	0·327
8	7·947	0·797	7·958	0·709	7·966	0·639	7·972	0·582
10	9·896	1·244	9·918	1·107	9·933	0·997	9·944	0·907
12	11·82	1·787	11·86	1·591	11·89	1·433	11·91	1·303
Normal .	0·454		0·403		0·362		0·329	
Check .	1·870		1·65		1·475		1·335	

Length of Curve	R = 60		R = 65		R = 70		R = 80	
	Absc.	Ord.	Absc.	Ord.	Absc.	Ord.	Absc.	Ord.
2	2·000	0·033	2·000	0·031	2·000	0·029	2·000	0·025
4	3·997	0·133	3·997	0·123	3·998	0·114	3·998	0·100
6	5·990	0·300	5·992	0·276	5·993	0·257	5·994	0·225
8	7·976	0·533	7·980	0·491	7·983	0·457	7·987	0·400
10	9·954	0·831	9·961	0·767	9·966	0·713	9·974	0·624
12	11·92	1·196	11·93	1·103	11·94	1·026	11·96	0·898
Normal .	0·301		0·278		0·258		0·226	
Check .	1·220		1·125		1·041		0·909	

FOR RADII AND DISTANCES IN FEET AND DECIMALS.

Length of Curve	R = 200		R = 250		R = 300		R = 350		R = 400		R = 450		R = 500	
	Absc.	Ord.	Absc.	Ord.	Absc.	Ord.	Absc.	Ord.	Absc.	Ord.	Absc.	Ord.	Absc.	Ord.
25	24·94	1·56	24·96	1·25	24·97	1·04	24·98	0·89	24·98	0·78	24·99	0·69	24·99	0·62
50	49·48	6·22	49·67	4·98	49·77	4·16	49·83	3·57	49·87	3·12	49 90	2·78	49·92	2·50
75	73·25	13·90	73·88	11·17	74·22	9·33	74·43	8·01	74 56	7·01	74·65	6·24	74 72	5·61
100	95·89	24·48	97·35	19·73	98·16	16 51	98·65	14·19	98·96	12 44	99·18	11·07	99·33	9 97
125	117·02	37·81	119 86	30·60	121·41	25 67	122·36	22 09	122·98	19·37	123·40	17·25	123·70	15·54
150	136·33	53·66	141·16	43·67	143·83	36·73	145·45	31·65	146·51	27·80	147·24	24·77	147·76	22·33
Normal .	. 14 94		11·69		9 63		8·26		7·14		6 32		5·68	
Check .	. 73 34		52 91		41·85		34 79		29 87		26 21		23 38	

Length of Curve	R = 500		R = 600		R = 700		R = 800		R = 900		R = 1000		R = 1100	
	Absc.	Ord.	Absc.	Ord.	Absc.	Ord.	Absc.	Ord.	Absc.	Ord.	Absc.	Ord.	Absc.	Ord.
50	49·92	2·50	49·94	2·08	49·96	1·79	49·97	1·56	49 97	1·39	49·98	1·25	49·98	1·14
100	99 33	9·97	99·54	8·31	99·66	7·13	99 74	6·24	99·79	5·55	99·83	5·00	99 86	4·54
150	147·76	22·33	148 44	18·65	148·86	16·01	149·12	14·02	149·31	12·47	149·44	11 23	149·53	10·21
200	194·71	39·47	196·32	33·03	197·29	28 38	197·92	24·87	198·36	22·13	198 67	19·94	198 89	18·13
250	239·71	61·21	242·83	51·33	244·72	44·17	245·95	38·75	246·80	34·50	247·40	31·09	247·77	28 27
300	282·32	87·33	287·66	73·45	290·90	63·31	293·02	55·59	294·48	49·54	295·52	44 66	296·28	40·65
Normal .	23·37		19·25		16·57		14·27		12·65		11·35		10·31	
Check .	. 105·81		83·73		69·59		59·75		52·42		46 75		42·21	

FOR RADII AND DISTANCES IN FEET AND DECIMALS (*continued*).

Length of Curve	R = 1200		R = 1300		R = 1400		R = 1500		R = 1600		R = 1800		R = 2000	
	Absc.	Ord.	Absc.	Ord.	Absc.	Ord.	Absc.	Ord.	Absc.	Ord.	Absc.	Ord.	Absc.	Ord.
50	49·99	1·04	49·99	0·96	49·99	0·89	49·99	0·83	49·99	0·78	49·99	0·69	49·99	0·63
100	99·89	4·16	99·90	3·84	99·92	3·57	99·93	3·33	99·94	3·12	99·95	2·78	99·96	2·50
150	149·61	9·36	149·69	8·65	149·72	8·03	149·75	7·49	149·77	7·04	149·88	6·24	149·86	5·62
200	199·08	16·63	199·24	15·36	199·32	14·26	199·41	13·31	199·48	12·48	199·59	11·10	199·67	9·99
250	248·20	25·95	248·50	23·97	248·68	22·26	248·84	20·79	248·99	19·09	249·20	17·33	249·35	15·60
300	296·88	37·31	297·38	34·47	297·71	32·02	298·00	29·90	298·24	28·04	298·60	24·94	298·88	22·45
Normal . .	9·44		8·70		8·07		7·53		7·06		6·27		5·64	
Check . .	38·50		35·40		32·77		30·51		28·55		25·29		22·71	

Length of Curve	R = 2000		R = 2250		R = 2500		R = 2750		R = 3000	
	Absc.	Ord.	Absc.	Ord.	Absc.	Ord.	Absc.	Ord.	Absc.	Ord.
100	99·96	2·50	99·97	2·22	99·97	2·00	99·97	1·81	99·98	1·67
200	199·67	9·99	199·74	8·88	199·79	8·00	199·81	7·27	199·85	6·66
300	298·88	22·46	299·11	19·97	299·28	17·98	299·38	16·35	299·50	14·99
400	397·34	39·87	397·90	35·46	398·29	31·93	398·56	29·04	398·82	26·63
500	494·81	62·18	495·89	55·33	496·67	49·83	497·22	45·32	497·69	41·57
600	591·04	89·33	592·91	79·53	594·26	71·65	595·21	65·19	596·01	59·80
Normal . .	22·71		20·15		18·11		16·44		15·07	
Check . .	93·50		82·44		73·77		66·78		61·02	

FOR RADII AND DISTANCES IN FEET AND DECIMALS (*concluded*).

Length of Curve	R = 3500		R = 4000		R = 4500		R = 5280		R = 5000	
	Absc.	Ord.	Absc.	Ord.	Absc.	Ord.	Absc.	Ord.	Absc.	Ord.
100	99·98	1·43	99·99	1·25	99·99	1·11	99·99	1·00	100·	0·95
200	199·89	5·71	199·92	5·00	199·93	4·44	199·95	4·00	200·	3·79
300	299·63	12·85	299·72	11·24	299·78	9·99	299·92	9·00	299·90	8·52
400	399·13	22·83	399·33	19·98	399·47	17 76	399·57	15·99	399·	15·14
500	498·30	35·65	498·70	31·21	498·97	27·75	499·17	24·98	499·34	23·67
600	597·06	51·73	597·75	44·92	598·22	39·94	598·56	35·96	599·	34·07
Normal . .	12·89		11·29		10·04		9·02		8·54	
Check . .	52·07		45·43		40·30		36·22		34·27	

7th. *The single chord system.*
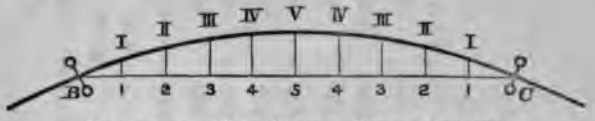


FIGURE 33. THE SINGLE CHORD SYSTEM.

This method may be occasionally useful in small curves of roads, paths, small channels, etc. The chord of the intended whole curve is measured, the angle of deflection observed roughly with a compass, and the suitable radius of curvature calculated. See Formulæ, page 214. $C = 2R \sin \gamma$.

Set out nine points along the chord at equal intervals, each being one-tenth of the chord; calculate the necessary offsets, which will only be five in number, as they recur in the second half of the curve, and set them out, thus obtaining nine points in the curve, which are sufficient for rough purposes of this description.

For example. Let the whole chord be found to be 131 feet, the angle of deflection 17 degrees (minutes are unnecessary), then the radius suitable to the case will be $= \frac{131}{2 \times 0{\cdot}2923} = 224$ feet.

In the attached table the five ordinates are given to a radius of 1000 feet, and can be thus reduced to the radius of 224 feet:—

1st ordinate	$= 15{\cdot}96 \times 224 \div 1000 =$	3·6 feet
2nd ordinate	$= 28{\cdot}19 \times 224 \div 1000 =$	6·3 feet
3rd ordinate	$= 36{\cdot}83 \times 224 \div 1000 =$	8·2 feet
4th ordinate	$= 41{\cdot}98 \times 224 \div 1000 =$	9·5 feet
middle ordinate	$= 43{\cdot}70 \times 224 \div 1000 =$	10·8 feet

The setting-out is now effected by measuring ten

distances along the chord from one tangent point, each distance being a tenth of the chord, that is 13·1 feet; and these points are marked. At the first of these points set-out the offset 3·6 feet at right angles to the chord, at the second 6·3 feet, at the third 8·2 feet, at the fourth 9·5 feet, at the fifth or middle point 10·8 feet, at the sixth 9·5 feet, at the seventh 8·2 feet, at the eighth 6·3 feet, at the ninth 3·6 feet. The extremities of the offsets will be points in the curve.

This system, though not a very good one, is preferable to the method of versed sines, which is certainly troublesome in calculation.

OFFSETS FROM THE CHORD FOR A RADIUS OF 1000.

Angle of Deflection	1st Ordinate	2nd Ordinate	3rd Ordinate	4th Ordinate	5th Ordinate	Angle of Deflection
°						°
1	·054	·097	·127	·146	·152	1
2	·219	·390	·511	·584	·609	2
3	·493	·877	1·151	1·315	1·370	3
4	·877	1·560	2·046	2·338	2·436	4
5	1·371	2·437	3·197	3·653	3·805	5
6	1·975	3·509	4·604	5·260	5·478	6
7	2·502	4·777	6·265	7·157	7·454	7
8	3·515	6·239	8·181	9·344	9·732	8
9	4·451	7 898	10·353	11·822	12·312	9
10	5·495	9·749	12·777	14·589	15·192	10
11	6·654	11·798	15·456	17·645	18·373	11
12	7·922	14·040	18·388	20 987	21·852	12
13	9·304	16·480	21·574	24·617	25·630	13
14	10 794	19·113	25 011	28·533	29 704	14
15	12·403	21·943	28·701	32·733	34·074	15
16	14·123	24·968	32·641	37·217	38·738	16
17	15·956	28·188	36·833	41·984	43·695	17
18	17·904	31·604	41·274	47·031	48·943	18
19	19·967	35·216	45·947	52·359	54 481	19
20	22·146	39·024	50·905	57·965	60·307	20

Checking equidistant points already ranged.

When a number of equidistant points have been

ranged on a curve, they may be rapidly checked by either of the two following methods :—

1. By equal rectangular offsets.

Produce the chord between any two points, until a rectangular offset hits off the next point; such rectangular offsets should all be equal, and when the chords are small compared with the radius, the offset is equal to double the ordinate from the tangent of the first point set out.

2. By oblique offsets.

Produce the chord between any two points to a distance equal to the chord itself, and mark the point; its distance from the next point on the curve is nearly equal to the square of the chord divided by the radius, but in any case such oblique offsets should be all equal.

The bisection of such an oblique offset will give the direction of the tangent from the last point on the curve.

These two methods are sometimes adopted for ranging curves, when accuracy is unimportant: but they are much better adapted for checking with precision.

Curves of varying Curvature.

Although ordinary circular curves, as before described and applied, are the most simple and in general the most convenient for almost all purposes, for roads, canals, or railways; yet in the case of a railway, their suddenness of departure from and return to a piece of straight is an acknowledged defect, and in cases where there is danger of a train leaving the rails, these are most dangerous points. In practice, or, more properly speaking in actual fact, in which theory does not inter-

fere, the sudden changes of cant are merely eased by the plate-layers on laying the rails, in an empirical way.

Two methods have been proposed to remedy this, both substituting other and more complicated curves for the circular one;—the one, by Gravatt, gives a varying curvature, the ordinates of which vary with the sine of a theoretical and abstract angle, of which the abscissa, the half-chord, and $\frac{\pi}{2}$ are the factors; the resulting curve is supposed nearly to meet the requirements of the cant, which again must be afterwards adapted to it; the other, by Froude, is far more practical in tendency, having a curvature made to depend on the maximum cant and the length of the radii of the adjoining circular portions of curve; the equation used is that of a cubic parabola. In this latter case, working out the ordinates for each single case would take up time, and apparently the setting-out of the curve would also be complicated and tedious.

The Compound Circular Curve.

There is, however, no absolute necessity under any circumstances for setting-out curves of varying curvature by making use of complicated expressions for ordinates, nor would there be any advantage in applying curves of a high order, if the same object can be obtained by simpler means. Now, in actual fact, a real curve is a comparative rarity on a railway, for it generally is nothing but a succession of small pieces of straight rail; cambering rails being still more the exception than the rule; so in practice we can make a curve of varying curvature to consist of a large number of equal portions of circular curve, described with different radii, decreasing from the

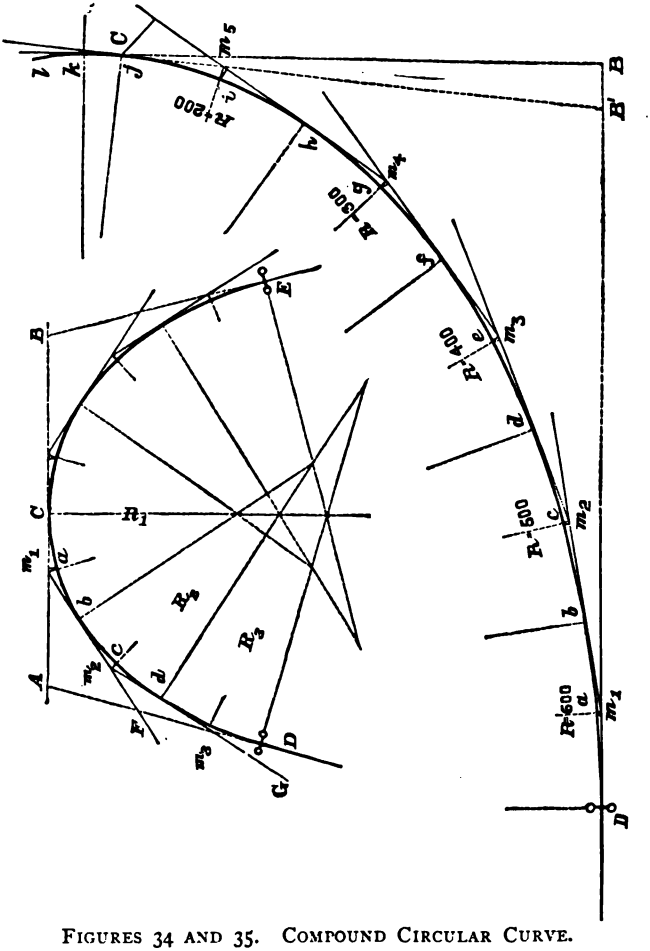
tangent points to the middle point of the curve, where the radius used would be a minimum.

It would, of course, be necessary to make a small preliminary sketch before setting-out on the ground, showing the radii used and the number of points set-out to each radius. The least radius would be used at the middle point of the curve, describing a small arc on each side of it, for example 200 feet or 300 feet; the next arc on each side would be described with a larger radius, and so the radii would increase for any number of arcs until the last and largest radius which describes an arc tangential to the straight or next curve is arrived at.

In fixing the data for such a curve, the position of the middle point of the curve is in one direction purely arbitrary; the lengths of the different arcs also are purely optional, although in practice it will be found most convenient to make all these arcs equal, and of some convenient length, as 100 or 200 feet; the increments of the radii may be made very gradual or very rapid, or may be made to follow any proposed mathematical law, thus giving great adaptability to the curve; in practice, however, a simple additive increment, equal in every case except in that of the last or tangential radius, is found most convenient; and even for the last radius the distances will be found generally to vary very slightly in setting-out.

In order to carry out this principle with as little labour as possible, the following Table has been constructed; it enables arcs of 100 to be set-out by various radii from 500 to 2640, the radii increasing by 100; this, however, does not preclude the variation of curvature being made more rapid, if wanted, as every alternate radius may be neglected, or radii differing by 300 or 400 used.

In one plan of setting-out this curve with the help of the Table (see figure 34), the line AB making equal



FIGURES 34 AND 35. COMPOUND CIRCULAR CURVE.

angles with the tangents DA, EB, and determining the middle point C of the curve, is first set-out. Obtain

from the Table the abscissæ due to the radius R_1 for the points $a\,b$, $a'\,b'$, which occur at 50-feet intervals along the curve, and measure them both ways from C along CA, CB; take the ordinates in the same way, and measuring them at right angles obtain the points $a\,b$, $a'\,b'$.

To proceed with the next portion of the curve or arc described with radius R_2, a common tangent to the two arcs is required at the point b. To obtain this, use the quantity mentioned as (normal) in the Table for R_1 and measure it from a in such a way that its other end m_1 may fall exactly in AC, then $m_1\,bF$ is the common tangent required.

From this new tangent bm_2F the points c and d are set-out by means of abscissæ and ordinates taken from b and due to the new radius R_2 in a similar way to those previously set-out from C. To obtain a new common tangent, a new normal cm_2 due to R_2 is set-out like am_2 and the direction $m_2\,dm_3$ obtained. Thus new common tangents are made at every change of radius until the final tangent point is reached. In case these final radii are large, it will be found that their abscissæ, ordinates, and normals differ so slightly from those for other radii that differ comparatively little from them in magnitude, that no unnecessary trouble on the score of exactitude is needed. The other half of the curve is of course set-out in the same manner and with the same data.

The other plan adopted in figure 35 illustrates the same principle, but commences from a tangent point; leaving the extra small portion of curve to occur in the middle instead of near the tangent.

Ranging this curve can be conveniently done on the

principle of the author's second polygonal system, but using a two-point instead of a six-point arrangement of figures, as given in the Table following this paragraph.

For example. Let a compound curve be set-out from two given tangents, the flattest curvature to be one of a radius of 600 feet. Using the following figures for $R = 600$:

Curve	Abscissa	Ordinate	Normal
50	49·94	2·08	2·09
100	99·54	8·31	

Starting from the tangent point, set-out two points on the curve a_1 and b; now measure the normal am_1, so that m_1 falls in DB, and sight in the new tangent $m_1 bm_2$.

Next using the following figures for $R = 500$:

Curve	Abscissa	Ordinate	Normal
50	49·92	2·50	2·51
100	99·34	9·97	

Set-out the points c and d with them, and from c measure the normal cm_2, so that m_2 may fall in the tangent, and thus obtain a new tangent m_2dm_3.

From this new tangent set-out two points with the figures for a radius of 400 feet, and proceed in this manner until arriving near the middle of the curve. Let 200 be the smallest radius used before arriving there, the points i and j being set-out, continue with the same curvature and set-out d' third point at l beyond the middle of the curve. The middle point of the curve can then be marked wherever it may occur on the arc between j and l, by setting-out equal angles to it from the two original tangent-points. One branch of the curve is thus set-out, and the other can be ranged in the same way, commencing at the other original tangent-

TABLE FOR SETTING-OUT COMPOUND CIRCULAR CURVES, VARYING THE CURVATURE AFTER EVERY TWO POINTS SET-OUT.

Curve	R = 200 Abscissa	Ordinate	R = 300 Abscissa	Ordinate	R = 400 Abscissa	Ordinate	R = 500 Abscissa	Ordinate	R = 600 Abscissa	Ordinate
50	49·481	6·218	49·769	4·157	49·870	3·121	49·917	2·498	49·942	2·082
100	95·885	24·484	98·158	16·513	98·961	12·435	99·335	9·967	99·538	8·314
Normal	6·417		4·216		3·145		2·513		2·089	
	R = 700		R = 800		R = 900		R = 1000		R = 1100	
50	49·958	1·786	49·968	1·562	49·974	1·389	49·978	1·249	49·983	1·136
100	99·660	7·131	99·740	6·242	99·794	5·550	99·831	4·996	99·862	4·542
Normal	1·789		1·565		1·393		1·251		1·138	
	R = 1200		R = 1300		R = 1400		R = 1500		R = 1600	
50	49·986	1·042	49·988	·961	49·989	·892	49·991	·835	49·992	·784
100	99·884	4·164	99·901	3·844	99·915	3·570	99·926	3·332	99·935	3·124
Normal	1·044		·962		·893		·835		·784	
	R = 1700		R = 1800		R = 1900		R = 2000		R = 2100	
50	49·993	·735	49·993	·694	49·994	·658	49·995	·625	49·995	·595
100	99·942	2·940	99·948	2·777	99·953	2·631	99·958	2·499	99·963	2·380
Normal	·734		·694		·658		·625		·595	
	R = 2200		R = 2300		R = 2400		R = 2500		R = 2640 ft. = ½ mile	
50	49·996	·567	49·996	·543	49·996	·521	49·997	·500	49·997	·473
400	99·965	2·272	99·968	2·173	99·970	2·083	99·973	2·000	99·975	1·894
Normal	·567		·543		·521		·500		·473	

point. The two closing distances *lQ* and *jQ* should be equal, if the ranging has been very carefully done; and this serves as a check on the operation.

In this example the constant decrement of radius of curvature used is 100 feet, but there is no necessity for adhering to such a decrement, the principle can be applied with any other.

Remarks on Ranging Curves for various objects.

Any one of the methods of ranging curves just explained may become suitable under some circumstances.

The centre-lines of railways require the most accurate ranging in these days of sharp curves, steep gradients, and high speed; for such purposes, when simple circular curves are adopted, either the two-theodolite or the one-theodolite and chain systems are generally preferred; but if theodolites are not available at the time, or if very windy or extremely cold weather interferes with their convenient use, the equidistant six-point system is probably the most suitable method for ranging; the checking can then be done by oblique offsets.

The curved portions of centre-lines of canals do not demand so much accuracy in ranging as those of railways, for the reasons that an imperfect curve in this case is not a source of positive danger, and that the regularity of the curve is of less importance than keeping close to the intended level, and is liable to alteration from the flow of water. Intervening pieces of straight between two curves are hence frequently dispensed with on canals, and the arrangement and design of curves are made as simple as possible; the radii are always convenient round numbers, and if the two tangent-points are shifted a little forward or back to suit a curve, it does not

become an important matter. The author's equidistant six-point system is well suited to ranging centre-lines of large canals on the curve; and Baker's tangential system may be used for them when the length of curve is comparatively short, but never in very long curves. For centre-lines of small channels, which rarely have long curves, Baker's method is sufficiently accurate; but in sharp curves, which have to run in at some fixed point dependent on level, it is best to use the chord-system, as it ensures the use of the appropriate curvature, and is not practically confined to the use of the radii in round numbers given in ranging tables.

Curves of main-roads and bye-roads correspond to those of main-canals and channels with regard to systems of ranging.

In bridges and viaducts, etc., as it would involve extra labor to set-out and construct a masonry work actually according to the curve, the following plan is

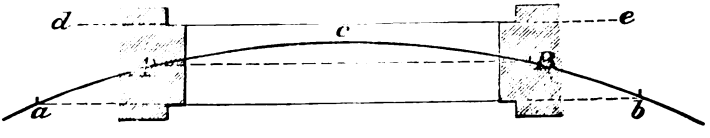


FIGURE 36. CURVES ON BRIDGES.

generally adopted. Let *acb* be (figure 36) the curve which has to be taken over a road by a railway bridge, and which on earthwork would be the centre line used. At *c*, the middle of the proposed bridge, take a tangent *dce*, and draw the chord *ab* parallel to it; the points *a* and *b* being sufficiently beyond the two proposed abutments to enable the chord *ACB* to be so placed that the points *A* and *B* will be also a little beyond the abutments; the chord *ACB* is between the tangent and the

chord *acb* and is parallel to both;—*ACB* is the centre line to be used for the bridge, all dimensions being set-out square from it.[1]

When a viaduct or aqueduct consisting of several spans occurs on a curve, it is necessary that each of the arches should not be conoidal but cylindrical; in order to effect this, the sides of the piers need not be radiated to the centre of the curve, but may be so arranged that each span will be equal on the centre line on the outside and on the inside—as in figure 37; where *AB* is the chord of the centre line of one opening; instead of using the radial lines *cd*, *ef* for the sides of the piers, they are increased and diminished on inside and outside, so that *CAD*, *EBF* at right angles to *AB*, being taken for the pier edges, the spans *CE*, *AB*, *DF* are all equal, and the arch can be made cylindrical; the same method being used for each span of the viaduct or aqueduct.

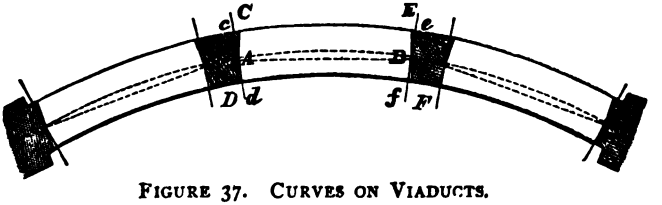


FIGURE 37. CURVES ON VIADUCTS.

In earthwork on the curve, the curve-points often have to be reset many times; to avoid this, it is usual to set-out two points at equal distances from the required point, and in a straight line with it, at right angles to the tangent at that point. The required point can then be found at any time by bisecting the distance between these two side points, which should of course be

[1] In figures 36 and 37 the curves are made very sharp for convenience in illustration.

clear of the works. It is usual to set them 100 feet apart. In this manner each point on the centre line, that is liable to be often disturbed during construction, can be found from the two points radially set-out from this point.

In setting-out curved centre-lines on bridges and viaducts the theodolite-and-chain system is generally to be preferred, as want of space may interfere with other methods.

The curved wing-walls of bridges and abutments can be best set-out on the chord system already explained; the radius and the chord can be obtained from the design, and the ordinates from the chord calculated with the help of the table given; sometimes three or four are sufficient instead of nine; and the points set-out with them at foundation-level, and again at plinth-level. Splaying wing-walls of compound curvature may be set-out on the same principle; each portion being treated separately in accordance with its special curvature as shown on the design; but if the curves are sharp, they may be struck from successive centres pegged on the ground.

Section 4. Details in connection with Railway Curves.

Resistance to Motion on a Curve.—A portion of this resistance will also occur on a straight level line. According to Mr. Scott Russell's experiments, the following empirical formulæ represent the amount of the various resistances on a straight level.

1st. The friction of the wheels and axles; this is constant at all velocities, and amounts to six pounds per ton weight of train.

2nd. The resistance of the air, which is proportional to the surface of the front of the train, and to the square of the velocity, amounts to one-fifth of a pound at one mile per hour for the usual frontage of eighty feet.

3rd. The result of concussions, oscillations, flexures, imbedding of wheels in rail, friction of air against sides, etc., when the permanent way is in fair order, equals one-third of a pound per ton per mile per hour, taking it proportional to the weight of the train and the velocity.

The sum of these will give the total resistance, and is true for passenger trains of from 20 to 64 tons at speeds from 30 to 60 miles per hour. For lower velocities, its results are too large, and for carriages or permanent way in bad repair, the results will be too small.

The total resistance to traction at low speeds on a straight may be taken at 1-280th of the load;

at 12 miles an hour, at 1-224th,
at 60 miles an hour, at 1-45th.

It might have been more correct to estimate the resistance of the air in proportion to the bulk of the train, instead of the frontage, but results do not appear to be so. A head or a side wind would add to the total resistance; the former can be accounted for by adding or subtracting its amount from the velocity of the train in the foregoing formulæ.

A further portion of the resistance to motion on a curve may be due to the gradient.

Should there be any inclination, the resistance due to this will be $= F + \frac{H.W.}{L}$

where F = resistance in pounds for the same load on a level, and any specified velocity.

W = load in pounds.

H and L = height and length of incline in feet.

The remaining portion of the resistance is due to curvature alone. With cylindrical wheels, and when no superelevation is given to the outer rail the following are the sources of resistance.

1st. The friction incidental to the sliding of the wheels on the rails, as one wheel must travel farther than the other.

2nd. The friction of the flanges on the outer rail due to centrifugal force.

3rd. The friction due to the axles being parallel instead of convergent.

4th. The resistance due to the obliquity of the moving power.

The first cause of friction is partly met by the employment of running tyres, the second by superelevation of the outer rail, the third by placing the axles nearer together or employing bogie-trucks, the fourth is considered to be a very small cause of resistance, partly counteracted by centrifugal effect.

When a train is once on the curve the risk of leaving the rail is not increased ; the departure from, and the return to, the straight cause all danger in this respect ; to obviate this a gradually increasing curvature might be adopted, as the parabolic curves used on some French railways, or the compound circular curve.

According to Mr. Latrobe's experiments, in cases where superelevation is given, the loss of power due to curvature for each degree of deflection subtending an

arc of 100 feet was equivalent to that due to an ascent of two and a half feet per mile. This is approximately expressed in these figures:

Radius in Feet	Ascent One in	Radius in Feet	Ascent One in	Radius in Feet	Ascent One in
5280	3893	**2000**	1474	**600**	442
5000	3686	**1750**	1290	**500**	369
4500	3318	**1500**	1106	**400**	295
4000	2949	**1250**	922	**300**	221
3500	2580	**1000**	737	**250**	184
3000	2212	**900**	664	**200**	147
2640	1946	**800**	589	**150**	111
2500	1843	**700**	516	**100**	74

Superelevation.

The superelevation of rail due to a curve is unfortunately a subject that is usually left uninvestigated; the engineering student, after learning that centrifugal force has to be resisted for different speeds and different curvatures, makes use of a table obtained from the nearest handbook of information regardless of the form of wheel that will have to run on those rails. The following investigation and example given by Oliver Byrne, being the clearest and yet the most concise that can be met with, cannot be improved on at present, and is therefore fully given.

To find first the centrifugal force tending to force the weight off the curve.

Let W = weight of mass in motion.

R = radius of curve.

v = velocity of train in feet per second.

f = centrifugal force.

g = force of gravity at earth's surface = 32·2 feet in a second.

Then $f = \frac{Wv^2}{gR}$.

When the wheels are cylindrical, which is the more usual form at present, the superelevation necessary is very easily obtained.

Let x=this superelevation, of which half is actual elevation of outer wheel, and half is actual depression to the inner wheel; and let b=gauge of rails;

then the centripetal force resulting $=\frac{W.x}{b}$,

and for equilibrium $\frac{W.v^2}{gR}$ must $=\frac{W.x}{b}$,

hence $x=\frac{bv^2}{gR}$.

For example. Required the superelevation for a radius of 500 feet when the gauge is 4·5 feet, and a speed of 60 feet per second.

$$x=\frac{4{\cdot}5\times 3600}{32{\cdot}2\times 500}=\frac{16{\cdot}2}{16{\cdot}2}=1 \text{ foot},$$

or six inches of elevation and six inches depression.

The following table of superelevations due to cylindrical wheels is calculated with this formula; any elevation less than one inch on the whole may generally be neglected.

TABLE OF SUPERELEVATION IN INCHES DUE TO VARIOUS SPEEDS AND CURVES.

Radius in Feet	Speed in Miles per Hour 10	20	30	40	50
BROAD GAUGE 7·06 FEET.					
500	1·13	4·52	10·17	18·08	28·25
573	·98	3·92	8·82	15·68	24·50
600	·94	3·76	8·46	15·04	23·50
700	·81	3·24	7·29	12·96	20·25
716¼	·79	3·16	7·11	12·64	19·75
800	·71	2·84	6·39	11·36	17·75
900	·63	2·52	5·67	10·08	15·75
955	·59	2·36	5·31	9·44	14·75
1000	·56	2·24	5·04	8·96	14·00
1432½	·39	1·56	3·51	6·24	9·75
1500	·38	1·52	3·42	6·08	9·50
1910	·29	1·16	2·61	4·64	7·25
2000	·28	1·12	2·52	4·48	7·00
2500	·23	·92	2·07	3·68	5·75
2865	·20	·80	1·80	3·20	5·00
INDIAN GAUGE 5·5 FEET.					
500	·88	3·52	7·92	14·08	22·00
573	·77	3·08	6·93	12·32	19·25
600	·73	2·92	6·57	11·68	18·25
700	·63	2·52	5·67	10·08	15·75
716¼	·61	2·44	5·49	9·76	15·25
800	·55	2·20	4·95	8·80	13·75
900	·49	1·96	4·41	7·84	12·25
955	·46	1·84	4·14	7·36	11·50
1000	·44	1·76	3·96	7·04	11·00
1432½	·31	1·24	2·79	4·96	7·75
1500	·29	1·16	2·61	4·64	7·25
1910	·23	·92	2·07	3·68	5·75
2000	·22	·88	1·98	3·52	5·50
2500	·17	·68	1·53	2·72	4·25
2865	·15	·60	1·35	2·40	3·75

SUPERELEVATION IN INCHES DUE TO VARIOUS SPEEDS AND CURVES (*continued*).

Radius in Feet	Speed in Miles per Hour 10	20	30	40	50
IRISH GAUGE 5·25 FEET.					
500	·84	3·36	7·56	13·44	21·00
573	·73	2·92	6·57	11·68	18·25
600	·70	2·80	6·30	11·20	17·50
700	·60	2·40	5·40	9·60	15·00
716¼	·59	2·36	5·31	9·44	14·75
800	·53	2·12	4·77	8·48	13·25
900	·46	1·84	4·14	7·36	11·40
955	·44	1·76	3·96	7·04	11·00
1000	·42	1·68	3·78	6·72	10·50
1432½	·29	1·16	2·61	4·64	7·25
1500	·28	1·12	2·52	4·48	7·00
1910	·22	·88	1·98	3·52	5·50
2000	·21	·84	1·89	3·36	5·25
2500	·17	·68	1·53	2·72	4·25
2865	·15	·60	1·35	2·40	3·75
NARROW GAUGE 4·71 FEET.					
500	·75	3·00	6·75	12·00	18·75
573	·66	2·64	5·94	10·56	16·50
600	·62	2·48	5·58	9·92	15·50
700	·54	2·16	4·86	8·64	13·50
716¼	·53	2·12	4·77	8·48	13·25
800	·47	1·88	4·23	7·52	11·75
900	·42	1·68	3·78	6·72	10·50
955	·39	1·56	3·51	6·24	9·75
1000	·38	1·52	3·42	6·08	9·50
1432½	·26	1·04	3·34	4·16	6·50
1500	·25	1·00	2·25	4·00	6·25
1910	·20	·80	1·80	3·20	5·00
2000	·19	·76	1·71	3·04	4·25
2500	·15	·60	1·35	2·40	3·75
2865	·13	·52	1·17	2·08	3·25

SUPERELEVATION IN INCHES DUE TO VARIOUS SPEEDS AND CURVES (*continued*).

Radius in Feet	Speed in Miles per Hour 10	20	30	40	50
GAUGE 4·5 FEET.					
500	·72	2·88	6·48	11·52	18·00
573	·63	2·52	5·67	10·08	15·75
600	·60	2·40	5·40	9·60	15·00
700	·51	2·04	4·59	8·16	12·55
716¼	·50	2·00	4·50	8·00	12·50
800	·45	1·80	4·05	7·20	11·25
900	·40	1·60	3·60	6·40	10·00
955	·38	1·52	3·42	6·08	9·50
1000	·36	1·44	3·24	5·76	9·00
1432½	·25	1·00	2·25	4·00	6·25
1500	·24	·96	2·16	3·84	6·00
1910	·19	·76	1·71	3·04	4·75
2000	·18	·72	1·62	2·88	4·50
2500	·14	·56	1·26	2·24	3·50
2865	·12	·48	1·08	1·92	3·00
HILL OR TRAMWAY GAUGE 3·5 FEET.					
500	·58	2·32	5·22	9·28	14·50
573	·48	1·92	4·32	7·68	12·00
600	·47	1·88	4·23	7·52	11·75
700	·40	1·60	3·60	6·40	10·00
716¼	·39	1·56	3·51	6·24	9·75
800	·35	1·40	3·15	5·60	8·75
900	·31	1·24	2·79	4·96	7·75
955	·29	1·16	2·61	4·64	7·25
1000	·28	1·12	2·52	4·48	7·00
1432½	·19	·76	1·71	3·04	4·75
1500	·19	·74	1·71	3·04	4·75
1910	·15	·60	1·35	2·40	3·75
2000	·14	·56	1·26	2·24	3·50
2500	·11	·44	·99	1·76	2·75
2865	·09	·36	·81	1·44	2·26

Junctions and crossings.—The greater portion of the alignment of the rails is simply set-out by the platelayers in accordance with the staked centre-line and proper gauge, a few extra levels being required wherever super-elevation is necessary. Junctions and connections, which are adopted in preference to turntables and traversers for the transfer of rolling stock from one pair of rails to another, demand special attention in setting-out.

A junction occurs under the most simple circumstances when a curved pair of rails depart from a straight main-line, and after clearing the latter run into a straight branch-line, or into any intermediate piece of straight; the actual junction of the rails can be disconnected at will by means of a moveable rail or switch and fixed point, the last fixed rail being cut to form a closure; most rails are from 15 to 27 feet long, the closing rail should be cambered, and should not be less than 7 feet, and if transverse sleepers are used, should rest on three of them.

The work of the setter-out consists in marking the junction-point and the heel of the switch, and afterwards the point of the crossing and its lead: the latter will depend on the sharpness of the curve adopted and on the gauge. In stations where it is advantageous to clear the main-line in a short length, sharp curves, having radii less than 500 feet, are used. The curve either runs quite up to the main line, and includes the switch as a nominal part of it, although this is practically straight, or, which is more usual, the curve runs merely up to the switch and is tangential to its outer end; while the switch is set straight, having a regulated length and clearance. The latter mode is preferable from its allowing greater scope for adjustment; as any slight error of sharpness in the setting-out of the curve is remedied by the flatness

of the switch; also because either the clearance or the length of the switch may be slightly modified while retaining the angle of departure unchanged; this method is hence adhered to in calculations in this book.

Next as regards crossings; in the case of a simple turnout, when both main and branch lines each consist of a single pair of rails, there is only one crossing, or point at which one rail of the main line has to be cut through. Crossings are denoted by their leads, such as 1 in 6, 1 in 10, etc., the lead being equal to half the cosecant of half the angle of crossing. For rough purposes and when the value of the lead is not subsequently used for calculation of distance, it may be taken as equal to the cotangent of the whole angle; even in the case of a large angle, as 11°28′42″ the result is 1 in 4·92 instead of 1 in 5, the error being less than a tenth as far as this. Leads are generally given to quarters and sometimes to eighths; in the following table, giving equivalent angles and leads, they are given to tenths, although they are not commonly required with such exactitude. There are also less convenient modes of designating leads. The most simple mode of obtaining the lead from a plan is to apply a scale, as to the next following figure, at or about ED, so that $ED=1$, one unit on the scale; then with a compass describe the arc ED from the centre C; now measure either CE or CD with the scale of which ED is the unit, and this measurement (m) is the lead of the crossing, and is equal to $\frac{CE}{ED}=\frac{1}{2}\operatorname{cosec}\frac{1}{2}DCE$.

It should be remembered that the angle of the crossing is that formed by the middle-lines of the two rails, which should be chalked to show it, and not the angle formed by the metal at the point where the rail is

Reduction of Angles of Crossings.

Lead One in	Angle			Lead One in	Angle		
5·0	11°	28′	42″	10·0	5°	43′	56″
5·1	11	15	8	10·1	5	40	30
5·2	11	2	8	10·2	5	37	10
5·3	10	49	34	10·3	5	33	54
5·4	10	37	32	10·4	5	30	40
5·5	10	25	54	10·5	5	27	32
5·6	10	14	42	10·6	5	24	26
5·7	10	3	50	10·7	5	21	24
5·8	9	53	28	10·8	5	18	26
5·9	9	43	22	10·9	5	15	30
6·0	9	33	38	11·0	5	12	38
6·1	9	24	12	11·1	5	9	48
6·2	9	15	4	11·2	5	7	2
6·3	9	6	14	11·3	5	4	20
6·4	8	57	42	11·4	5	1	40
6·5	8	49	24	11·5	4	59	2
6·6	8	41	22	11·6	4	56	28
6·7	8	33	34	11·7	4	53	55
6·8	8	26	0	11·8	4	51	26
6·9	8	18	40	11·9	4	48	58
7·0	8	11	32	12·0	4	46	34
7·1	8	4	36	12·1	4	44	12
7·2	7	57	50	12·2	4	41	52
7·3	7	51	18	12·3	4	39	34
7·4	7	44	54	12·4	4	37	18
7·5	7	38	42	12·5	4	35	6
7·6	7	32	40	12·6	4	32	52
7·7	7	26	46	12·7	4	30	46
7·8	7	21	2	12·8	4	28	38
7·9	7	15	26	12·9	4	26	34
8·0	7	10	0	13·0	4	24	31
8·1	7	4	42	13·1	4	22	30
8·2	6	59	30	13·2	4	20	30
8·3	6	54	26	13·3	4	18	32
8·4	6	49	30	13·4	4	16	36
8·5	6	44	40	13·5	4	14	42
8·6	6	39	58	13·6	4	12	50
8·7	6	35	22	13·7	4	11	0
8·8	6	30	52	13·8	4	9	10
8·9	6	26	28	13·9	4	7	22
9·0	6	22	10	14·0	4	5	36
9·1	6	17	58	14·2	4	2	10
9·2	6	13	52	14·4	3	58	47
9·3	6	9	50	14·6	3	55	30
9·4	6	5	54	15·0	3	49	14
9·5	6	2	2	15·5	3	41	50
9·6	5	58	16	16·0	3	34	54
9·7	5	54	34	16·5	3	28	22
9·8	5	50	56	17·0	3	22	16
9·9	5	47	24	18·0	3	11	0

cut through. The point of crossing once fixed should not be altered for accommodation's sake, although the point of the switch may be shifted a little with that object. The leads of crossings on the English narrow gauge generally vary between 1 in 5 and 1 in 12, 1 in 10 is the lead most commonly preferred, but in stations where the speed is small 1 in 7 is not too sharp; they are distinguished as acute or obtuse. In steel crossings, the portions of rail that meet at a point are, for a few feet from it, all cast in one piece of steel and fixed to a block of wood; their sizes and forms vary greatly according to the maker.

The simple turnout from a straight main line.

To determine the data necessary for setting-out a crossing; let AP, LE be the two rails of the single main line; $ABCD$ and LM those of the branch. The point A is fixed as may be wanted, the length of switch (b) and its rectangular offset (a) are fixed quantities: hence also the angle $A = \sin^{-1}\frac{a}{b}$; the gauge (g) is also given, and the radius of curvature r of the turnout, is a predetermined quantity. The two quantities required for setting-out are the chord BC, by which the point C may be marked on the rail LE, and the angle of crossing ϕ from which the lead can be calculated.

Completing the figure for purposes of trigonometrical calculation

$$\text{we obtain } \cos\phi = \cos A - \frac{g-a}{r+\frac{1}{2}g};$$

$$\text{and the chord } BC = \overline{g-a} \,.\, \operatorname{cosec}\frac{\phi+A}{2}.$$

If ϕ be a predetermined angle, the radius of the branch, $r=\frac{1}{2}\left(BC.\text{cosec}\ \frac{\phi-A}{2}-g\right)$.

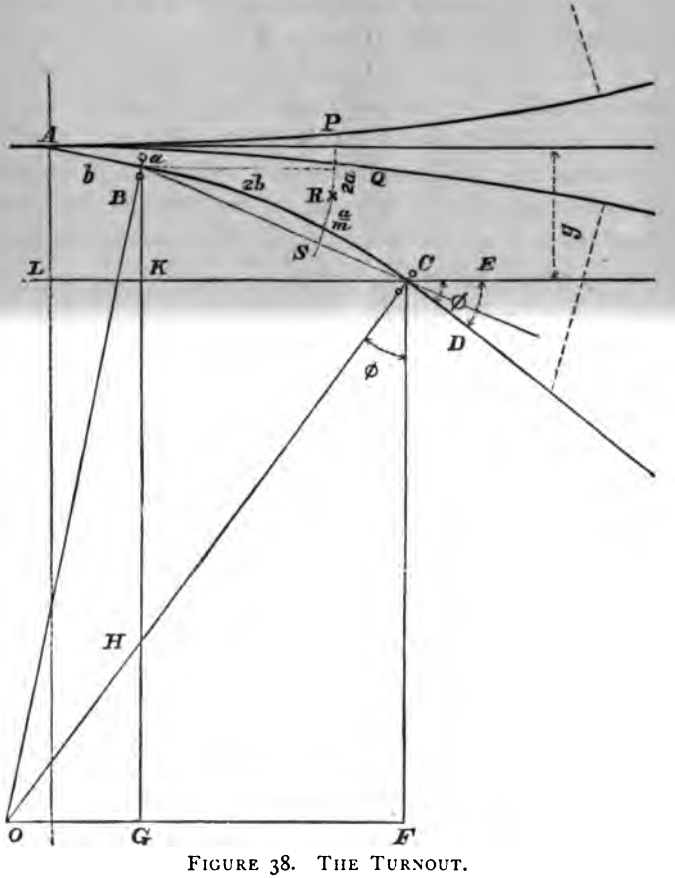


FIGURE 38. THE TURNOUT.

The results of the formulæ are given in the following table for usual gauges and lengths and clearances of switch.

TABLE FOR SETTING OUT CROSSINGS, GIVING ALSO THEIR LEADS.

(Turnout from a straight track.)

Radius in Feet	Broad 7·06		Indian 5·5		Irish 5·25	
	Lead One in	Distance	Lead One in	Distance	Lead One in	Distance
2000	11·7	122·1	13·2	101·7	13·5	98·4
1500	10 3	109·1	11·6	92·2	11·9	89 2
1400	9·9	106·4	11·1	91·3	11·5	87·1
1300	9·6	103·4	10·9	87·5	11·1	84·8
1200	9·2	100·3	10·5	85·0	10·7	82·4
1100	8·9	96·9	10·0	82·1	10·3	79·7
1000	8·5	93·3	9·6	79·4	9·8	77·0
900	8·1	89·7	9·1	76·3	9·4	74·0
800	7·6	85·5	8·6	72·9	8·9	70·7
700	7·1	81·0	8·1	69·1	8·3	67·1
600	6·6	76·2	7·5	65·0	7·7	63·1
500	6·1	70·5	6·9	60·3	7·1	58·6
400	5·4	64·0	6·2	55·0	6·3	53·4
300	4·7	56·6	5·4	48·7	5·5	47·3
200	3·8	47·4	4·4	40·8	4·5	39·7

Radius in Feet	Narrow 4·71		Australian 3·5		Metric 3·281	
	Lead One in	Distance	Lead One in	Distance	Lead One in	Distance
2000	14·2	90·9	16·1	74·5	16·5	71·0
1500	12·5	82·6	14·2	68·2	14 6	65·0
1400	12·2	80·9	13·8	66·7	14·2	63·2
1300	11·7	78·6	13·3	65·0	13·8	62·1
1200	11·3	76·4	12·9	63·3	13·3	60·5
1100	10·9	74·0	12·4	61·5	12·8	58·7
1000	10·4	71·5	11·9	59·5	12·1	56·8
900	9·9	68·8	11·3	57·3	11·7	54·8
800	9·4	65·8	10·7	54·8	11·0	52·5
700	8·8	62·4	10·1	52·3	10·4	50·0
600	8·2	57·4	9·4	49·3	9·7	47·2
500	7·5	54·7	8·6	45·9	8·9	44·0
400	6·7	49·9	7·7	42·0	8·0	40·3
300	5·8	44·2	6·7	37·4	7·0	35·9
200	4·8	37·2	5·5	31·5	5·7	30·3

For the four larger gauges, the length of switch is 15 feet, the throw $4\frac{1}{2}$ inches.

For the two small gauges, the length of switch is 10 feet, the throw 3 inches.

The setting-out may be constructively set-out on the ground as follows, without either tables or calculation, whatever the conditions of the case may be.

Mark the points A and B on the ground according to any predetermined length (b), and clearance (a) of switch; determine also by choice a suitable lead for the intended crossing, let it be 1 in m, represented in the figure by the ratio of ED to EC. From a centre at B, with a radius$=2b$, or twice the length of the switch, describe an arc $PQRS$, meeting the rail at P. From P measure $PR=2a$ on this arc: from R measure a chord $RS=\frac{a}{m}$, thus fixing the point S. The direction BS will give the position of C, or point of crossing. By drawing BQ parallel to AP in the figure it will be seen that the angle $QBS=\frac{\phi+A}{2}$ as required. The radius r of the branch can afterwards be found by the foregoing formula.

The simple turnout from a curved main line.—The two cases correspond generally to the foregoing. In one case the turnout curves in a direction similar to that of the main line, in the other it curves in a reverse direction.

The corresponding formulæ for these two cases are thus. Let R be the radius of the main track, and N the angle at the centre of the main-track curve, which is subtended by the required chord-distance BC; the terms correspond in all cases; and the value of the half of the angle N must first be found.

With similar curves: $\tan\frac{N}{2}=\frac{g-a}{2R-a}\cdot\operatorname{cotg}\frac{\phi+A}{2}$

With reverse curves: $\tan\frac{N}{2}=\frac{g-a}{2R+a}\cdot\operatorname{cotg}\frac{\phi+A}{2}$

And in either case, the required chord distance

$$BC=\overline{g-a}\ \text{cosec}\ \frac{\phi+A}{2}.\ \cos\ \frac{N}{2}.$$

If ϕ be the predetermined quantity, as is usually the case, the value of r, the radius of the branch, is :—

With similar curves,

$$r=\frac{1}{2}\left(BC\ \text{cosec}\ \frac{\phi+N-A}{2}-g\right)$$

With reverse curves,

$$r=\frac{1}{2}\left(BC\ \text{cosec}\ \frac{\phi-N-A}{2}-g\right).$$

But if r be the predetermined quantity, and ϕ be required. In either case first calculate the side ON of the triangle ONC, where N is the centre for the main curve, and O is the centre for the branch curve. Then for similar curves, as $OC=r+\frac{g}{2}$, and $CN=R-\frac{g}{2}$, the three sides of the triangle are given, viz. c, n, o, and their half sum $=s$,

$$\text{and tan}\ \frac{\phi}{2}=\sqrt{\left(\frac{\overline{s-o}.\ \overline{s-n}}{s.\ \overline{s-c}}\right)}.$$

Also for reverse curves ;—having in a corresponding triangle calculated the side ON; then $OC=r+\frac{g}{2}$, and $CN=R-\frac{g}{2}$; thus the three sides of the triangle, viz. c, n, o, are given, and their half sum $=s$.

$$\text{then cotg}\ \frac{\phi}{2}=\sqrt{\left(\frac{\overline{s-o}\ \overline{s-q}}{s.\ \overline{s-c}}\right)}.$$

In either case the angle ϕ can be obtained if required. From the variety of curves that may be given

either to the main line or the branch, and the number of combinations that might be formed, tables for these cases would be rather lengthy, and require a book specially devoted to them; the calculations however are very simple.

It is evident that any formulæ for setting-out crossings of turnouts can only be modifications of those given for the three cases here dealt with. Simple, or even complicated sets of turnouts may be set-out on the ground by anyone with the aid of the above formulæ and tables; but when they form part of a cross-over road, or of a series of them, special designs must be made, and the setting-out done in accordance with them. Designing is not within the scope of this work, and none of the problems involving complicated curved centre-lines affect the setter-out beyond that he has to set-out the work, and mark switches and crossing points, in accordance with the design laid down on a plan.

The designs for cross-over roads are frequently made without the introduction of intermediate straights between reverse curves of comparatively small radius; in such cases it rests with the designer, to correct this; but when the design shows tangent points, independently of the length of switch, this matter can be adjusted by the setter-out, without altering the radii used, either by setting the switch-point back or by cambering the closure.

Setting back a switch-point.

The distance to which the point should be set back is thus obtained. Let b and a be the length and throw

of the switch, and c the tangential abscissa to the given radius of curve due to an ordinate a; d be the distance for setting back the tangent point along a straight line, in order to make the curve tangential at the heel;

then $d = \sqrt{b^2 - a^2} - c$; where $c = \sqrt{R^2 - (R-a)^2}$.

In case the curvature is such that c is greater than $\sqrt{b^2 - a^2}$, it is better to put the switch-point at the exact tangent point of design, and to adjust the closure or last fixed rail by cambering so as to fit up to the switch-heel, thus allowing the error in flatness to remain.

Corresponding adjustments in setting-out can be made for turnouts from curved main tracks, setting back for errors of sharpness, and cambering closures for those of flatness.

Radii of centre-lines of constructed railways.

It having been suggested that formulæ for the purpose of finding the radius of the centre-line of an actual railway-curve would be useful both to surveyors and engineers in the field, the following is appended to this part of the book.

Let C be the chord of the outer rail, measured tangential to the inner rail, to the middle of the metals everywhere.

R, the radius of the centre line in any case, whether the line be of single or of double track.

g, the gauge of the track, and s the six-foot interval between tracks, which frequently varies, and requires measurement.

The formulæ, when symmetrically arranged for purposes of calculation, direct and converse, are (1) for the

chord of a single track, (2) for that of the inner track of a double line, (3) for that of the outer track of a double line.

$$R=\frac{1}{2}\times\frac{C^2}{4g} \qquad \text{or, } C=2\sqrt{g}.\sqrt{2R} \qquad (1)$$

$$R=\frac{1}{2}\left\{\frac{C^2}{4g}+\overline{s+g}\right\} \text{ or, } C=2\sqrt{g}.\sqrt{2R-\overline{s+g}} \quad (2)$$

$$R=\frac{1}{2}\left\{\frac{C^2}{4g}-\overline{s+g}\right\} \text{ or, } C=2\sqrt{g}.\sqrt{2R+\overline{s+g}} \quad (3)$$

The first set of formulæ is applicable to any single case required in the field, and the second set to the tabulation of chords corresponding to given radii in round numbers ; the same unit of measurement, whether feet or chains, being adhered to throughout.

PART IV.

ROUTE-SURVEYS.

Section 1. Traverses of Routes by Land and Sea.

Route-surveying may be divided into two classes; the one being exploration or itinerary surveys on comparatively unknown routes by land, the other being what is familiarly called Navigation, which deals in corresponding manner with the course of vessels at sea. They both follow the principles of a large Compass Traverse, already explained and exemplified (see pages 105 to 110 and Record No. III.), and vary from it mostly in that the distances from station to station are much greater, and more coarsely measured or estimated.

The portions of traverse in route-surveys are generally divided into journeys of one day, or of a day's run at sea; the traverse-stations have their distances apart fixed by local considerations on land, such as bends of valleys, ascents, passes, positions of villages, while at sea these distances depend on the times, or periods between changes of course arbitrarily adopted by the sailing-master. The final closing of such surveys, or traverse-work, depends on such previously well-known, or predetermined points, as towns, villages, etc., or at sea, rocks, ports, harbours and islands, whose positions are accurately known, and laid down on trustworthy

maps or charts, or recorded authentically. Some such points also serve for checking intermediate places in the route; but the principal means of daily verification in all such work is direct astronomical observation aided by good chronometers for giving differences of longitude. Hence the necessity in route-surveys of having men capable of taking such observations, of having portable instruments for that purpose, and of also taking copies of the Nautical Almanac for the current and the following year, as these give the positions of all the principal celestial bodies for Greenwich mean time. In route-surveys the necessary astronomical observations, for the determination of latitude, longitude, and the direction of the meridian, are made at least once, or twice, a day; the intervening traverse-work is calculated and reduced to a single meridian, and the errors determined; the traverse-work can then be plotted on a plan, to some convenient scale, with names of places, dates, times, and everything desirable or suitable for record.

Such surveys are not generally the principal object of an exploration, but are more frequently subservient to obtaining knowledge on other points; such as for military purposes, in which case the gradients of the country or of the roads must be noted as suited to artillery or not; the condition of the country as suited for cavalry to act, passes, fords, and strong positions, places suited for ambuscade or cover for troops, have to be noted; the amount of shelter, food, fuel, and forage that might be obtained; the amount and quality of the water all along the route, the times and seasons of crops, their preparation and harvesting; the amount of timber and stone available for hutting and bridges;

and the number and disposition of the inhabitants, their arms, resources, and modes of organisation, attack, and defence. Information on most of these points is generally useful under any circumstances, apart from purely military considerations; and although it is essential to a military route-report, it is also of service in the event of any engineering or mining works being contemplated. In drawing up such reports of supplies, shelter, and accommodation, care should be taken to distinguish between the amount available on an emergency, or for a permanence, also to note whether the people are disposed to trade or otherwise, and whether they have been spoilt by gratuitous gifts, or deceived by false promises or inducements.

Sometimes route-surveys form part of a scientific exploration, in which case the geological features, the mineral resources, the fauna and flora, the climate and meteorological conditions of the country are sometimes the most important subjects of observation; while in others the anthropological data, the personal peculiarities of the people, their habits, history, customs, superstitions, traditions, ideas, cleanliness, prevalent diseases, virtues and vices, are the main points of interest. Under such circumstances the duties of the expedition generally fall rather heavily on the members composing it, especially as those in connection with the mere transit and arrangements for passage through the country and intercourse with the natives form serious part of them.

It frequently happens too that the mere sight of surveying instruments exercises a most irritating effect on the inhabitants, who may look on them as forerunners of annexation, loss of independence, or religious interference. Under these circumstances the survey-work

and the observations have to be partly carried on by stealth, and frequently with a great deal of hurry, which is very prejudicial to good work. Sometimes armed opposition or insatiable demands cannot be avoided, but are rather increased by any amount of tact and forbearance; in this case prompt and severe measures must be relied on, without regard for the opinions of those who are not fully acquainted with the circumstances, or are opposed to the success of the expedition.

These and other similar interruptions and difficulties, such as illness, breakage of instruments, loss of baggage, non-arrival of supplies, detention by floods, etc., may interfere very seriously with the work of the expedition, and render it necessary that the person to whom the survey is entrusted shall be not only capable of surveying, and taking astronomical observations, but shall also be a scientific man in other respects, a fair linguist, energetic, active, enduring, courageous, and fertile in resources and expedients.

The Traverse-work of an itinerary survey being dependent on compass-bearings and measured or estimated distances; the direction of the meridian, the magnetic variation of the compasses, and the latitude and longitude should be daily determined at starting and closing points or other stations. As observations or estimations of longitude are far less accurate than those of latitude, the more nearly the route follows a meridian, the more correct the verification; and for the same reason the checks in longitude afforded by them may frequently be disregarded, while those in latitude may not. The traverse-work should be made to involve the smallest possible number of sights, or changes of course, within practical limits, thus making

the lengths of straight course as long as possible. For measurement of distance on such routes, various forms of pedometer have to be depended on; and as for bearings two compasses should always be used to check each other for both forward and back bearings, so also for distances two pedometers should be used. As to patterns of pedometer, the wheel-pedometer is good on smooth and level ground, while on rough and hilly routes it records distances in excess of the truth; the suspended or watch-pedometer, or passometer, which records the number of paces taken, gives results which have to be reduced or augmented in accordance with the nature of the ground; and the length of pace adopted under varying conditions; it is better therefore to have such instruments made to record the number of paces, than the number of miles estimated by the instrument-maker, and then suitable allowances can be made for correction, and a more correct mileage may be estimated.

In time-itineraries, where the distances are merely estimated from the recorded times of departure from and arrival at certain places or stations in the route, special attention should be paid to the variation of speed in journeying; some measurements should occasionally be taken to test it, both on the level, and on ascents and descents. The beginnings and endings of ascents and descents, also any peculiarities of the ground and circumstances affecting the speed, and halts and delays, should be invariably noted in the record; for such causes may combine to vary an otherwise equable speed by as much as a half.

Again, as the observed bearings, whether taken in degrees or more roughly in points of $11\frac{1}{4}°$, should fairly

represent the actual directions of the main lines traversed, it is advisable to take these when clear of a town or of any exceptional windings of route, and thus reduce errors due to such causes to a minimum. In any case, however rough the mode of estimating distance and bearing may be, the observations themselves should be correct within the capability of the means employed; otherwise the results may be valueless and unfit for reduction to plan or map.

The observations for latitude, etc., should invariably be taken with the utmost precision that the instruments admit, and as some of them are sure to be defective and necessitate rejection, the number of observations should be more than may appear exactly sufficient. The elevations of various points on or near the line of route are determined by detached observation with aneroids or hypsometers; they should be correct within 20 or 25 feet of the truth. A small amount of hill-sketching in the bolder portions of the country is also necessary, the bearings of principal peaks, passes, and marked features being taken from several points to determine their positions.

The principal cause of error in all such compass traverses is the neglect of allowances for corrections under the before-mentioned heads, also for magnetic storms, local magnetic attraction, and general diurnal or other change of magnetic variation. On some of these points information may afterwards be obtained at the nearest magnetic observatory. The day's traverse, or run and course, should invariably be plotted daily on a sufficiently large scale on one plan, and the corrected traverse, after reduction to fit in with the observed latitudes and estimated longitudes on another. At the close of the

expedition, when further information may be forthcoming, the whole can be finally worked up to a scale on which the actual errors become invisible, thus making a final correct plan of the route.

The formulæ, and the method used in the reduction of compass traverses to a single meridian, already fully explained at page 106, are adopted in reducing the set of traverses of a day's journey to a resulting total difference of latitude, and total difference of departure in geographical miles; other formulæ in connection with geodetic arcs and latitudes are given at pages 53, 54, and exemplified at pages 63 to 66.

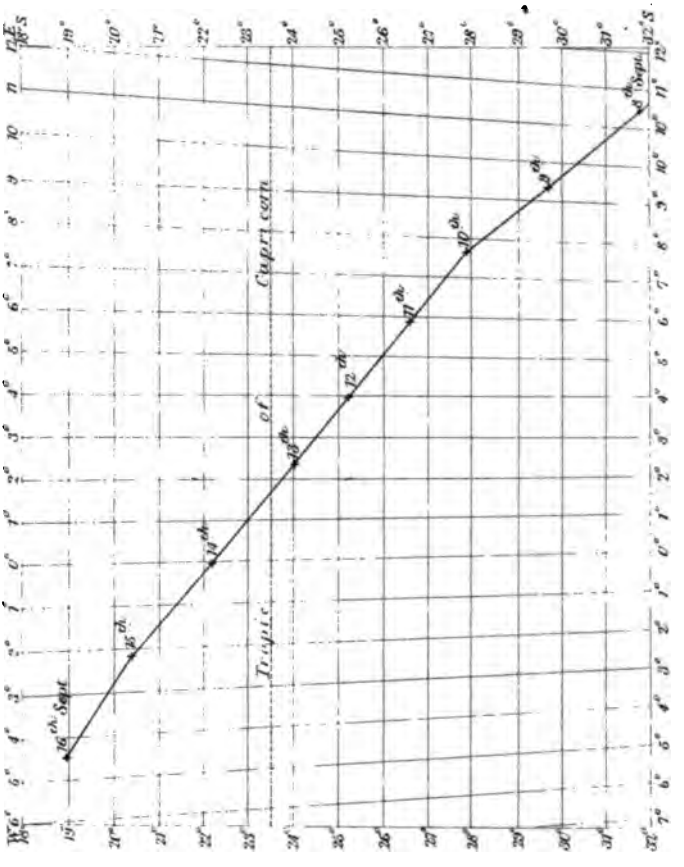
The following sketch of a route-survey by land comprises a portion of Thibet explored by three pandits; the corresponding records Nos. VI. and VII. of the courses traversed, and the latitudes and elevations observed, are given at the end of the book. The whole has been taken from the Indian official records, as affording a better illustration of such operations in unexplored regions than anything in the author's possession or experience. The tract explored includes an area of 18,000 square miles, the total distance traversed being 850 miles. The error in longitude amounted to only $2\frac{1}{2}$ minutes on the direct traverse of 160 miles.

The values of the pace used, as tested by difference of latitude, were, that of the first pandit between Badrinath and Gartok, 2·495 feet; that of the same, between Gartok and Thok-Jalung, 2·512 feet; that of the second pandit, 2·634; and that of the third pandit, 2·495 feet. The observations for latitude were 190 in number, taken at 75 points. Eighty observations of altitude were also made. The check-bearings on the various mountain peaks are omitted in the field-records issued. The

time occupied on the actual work was about five months, and from the dangerous and very trying climate and circumstances under which it was executed, it is considered one of the exploits in surveying of modern times.

In Navigation the traverse representing the vessel's course commences on leaving sight of land of known position, and closes or is checked under similar conditions; the course, or courses, are set according to two or more compasses, the direction being most generally estimated roughly in points of $11\frac{1}{4}°$, while for the actual true course allowance has to be made for deviation, leeway of the ship itself, winds and set of current, the estimation for these allowances requiring a knowledge of the sailing qualities of the vessel, and of the local currents at various places in various seasons; the speed of the vessel through the water is observed by means of the patent log, and to this also correction must be applied for contrary or favorable sea-drift and incidental currents, in order to obtain the actual absolute speed from time to time. The calculation or reduction of the day's set of traverses is then made as in a compass traverse (for which see page 106), either with the aid of Gen. Boileau's well-known traverse-tables, or with a table of sines and cosines; the results taking the form of a difference of latitude and a difference of departure, both being denoted in geographical miles.

The additional calculations at sea, beyond those of the mere reduction of traverse, to differences of latitude, and departure in miles, are to obtain the difference of longitude, the day's course, or mean direction followed in the traverse, and the day's run, or distance estimated in this mean direction; these may be calculated on either of two principles, the middle latitude or the Mercatorial method.



Sketch of Ships Course,

in accordance with recorded observations.

LONDON, CROSBY LOCKWOOD & Co^y 7, STATIONERS HALL COURT, LUDGATE HILL.

1863



By the middle latitude method, the formulæ are:

$$\text{difference of longitude} = \frac{\text{difference of departure} \times \text{radius}}{\text{cosin mid-latitude}};$$

$$\text{tang. course} = \frac{\text{diff. of long} \times \text{cosin mid-latitude}}{\text{difference of latitude}};$$

$$\text{distance} = \frac{\text{difference of latitude} \times \text{radius}}{\text{cosin course.}};$$

If the Mercatorial method be adopted, a table of Mercatorial or meridional differences of latitude must be used, and the formulæ are:

$$\text{difference of longitude} = \frac{\text{mer.diff. lat} \times \text{diff. of departure}}{\text{difference of latitude}};$$

$$\text{tangent course} = \frac{\text{diff. of longitude} \times \text{radius}}{\text{mer. diff. latitude}};$$

$$\text{distance} = \frac{\text{diff. of latitude} \times \text{radius}}{\text{cosincourse}}.$$

Another mode is to obtain the course first, thus:

$$\text{tangent course} = \frac{\text{diff. of departure} \times \text{radius}}{\text{diff. of latitude}};$$

and afterwards obtain

$$\text{difference of longitude} = \frac{\text{mer. diff. lat.} \times \text{tang. course}}{\text{radius}};$$

the distance being obtained as in the former case.

The Mercatorial method is well suited to the long distances performed at sea in 24 hours by steamers, and ships under favorable circumstances; but for the shorter distances travelled by land in journeys of eight or ten hours, the middle latitude method is sufficiently exact.

The resulting differences of latitude and longitude are checked by the results of astronomical observation, and by chronometers keeping Greenwich time; and thus

the error in position can be known within certain limits, even if the position itself is not fixed with sufficient certainty by these two modes of determination.

The method of reducing and calculating the daily courses of a vessel is illustrated by the examples actually worked out on the voyage which are given in Record No. V. at the end of the book.

Plotting the Route.

The plotting of a series of days' journeys may be made for nautical purposes on charts or chart-paper, having latitudes and longitudes drawn on the Mercatorial principle. These however possess, what is in the eyes of the landsman the defect, of not being scaleable anywhere with any scale of equal parts : hence for the traveller's purpose it is generally better to plot routes on maps or plans with the latitudes and longitudes laid down on the more simple principle shown in the attached sketch.

In this the lines of latitude are all equidistant parallel straight lines, while the lines of longitude are converging straight lines, the middle meridian alone being at right angles to the lines of latitude ; the convergence is laid down thus. The value of a degree of longitude in miles at the upper latitude L is calculated $= 60 \cos L$, and these degrees are laid down to scale on the upper edge of the map ; the value of a degree of longitude in miles at the lower latitude L' is calculated $= 60$ cosin L', and these degrees are laid down in the same way at the lower edge of the map. The corresponding degrees of longitude above and below are then joined by converging straight lines. Further exactitude may be obtained by

ROUTE SURVEY.

Scale, 30 Miles to an Inch.

Field record. N^os VI and VII

... LUCKWOOD & CO.'S STATIONERS HALL COURT, E.C. 1884



laying down the values of the degree of longitude at several latitudes, and curving the meridians.

Such maps admit of scaling off distances in geographical miles, and afford a check on the calculation of course and distance; they are amply accurate for the ordinary purposes of route-survey on the scales usually adopted, if kept within the limits of 10° or 12° of longitude.

Expeditionary Surveys.

Although Route-Surveys by land are generally of the foregoing type, there is yet also a type of General Survey, based on the principles of triangulation and measured base lines, that is sometimes adopted in connection with expeditions. Of this nature was the Expeditionary Survey conducted during the Abyssinian campaign by Captain Carter, with some other surveyors of the Indian Survey Department. In that case the tract of country covered about 7000 square miles, extending from ten to fifteen miles on each side of a route from Annesley Bay to Magdala. The method adopted was to measure a base line at about every forty miles along the course, to triangulate in its vicinity, to connect these portions by lines of traverse, and to fill in detail where required by plane-table work. The scale adopted was four miles to an inch. The first base line, measured at Zumaila, was about three miles in length; from this a calculated base, about eight miles in length, beyond the hills, was arrived at by triangulation, and triangulation thence made to points ten, and even twenty miles off. The instruments used were a portable transit instrument, three six-inch theodolites with eyepiece micrometers, prismatic compasses, plane-tables, a set of

chronometers, an aneroid barometer, and some hypsometers. The work effected amounted to eight observations of latitude by circum-meridian altitudes, and six by observation on the pole star, five azimuths, fifty time observations, one longitude by moon-culminating star, and one by the moon's zenith distance.

Five base lines were measured, eight triangles completely determined, fifty intersected points fixed, and four hundred miles of traversing done with the subtense instrument; thirty observations of height taken with the theodolite, and fifty with the thermometer. The time occupied was about three months.

It will be evident to anyone who has read the foregoing chapters that such combinations of ordinary triangulation with route-surveying call for no special explanation or information that may not be equally well treated under those separate heads, and that anyone acquainted with the two branches of practice severally is equally competent to undertake the combination.

Section 2. Rough Astronomical Observations.

On route-surveys, as well as occasionally on others, a certain amount of astronomical observation of a coarse order is necessary, and this also involves the necessity of a fair but correct smattering of knowledge of the subject, and of sufficient practical ability to apply it within the required limits. The following remarks are accordingly drawn up with a view to supply the minimum of knowledge with which it may be done.

Astronomical observations are based on measurements of time and space in arc; the observations on which the greater part of the more directly practical and

useful calculations depend are altitudes of heavenly bodies and their times of transit over the meridian of a place, or of arriving at any position in the celestial concave convenient for special observation. It hence is necessary for the practical observer to be able to observe altitudes and correct or clear them of all sources of error; and secondly, for him to have the means of observing and recording transits, which may be done either by recording the exact moment of an observed transit, or the intervals of time between the transits of two bodies. In the former case the observer must be possessed of the correct absolute time, and in the latter must possess the means of estimating time correctly for comparatively short intervals only; or in the one case he must have a tolerably correct chronometer of known error in good order, in the other a good watch. A chronograph recording to tenths of seconds is also a useful adjunct, and is almost indispensable to those unaccustomed to subdivide time between chronometer beats.

In permanent observatories meridian transits and zenith distances may be observed with the mural circle, but on voyages the transit is generally observed with one instrument and the altitude with another. The instruments absolutely necessary under the latter circumstances consist of a good reflecting repeating circle or a sextant in good adjustment, an artificial horizon, barometer and thermometer, and a portable transit instrument with two stands, the second one being for use in the prime vertical, or a good transit theodolite, of sufficient width between the supports of the horizontal axis, having the means of illuminating the cross-wires.

At sea such instruments for observing transits are dispensed with, and angular measurement in any direc-

tion with the sextant acts as a substitute. There is no attempt made in this book to describe or explain the handling of instruments, for the reason that it would be a waste of time for anyone to read them. A few minutes' experience with the instruments, aided by a competent person to explain their manipulation, is more efficacious than hours spent in poring over written explanations of them.

In addition to the above instruments and appliances, the observer should be provided with the Nautical Almanac for the year, which is published four years in advance, and gives the positions of the more important celestial bodies computed to Greenwich mean time, and tables of some of the more useful corrections, also with Tables of Six- or Seven-figure Logarithms of numbers and trigonometrical ratios for calculating to seconds; and with Four-figure Tables of Logarithms of trigonometrical ratios for the calculations in which seconds may be dispensed with; also with Star-Charts, and maps or charts of the locality.

It is of course a considerable advantage to be provided with a double set of instruments and appliances, as any or all of them are liable to damage or loss in travel.

The capabilities of the traveller or route-surveyor may be limited to the more simple observations and calculations for obtaining the time, the latitude, the longitude, and the direction of the meridian.

The Time.

If the observer has a good chronometer giving nearly Greenwich mean time, and either gaining or losing steadily, he can calculate Greenwich mean time for the moment of any observation he may take; if not, he has to be re-

peatedly and continually finding the time and correcting his timepiece by observation on celestial bodies whose apparent positions are given in the Nautical Almanac; while in the former case he need check his chronometer only occasionally.

The time adopted in the Nautical Almanac is astronomical time reckoned from and to mean noon at Greenwich; civil time is twelve hours in advance of astronomical time; and nautical time is twelve hours in advance of civil time. Hence the nautical day ends at the noon when the astronomical day of the same date begins, or, as the sailors say, the Nautical Almanac is always a day behindhand. The nautical day, like the civil day, is divided into two periods of twelve hours, distinguished by A.M. and P.M.; but the astronomical day consists of twenty-four hours, counting from noon to noon.

For example:

The civil time, April 20th, 9 o'clock A.M., is, by astronomical time, April 19th, 21 o'clock.

The civil time, April 20th, 4 o'clock P.M., is, by astronomical time, April 20th, 4 o'clock.

In the same way also:

The nautical time, July 20th, 10 o'clock A.M., is, by astronomical time, July 19th, 22 o'clock.

The nautical time, July 20th, 3 o'clock P.M., is, by astronomical time, July 19th, 3 o'clock.

(Nautical time will probably be eventually abolished.)

Mean solar time (whether civil, nautical, or astronomical), being a purely artificial arrangement, based on an average day in the year, or an average interval between two consecutive passages of the sun over the meridian of a place, instead of an actual real interval which gives apparent local time; the equation of time

(which is given in the Nautical Almanac), or difference between apparent and mean time, is the correction necessary in reducing observations made with apparent local time to mean local time; the latter may then be compared with Greenwich mean time for the same moment, and the difference between them is the longitude of the place of observation expressed in time. Apparent local time might be obtained most simply by observing the moment of transit of the sun's centre over the middle wire of a transit instrument fixed in the plane of the meridian; that moment is apparent local noon. (The observation, however, would require correction for instrumental adjustments; for this read the paragraph on the adjustment of the Transit Instrument.) The result would then be reduced to mean time by applying the equation of time; and if it is just before noon, by adding twelve hours and altering the date it is reduced to astronomical time; if, however, it is just after noon, the date and hour remain unchanged in astronomical as in civil time. At sea, where the moment of apparent local noon is important, the process is more rough; as the sun mounts in the sky to about the same altitude as it was at noon on the preceding day, the chief mate shouts to the sailing-master, 'It's topping, sir,' and the skipper gets ready his sextant; when the sun just appears to get stationary without ascending or descending, the altitude is observed with the sextant, the chronometer time is noted, and that is taking a meridian altitude at sea, and fixing local apparent noon. The more precise way of doing this is to take several observations, recording altitudes and times both just before and just after the sun tops, and to interpolate the mean as a check on the apparent noon indicated by a maximum altitude.

Sidereal time corresponds to right ascensions of celestial bodies in the same way as solar time corresponds to terrestrial longitudes, the former being reckoned from the first point of Aries, or the intersection of the ecliptic with the equator, the latter from the meridian of Greenwich; the one being convertible into the other by expressing hours, minutes, and seconds of time by their equivalents in degrees, minutes, and seconds of measure of the circle, thus 1 hour = 15°, 1 min. = 15′, 1 sec. = 15″.

The interval between two successive transits of a star over the meridian of a place is a sidereal day, and represents correctly the time of one revolution of the earth on its axis. A sidereal day of 24 hours sidereal time is thus equal to 23h. 56m. 4·092s. of mean solar time. As right ascensions are expressed in sidereal time, observatory clocks are regulated to keep to it, and they thus show the right ascension of the meridian of the place at any moment.

If therefore a transit-instrument be correctly adjusted to the meridian of a place, the transit of a star or planet across its mean wire fixes the moment when its right ascension in time is true sidereal time for the place. The right ascensions and declinations of the principal stars and planets are given in the Nautical Almanac for various intervals of time, and can hence be interpolated for any date; the annual variation of the right ascensions and declinations of stars being small, the proportional part for any date can be obtained by applying the fraction of the year corresponding to the date; this is also given in the Nautical Almanac.

The two above-mentioned processes afford the means of checking or correcting either local mean solar time,

or local sidereal time, or, using both operations, of checking one by the other and comparing the clock with the chronometer; they happen, however, generally to assume either that the meridional direction has been already carefully determined or approximately arrived at. In the case of mean solar time being required to be deduced from observations on the moon, stars, or planets, and their right ascensions given in the Nautical Almanac, it must of course be always borne in mind that the Sidereal Time of Mean Noon, or moment for comparison of time given in the Nautical Almanac for the given astronomical day, must be reduced to the local meridian by applying the acceleration due to longitude, adding it for west and subtracting it for east longitude. The reduction of the right ascensions in the Nautical Almanac to the given moment can be effected by simple proportion, even in the extreme case of the moon, whose right ascensions are given for every hour.

The method of observing the sun at equal altitudes before and after noon avoids this difficulty; it may be roughly done as follows. Set up a transit-theodolite in perfect adjustment with its vertical circle clamped to an altitude of some even number of degrees or minutes exceeding that of the sun at some time in the morning; when the sun rises to this altitude, note the time and altitude, and take some more similar observations in the morning at other convenient altitudes, without moving the instrument throughout the day; in the afternoon observe a set of times and altitudes corresponding to the former in number and altitude. The mean of the times of all the observations will be nearly apparent local noon, but a correction has to be applied on account of the change of the sun's declination in the interval between the mean

of the morning set of observations and the mean of the afternoon set.

The calculation of this correction is based on the following principle. Let SS' be a circle of declination, PZ a meridian of the place, and *aaa* a circle of altitude in the attached figure 39. If the declination remains un-

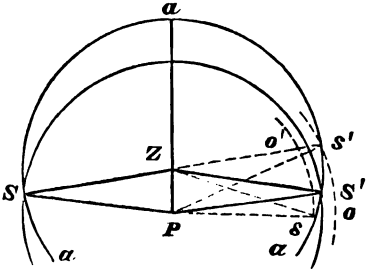


FIGURE 39. CORRECTION FOR EQUAL ALTITUDES.

changed in the interval under consideration, the polar distance PS' in the afternoon remains $= PS$, the polar distance in the morning; but if the declination has decreased, the polar distance Ps' has increased, and is greater than PS; while, under contrary circumstances, Ps, the new polar distance is less than PS; the corresponding changes of declination being represented by $o's'$ and os, and the corresponding required corrections being represented by the arcs $o'S'$, oS' on the circle of declination; while ZP is the complement of the latitude of the place.

Then if x = the required correction in angular seconds

d = change of declination in the interval,

P = the angle SPS', and Q = polar distance PS

$$x = d\left(\text{cotg colat} \times \text{cosec}\,\frac{P}{2} - \text{cotg}\,Q \times \text{cotg}\,\frac{P}{2}\right).$$

This result, after dividing by fifteen to reduce it to seconds of time, must be added to the half-interval between the times of observation in the morning and in the afternoon, when the distance of the celestial body from the elevated pole has increased, but subtracted when that distance has diminished.

As to the declinations themselves. First, for the sun, the declination is given in the Nautical Almanac both for apparent noon and for mean noon, with differences for one hour; the difference for any interval can then be applied to obtain the required reduced declination for any corresponding moment. The time used in the Nautical Almanac being Greenwich time, the local time of observation must first be reduced to the corresponding Greenwich time by applying the longitude expressed in time; the difference of declination for the interval will be either added to or subtracted from the noon declination used according as the declination is increasing or decreasing, or as the Greenwich date precedes or follows the nearest noon. Secondly, for the moon, whose declination is given for every hour, and for the planets, whose declination is given for every day at mean noon, the reduction is effected in the same way. Thirdly, the declinations of certain stars are given for January 1st, with their annual variations, to which the fraction of the year can be applied as for right ascensions, as before explained.

The method of equal altitudes is highly accurate, and is independent of previous observations.

A more common and coarser method of finding the time, assumes that the latitude is known, and is based on a single altitude. The observation is made on a celestial body of known declination, when on or near the prime vertical, that is, nearly due east or due west

of the observer, as under those circumstances the apparent motion in altitude is greatest, and thus gives most precision in result.

Two or three such altitudes[1] taken with a sextant or a reflecting repeating circle, either from the sea horizon or from an artificial mercury horizon, are corrected for index error, horizon, refraction, and parallax, and their mean obtained; the complement of this is then the co-altitude, or zenith distance, forming a side of the spherical triangle under consideration; the corresponding times of observation and their mean are also noted.

Now the quantity required is the interval in time between the azimuthal direction of the celestial body and the meridian of the observer, which is represented in angular measurement by the angle ZPS in the attached figure 40, where Z is the zenith, P the elevated pole, S the sun or star.

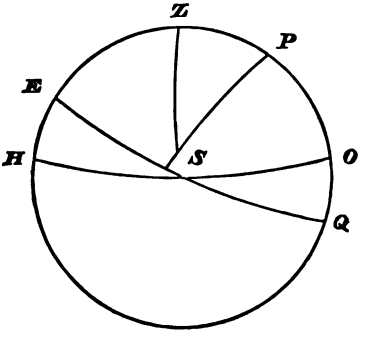


FIGURE 40. THE HOUR-ANGLE.

The three sides of the triangle ZPS are given, for ZS is the zenith distance or co altitude, ZP is the co-

[1] For explanation of the process of taking and correcting altitudes, see the following paragraph on *Latitude*.

latitude, and PS the distance of the celestial body from the elevated pole is equal to 90 $\pm$ its declination.

Then if the required hour-angle $ZPS = h$,

l = the latitude

p = the polar distance

a = the altitude

$$\sin^2 \frac{h}{2} = \text{radius}^2 . \frac{\cos \frac{1}{2}(p+l+a) . \sin \frac{1}{2}(p+l-a)}{\cos l . \sin p}.$$

The obtained hour-angle is then reduced to its equivalent in time, which is the interval between the time of observation and the time of apparent noon, whether earlier or later as the case may be, according as the celestial body was rising or falling at the time of observation. The difference one way or the other between twenty-four hours and the interval in time is the apparent local time of the observation, and if this be corrected by applying the equation of time, the result is the true local time of the observation. This, compared with the recorded time shown by the chronometer, shows the chronometer error at the moment of mean observation.

This last method differs from the former in that it utilises the altitudes observed, instead of neutralising them; they hence require careful correction. This subject is explained in the following paragraph on Latitude.

These methods of finding the time are illustrated by examples following the explanation of this branch of the subject.

In addition to the above, the following is a convenient and ready mode of checking from time to time the rate of a watch or chronometer that is known to go fairly. Having chosen some easily found familiar star (and a second one as a reserve when required) observe

and note its altitude when near the prime vertical by sextant or reflecting circle and artificial horizon ; and note the time by the watch, supposed in the first instance to be corrected by results of previous observations.

If the rate of watch-error is to be found every three, seven, ten, or fourteen days, set the angular instrument to the angle noted on the required day, and observing the same star, note the time of passage by the watch. For acceleration of the star in mean time subtract 3 minutes 55·908 seconds for every day in the interval between the two observations from the time noted at the first observation ; the difference will be the correct time for the second observation. Compare this calculated time with the observed time to obtain the error gaining or losing in the interval, and thence deduce the daily rate of gain or loss for the watch.

The Latitude.

The simplest mode of obtaining the latitude is by observing the meridian altitude of any celestial body whose declination is given in the Nautical Almanac ; the sum of, or the difference between, the meridian altitude and the declination, will give either the latitude or the complement of the latitude.

The following is a rule that avoids hesitation among the four cases. Subtract the corrected altitude from 90°, and thus obtain the true zenith distance, which is then marked as north or south, according as the zenith is north or south of the celestial body ; then, if the declination is similarly marked in the Nautical Almanac, the sum of the two is the latitude, marked in the same way, but if the declination is differently marked, the

difference will be the latitude, and will be marked either north or south like the greater of the two arcs employed.

This method of obtaining the latitude is the most common one; it depends on a single, or on a mean of several, closely following, observations of altitude, which hence must be fully corrected to obtain a true altitude.

The principles on which these corrections are based require examination in detail.

In the first place the instrumental correction for index error, if any, must be applied, although the sextant or circle may be in good adjustment. The resulting angle should then be divided by two if an artificial horizon has been used; but if the sea horizon has been used a correction for dip must be supplied, which may be obtained from tables, or calculated. The eye of the observer being at a certain height (h) above the true horizon, the horizon is hence depressed in proportion to that height, but this is also modified by terrestrial refraction, which renders the correction uncertain in amount, and hence to be avoided whenever possible. Its value is nearly

$$\frac{9}{10}\sqrt{\frac{2h}{r}},$$

when r is the radius of the earth, and it is subtractive.

The remaining corrections for altitudes to be applied afterwards are three in number; first for refraction, second for parallax, and third for semi-diameter.

Refraction is due to the effect of the atmosphere in bending a ray of light, thus forcing us to see a celestial object at an apparent elevation which is higher than its true position; hence the correction for refraction must be subtracted from the observed altitude. Its amount varies with the tangent of the zenith distance of the body observed, but also depends on the humidity,

density, and temperature of the air, and is hence very variable. In England the amount is nearly zero for observations in the zenith, and about 57·82 seconds for altitudes of 45°, and about 33 minutes for the horizon. It is generally obtained from tables, and modified by observation; hence the need of recording the meteorological condition at times of astronomical observation.

The correction for Parallax is the reduction of vertical angles observed on the earth's surface to a position at the centre of the earth, a mean terrestrial position convenient for reference. This correction, if necessary, is always to be added to the observed altitude. In observations of the stars it is inappreciable, but in those of the sun and planets the small angles by which the parallax is measured may be used without the intervention of their sines, though the correct formula for the parallax (p) at any altitude (A), is

$$\sin p = \sin H \times \cos A,$$

where H is the horizontal parallax, which is given in the Nautical Almanac for the sun for every seven or ten days, and for the planets for every day.

In observations of the moon, which is so much nearer the earth, the increase and diminution of the earth's radius at various latitudes has its effect on the parallax of observations taken at such latitudes. The equatorial horizontal parallax (P) for the moon is hence given in the Nautical Almanac for every Greenwich mean noon and midnight; while for any latitude (l) the true horizontal parallax must be computed by the formula

$$H = P\left(1 - \frac{\sin^2 l}{c}\right)$$

where c is the denominator of the fraction representing the compression of the earth at that latitude. Tables

exist which effect this computation; there is, however, the further necessity of modifying these results to suit the altitude, in accordance with the former formula.

The correction for Semi-diameter. – Altitudes being generally observed to the upper or lower edge or limb of the larger heavenly bodies, the angular value of the semi-diameter must be added or subtracted to reduce the observation to the centre.

The semi-diameters are all given in the Nautical Almanac for every day; but in the special case of the moon the semi-diameter (s) given is that when she is in the horizon; this has to be further corrected for the altitude of observation, as she draws nearer the observer as she mounts in the sky, by adding to it the following amount $= s^2(0{\cdot}000019 \times \sin A')$ where A' is the altitude already corrected for everything up to and inclusive of parallax.

These corrections must be applied to all similar observations of altitude whose values are utilised in calculation.[1] Apparent altitudes differ from true altitudes in being uncorrected for refraction and parallax. In the foregoing method of determining latitude, the altitude is dependent on the accuracy of a single observation at an exact time; it is better therefore to have two instruments and two observers acting simultaneously as checks on each other.

When, however, all the work is done by one observer, who does not wish his results to depend on a single meridional altitude, he may take several altitudes at a few minutes before and a few minutes after the meridional passage, and then deduce a mean altitude for a moment differing by a small interval of time from the mean time

[1] A small set of tables for correcting altitudes is given in the author's 'Pocket Logarithms' and reproduced in Section IV.

of apparent noon. A small extra-meridional correction has then to be applied, which is based on the approximate latitude (L) and the declination (D), and is equal to

$$\frac{\cos L \,.\, \cos D}{\sin (L \pm D)} \times \frac{\text{versin } h}{\sin 1''}.$$

where h is the hour-angle for the mean interval. In the event of the celestial object being the sun, moon, or any planet, the allowance for change of declination in the interval must also be made.

A second method of determining the latitude consists in observing the greatest and least altitudes of a circumpolar star ; the mean of these is the altitude of the pole which is the latitude of the place. If, however, the circumpolar star is one whose declination is given in the Nautical Almanac, and whose polar distance can thus be obtained, a single altitude at its inferior passage may be sufficient, in which case the latitude is equal to the sum of the corrected altitude and the polar distance.

A third method of determining the latitude, applicable to northern latitudes only, consists in observation of the altitude of the pole star at any time, whether in the meridian or not. Tables for this purpose, with explanation, are given in the Nautical Almanac, and are hence unnecessary here.

To determine the Meridian.

1. Bisect the horizontal angle formed by two positions of a star or of the sun when arriving at the same altitude in the east and in the west, and bring this direction down vertically by plumb-line or otherwise to the ground, and mark it firmly. At night a light showing through a hole in a board is convenient for this purpose.

2. The mean between the eastern and western elongations of a circumpolar star is the meridional direction.

3. At the moment when a truly vertical circle cuts both the pole star and the star Alioth (ε Ursæ Majoris) this direction is the meridian.

4. If it be preferred to make use of some easily fixed and found position for an instrument, and also of some conspicuous object, such as a church-spire, for reference at any subsequent time, the azimuth of this latter may be obtained by means of the mean of the angles made with it by the extreme positions of stars explained in methods Nos. 1 and 2.

5. If the latitude is known, and the sun or a star of known declination be observed, a single altitude will suffice, and the azimuth may be calculated by the same process, as time is found from a single altitude, the required angle in this case being Z instead of P (see the figure); for PZS is the supplement of the azimuth.

6. The variation of the compass being simply the variation of the magnetic from the true meridian, any of the above methods for obtaining either the true meridian or a correct azimuth of a fixed object enable it to be obtained from them; but at sea it is usually found by observing the bearing of the sun at sunrise and at sunset, or when it is a semi-diameter above the horizon, as this allows roughly for refraction.

7. If the true azimuth of the sun be required at any time at sea, it can be calculated from the altitude (a), the polar distance (p), and the latitude (l), in the same triangle as the hour-angle (see figure 40) by the formula

$$\sin^2 \frac{z}{2} = \frac{\cos \frac{1}{2}(p+l+a) \cdot \cos \frac{1}{2}(p-l-a)}{\sin a \sin l}.$$

The Longitude.

1st. The difference of longitude is most easily and, if the places are not very distant, also most exactly determined by means of good chronometers that gain or lose steadily and equably. If a chronometer giving true local mean time at one place be conveyed to another place whose local mean time is accurately observed and compared with that of the transferred chronometer, the difference between these two local times is the difference of longitude between the two places expressed in time, after making allowance for the gaining or losing of the transferred chronometer.

The longitude in time is reduced to longitude in arc by multiplying severally the hours, minutes, and seconds of time by fifteen, and thus converting them into degrees, minutes, and seconds of arc.

Comparing timepieces is a matter that requires a little practice, more especially when the number of beats to a second is different in each of them. The mode is to listen to the beats of both timepieces while watching them, and counting those of one of them up to the exactly coincident beat, when the time by the first timepiece is immediately recorded, and the counting of the beats of the other commenced and kept up, till by seeing its hand at some conspicuous division, the minutes and hours of the latter can be noted by eye. The exact moment of coincidence can then be fully recorded.

2nd. When chronometers fail or require checking, and when from any cause a telescope cannot be adjusted to the meridian, as at sea, the method of lunar distances may be employed; this, if all the observations and the rather long calculation be very accurately performed,

enables the longitude to be obtained with certainty to within twenty miles.

The direct observation of lunar distance may be true to 30 seconds with tolerable certainty; taking this amount of error as a guide to results, it will occasion a difference of one minute in time, or of fifteen angular minutes in longitude, while the errors inherent in the corrections for refraction will certainly produce more than five minutes' more error, or 20 minutes in all.

In the event of calculated altitudes being used instead of observed ones, there is then the liability to error in juxta-meridional cases from the assumed latitude on which they depend, or, in other cases, from error in time.

It has been asserted that by this method the longitude may be obtained from a set of such observations to within ten miles, on the supposition that the errors will compensate each other in the mean; this cannot, however, be justified, as means of sets simply have the result of diminishing the effect of the worst observations, while they also diminish that of the best. It is hence waste of labour to attempt by this method to get results nearer than the limit above mentioned.

In making use of the moon's motion to determine longitude under the above circumstances, its position must be determined at the moment of observation by its angular distance from some known heavenly body measured with a sextant or reflecting circle, while the true altitudes, both of the moon and of the star, sun or planet observed, are also necessary for the same moment. The Nautical Almanac gives for every three hours of Greenwich mean time the distances of the moon from the sun, the planets, and certain principal stars; hence

the observer can find the Greenwich mean time corresponding to the ascertained lunar distance, while the difference between this and the true local mean time is the longitude estimated from Greenwich expressed in time, which can then be reduced to angular measurement, as before explained.

The most convenient mode of conducting this observation at sea is to have three observers with three instruments, so that the two altitudes and the lunar distance may be perfectly simultaneous; the lunar distance requires correction for semi-diameter and for refraction, while the observed altitudes must be corrected as before explained in the paragraph on latitude. The lunar distance itself requires most precision, as one second of error in observation causes about two minutes of error in longitude. The calculation necessary is best illustrated by example, for which refer to the collection of examples at the end of this chapter.

When one observer alone is available, the lunar distance is observed, and the two altitudes may be calculated; the latitude in this case must be known, the right ascension and declination of the celestial object, and the sidereal time of mean noon, if the object is other than the sun; so also by previous observation the local time. Having these, the two zenith distances must be calculated for the same moment, and corrected for refraction and parallax to get the apparent zenith distances. This calculation is the converse of the problem before explained for determining the hour angle. (See figure 40.) In this case ZS is one true zenith distance required for the given time, ZP is the given meridian zenith distance for the same object obtained from the latitude and declination, and ZPS is the hour-angle; then ZP, PS, and the

hour angle P are given, and ZS is the side required. The formulæ[1] required and used in the Example are

$$\sin^2 D = \sin^2 \frac{ZPS}{2} \cdot \text{cos lat. cos decl.};$$

$$\sin^2 \frac{z}{2} = \sin^2 D + \sin^2 \frac{\text{mer.}ZP}{2}; \text{ whence } ZS = z,$$

and the required altitude $= 90 - z$.

The other method for the single observer involves his observing the altitudes also; in this case he must take several sets of observations, noting the times of each, and using the means of the observed times and lunar distances. Each set of observations will then include, first, an observed altitude of the star used; second, an observed altitude of the moon; third, the lunar distance itself, or several reduced to a mean distance for a mean time; fourthly, the altitude of the moon; and fifthly the altitude of the same star or celestial body. The required altitude of either moon or star corresponding to the time of the mean lunar distance can be obtained by interpolation from the differences of times and differences of altitude. The time intervening between the two observations of altitude of the moon should be as short as possible, on account of the rapid change of declination.

If it is required also to fix the time simultaneously, it is advantageous to have either the moon or the other celestial object near the prime vertical; and if several lunar distances are observed, it is also better to choose celestial objects that are about the same distance from the moon, but on opposite sides of it, thus neutralising the effect of certain errors in those pairs of cases.

[1] D is a mere intermediate angle in the calculation.

The lunar distance itself is measured between the moon's bright limb and the nearest limb of the other celestial body, and corrected for the semi-diameters of both. When the altitudes of the moon and the star are very unequal, the effect of refraction on each semi-diameter will be different, and the correction for semi-diameter does not admit of accurate determination even with the aid of tables. It is usual to avoid such cases when possible. Thus, if in the attached figure 41, RV is the difference of altitudes of M and V,

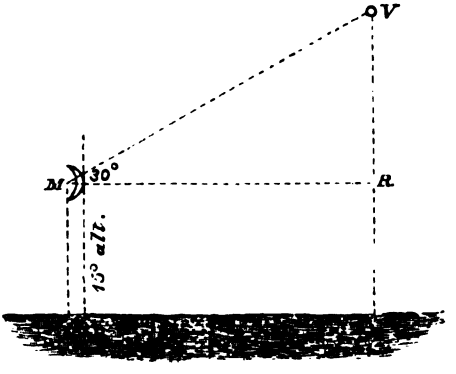


FIGURE 41. THE LUNAR DISTANCE.

representing the projection of moon and the planet or star, if the angle VMR is greater than 30°, or if either altitude AM or $A'V$ is less than 15°, the case is unsuited to very accurate calculation.

The calculation for lunar distances may be made with the aid of special Lunar Tables, which give many pages of special tables for this purpose; but these are generally very intricate and confusing. The more natural plan is to make use of ordinary tables of logarithms,

and a more self-evident mathematical method, as follows.[1] Let a and β be the apparent altitudes, and a' β' the true altitudes of the star and moon respectively, and γ the apparent lunar distance, and $\sigma=\frac{1}{2}(a+\beta+\gamma)$.

$$\text{Then } \sin^2 D=\cos\sigma \cdot \cos\overline{\sigma-\gamma} \cdot \frac{\cos a' \cos\beta'}{\cos a \cos\beta};$$

$$\text{and } \sin^2\frac{\gamma'}{2}=\cos^2\frac{a'+\beta'}{2}-\sin^2 D$$

whence γ', the true lunar distance, is obtained.

It is hardly necessary to add, that at sea an indistinct horizon would prevent the adoption of observed altitudes and render calculated altitudes necessary as before explained.

In this class of calculation, the involved computation of time is often unavoidably tedious; but as regards the clearing of the lunar distance itself, far too many difficulties have been made. The problem consists mathematically in the reduction of an arc that undergoes change due to the corrections in the two altitudes. The corrections are due to two causes, refraction and parallax. First as to refraction; the barometric and thermometric readings enable this to be calculated (see table of altitude-corrections in the Pocket Logarithms) within moderate limits of accuracy; beyond these, it must be noticed that such readings are true for the actual position of the observer only, while the visual ray, affected by refraction, passes through atmosphere under other conditions, hence the estimated corrections for refraction in altitude cannot be absolutely correct. It would, however, be an error to adopt mean refraction in preference, and thus arrive further from the truth; while it would be equally unwise to make use of such Lunar tables as

[1] D is a mere intermediate angle in the calculation.

merely arrive at corresponding results through a circuitous blind method.

Secondly, as regards the parallax of the moon, which varies with the position of the observer. The compression of the earth at any latitude is not absolutely known by formula, the earth being an irregular spheroid; but a close approximation may be used in the reduction of the parallax.

Thirdly, with respect to semi-diameters in inclination, the limit before mentioned regarding difference of altitudes can be generally adhered to, and a star may be observed in preference to the sun.

Finally, when a lunar distance is observed under favourable circumstances, so that calculated results may possibly be true to seconds of arc, seven-figure logarithms should be employed in reducing the calculation.

Under any circumstances, when a telescope can be set to an altitude or directed to the zenith, another method should be employed in preference.

3rd. *Lunar Transits.*—The use of a portable transit instrument, set up in the plane of the meridian, enables this more accurate method of determining longitude to be adopted on land, even if the timepiece available is rather faulty, as it merely requires the measurement of a short interval of time.

There are certain stars having nearly the same declination and right ascension as the moon, that are termed moon culminating stars, which may be seen and recognised when the sun happens not to be too near. The Nautical Almanac gives the right ascensions and declinations of these stars, also the right ascension of the moon's bright limb and the declination of her centre at the instant of their respective transits at Greenwich.

Since then we have the computed times of transit over the Greenwich meridian of both the moon and a moon culminating star, and can observe at any place with the transit instrument and a good watch the times of transit of the same two bodies, the difference between the computed Greenwich interval and the observed local interval in time is proportional to the longitude.

The moon's right ascension being discovered by observing her transit, and her position being given for each hour of mean time of the meridian of Greenwich in the Nautical Almanac, the corresponding Greenwich time may be found by a simple proportion; but as an observation would be effected on the bright limb, the right ascension recorded in sidereal time must be corrected for semi-diameter, either added or subtracted. This short period varies inversely with the distance of the moon and the cosine of her declination; but as this interval is given in the Nautical Almanac for the upper and lower transit on the Greenwich meridian, an intermediate quantity may be adopted in proportion to the approximate longitude of the place of observation.

These methods of obtaining longitude are illustrated by examples, in the next section.

The Adjustments of the Transit Instrument.

In the foregoing pages of this chapter, which are intended merely as a guide to such rough astronomical observations as may be useful on route-surveys, the use of the portable transit instrument in the plane of the meridian has been referred to as applied to the two simple cases of finding the sidereal time and observing lunar transits. The transit may, however, also be used for determining latitude by observation on the sun or a star in

the Prime Vertical, at right angles to the meridian; in this case a second stand is extremely useful, as it enables the instrument to be moved from one to the other.

In obtaining latitude by this method, a correct timepiece is unnecessary, but the approximate time is required, and the approximate longitude. At the moment that the sun or star arrives at the Prime Vertical its altitude is observed and corrected (A), its declination (D) is obtained with the help of the Nautical Almanac, and the latitude (L) is obtained by the formula:

$$\sin L = \sin D \times \operatorname{cosec} A.$$

Under any circumstances the transit instrument should be put in as good adjustment as possible, and even in route-survey work it is sometimes necessary to compute and apply corrections on account of defective adjustment.

First as to the adjustments themselves.

Collimation.—The line of sight through the centre wire must be exactly at right angles to the turning axis of the telescope; this is checked by observing on some distant object with the centre wire, and then reversing the instrument in its Y's or supports, and repeating the observation; half the difference in the relative position of the object should be corrected by using the collimating screws.

Level.—The turning axis of the telescope should be truly level. To check this, use the striding-level, whose scale is graduated usually to units equal to fifteen seconds of arc, and read the scale at both ends, then reverse the level and again read the scale at both ends, thus obtaining four readings; then the required correction to be applied by turning the levelling screw is one-fourth

of the difference between the sum of the two east readings and the sum of the two west readings.

Azimuth.—The two preceding adjustments having been made, the centre wire will describe a true vertical circle when the telescope turns on its turning axis; but even if the instrument was originally set up exactly to the meridian, these adjustments have affected this, and the resulting deviation in azimuth must be adjusted by turning the azimuth screw. This requires much precision, which can only be arrived at by measuring the length of the axis of the instrument and correcting the number of threads in an inch of the screw effecting the adjustment. This will enable the observer to judge of the effect of one revolution, and to know how much to turn the screw for the amount of movement required.

It is usual to set up or fix some distant terrestrial object (a pole bearing a serrated cross-bar), in the meridional direction by observation on a circumpolar star at its eastern and western elongations. To check this, when the instrument is set up it is usual to observe the upper and lower transits of a circumpolar star. The interval of time between them should be exactly twelve hours of sidereal time; if not the observed intervals will indicate to which side the deviation in azimuth lies; as, if the western interval (i.e. that between the upper and the lower transit) be too small, the deviation is to westward of the pole, and the converse.

To obtain the actual value of the angular error in azimuth, observe the upper transit of one circumpolar star S in the attached figure, and the lower transit of another circumpolar star s, not differing very considerably from the former in declination. If then ZSs be the incorrect meridian of observation of the instrument, and

ZPH the true meridian passing through the pole P; the angle Z represents the error in azimuth, Ps, PS, are the

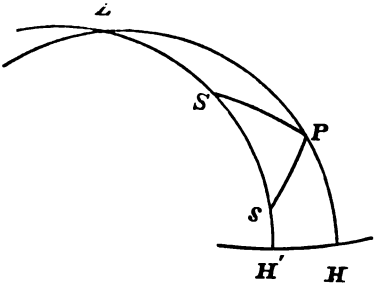


FIGURE 42. ERROR IN AZIMUTH.

polar distances of the stars S and s; ZP is the colatitude, then

$$\sin Z = \frac{\sin Ps \,.\, \sin PS \,.\, \sin SPs}{\sin (Ps + PS) \,.\, \sin ZP}.$$

The value of the angle SPs in this expression varies according as the deviation is east or west; if R, r be the right ascensions, and T, t the observed times of transit of the stars S, s respectively, for the case of a deviation east,

$$SPs = 24 - \{(R - r) - (T - t)\}$$

and for the case of a deviation west,

$$SPs = (R - r) - (T - t);$$

also since the angle Z is a very small angle, and the angle SPs a very large one, their sines may be dispensed with, and the angles or supplements used instead.

These three adjustments for collimation level and azimuth can be perfected only by repeated approximation; any errors left remaining should be noted for use in applying corrections to any observations that may be taken afterwards.

The correction (c) for collimation error to be applied

is thus obtained. Fix a permanent scale on the distant terrestrial meridian mark, which is suited to the distance; then fifteen seconds of arc to the radius expressed by the distance will be equivalent to one second of time in the error; the equatorial collimation correction (e) being thus obtained, the correction (c) for transits of other bodies having other declinations will be

$$c = e \cdot \sec D,$$

and must be marked either E or W, as the case may be.

The correction for level error (l) to be applied is thus obtained. Let n be the inclination of the turning axis to the horizon shown by the level; then the transit of any celestial body whose altitude is A and declination is D is affected to the extent of the following value,

$$l = n \cdot \sin A \cdot \sec D.$$

As, if n be expressed in units of fifteen seconds each, l will be in seconds of time, the divisions on striding-levels are generally made equal to $1\frac{1}{2}$ seconds of arc, or a tenth of a second in time.

The correction for azimuth error (a) affecting the transit of any celestial body having a declination D at an altitude A is expressed by the following formula, where d = the azimuthal deviation before determined.

$$a = d \cdot \cos A \cdot \sec D.$$

The care of chronometers requires special attention on journeys or route-surveys by land or sea. The following are the principal points in which care is required in transit and manipulation.

1. The chronometer should be kept as much as possible in an equable temperature, free from jerks, swinging motion, and changes of position.

2. It should always be wound up at the same hour, and in the same manner, by steadily turning the key

only for a sufficient number of counted times, the last two being wound very gently.

3. Every opportunity of checking its error and rate of gaining or losing should invariably be taken ; and the results, together with pressure, temperature, and weather, should be recorded for reference.

4. The method of comparing chronometer and time-pieces before described should be invariably adopted.

Section 3. EXPLANATORY EXAMPLES.

These examples may be worked out with the help of ordinary trigonometrical tables and formulæ; a collection of the latter is given at pages 54 and 55 of Part I. For altitude corrections, see Section 4, Part IV.

The Time.

1. By meridian observation of the sun.

February 23, 1860.

The moment when the sun arrived at its highest, or when it was on the meridian of the place, was shown by a watch or chronometer to be 12 h. 4 min. 54 sec.

Now the local time of mean noon was	12 h.	0 min.	0 sec.	
and the equation of time for the date was		13	41	
Hence the local time of apparent noon was	12	13	41	
And as the watch was then	12	4	54	
the error of the watch was		8	47	slow.

2. By equal altitudes of the sun.

The following are the times recorded by chronometer. Altitude 74° 20′; on February 10, 1860.

Wires	Preceding limb in Morning			Wires	In Afternoon.		
(1)	9 h.	27 min.	7·87 sec.	(5)	2 h.	44 min.	5·25 sec.
(2)	9	28	3·50	(4)	2	44	59·50
(3)	9	28	59·50	(3)	2	45	55·50
(4)	9	29	55·25	(2)	2	46	50·25
(5)	9	30	49·25	(1)	2	47	46·40
Mean wire	9	28	59·05	Mean	2	45	55·30

giving noon by chronometer to be 12 h. 7 min. 27·17 sec.

Wires	Following limb in Morning			Wires	In Afternoon.		
(1)	9 h.	32 min.	34·25 sec.	(5)	2 h.	38 min.	33·50 sec.
(2)	9	33	31·75	(4)	2	39	30·00
(3)	9	34	28·50	(3)	2	40	26·00
(4)	9	35	26·00	(2)	2	41	23·75
(5)	9	36	22·50	(1)	2	42	20·05
Mean wire	9	34	28·60	Mean	2	40	26·75

giving noon by chronometer to be 12 h. 7 min. 27·67 sec.

The mean of the two sets of observations gives	12 h.	7 min.	27·42
Instrumental corrections			−17·09
Corrected mean	12	7	10·33
And using the equation of time, app. noon was at	12	14	30·13
Hence the error of the chronometer was		7	19·80 slow

3. To determine by a single altitude the true mean time at sea on November 4, 1863; (I.) in the morning and (II.) in the afternoon.

I. At approximately 8.30 a.m., in latitude 46° 45′ N, the corrected altitude of the sun was 12° 59′; the equation of time 16 min. 17 sec.; the sun's declination 15° 19′ 23″ less 1′ 48″ = 15° 17′ 35″.

To calculate the hour-angle h.

Altitude	12° 59′	
L.cosec. polar distance . . .	105° 18′	0·01567
L.sec. latitude	46° 45′	0·16419
L.cos. half sum	82° 31′	9·11474
L.sin. remainder	69° 32′	9·97168
		19·26628
Log sin $\frac{h}{2}$ of half the hour-angle 25° 26′ 49″		9·63314

Equivalent hour-angle in time is	3 h.	23 min.	25 sec.
	24		
	20	36	25
Equation of time . . .		−16	17
True mean local time . .	20	20	08
or in civil time was . .	8	20	8 instead of 8.30

II. At approximately 3.30 p.m., in latitude 47° 22′ N., the corrected altitude of the sun was 10° 55′; the sun's declination 15° 19′ 23″ plus 3′ 29″ = 15° 22′ 52″.

To calculate the hour-angle h.

Altitude	10° 55′	
L.cosec. polar distance . . .	105 23	0·01585
L.sec. latitude	47 22	0·16922
L.cos. half sum	81° 50′	9·15245
L.sin. remainder	70 55′	9·97545
		19·31297
Log sin $\frac{h}{2}$ of half the hour-angle 26° 57′ 43″		9·65648

Equivalent hour-angle in time is .	3 h.	35 min.	42 sec.	
Equation of time		−16	17	
True mean local time . . .	3	19	25	
Or the civil time was, instead of 3·30 p.m.	3	19	25	p.m.

4. To determine error of chronometer by a single observation of the sun on February 22, 1860, in lat. 51°23′36″ N.

	h.	min.	sec.	
Sun's declination . .	=9	59	39·5	Following limb observed.
Time of transit over mean wire	22	24	30	
Corr. level, coll., and azimuth			− 0·23	
	22	24	29·77	
Passage of semi-diameter in sidereal time . . .		1	5·95	
	22	23	23·82	
Sun's Right ascension for date and place . . .	22	24	9·50	
Clock error . . .			45·68	Slow.

Comparison of Chronometer with Clock.

	h.	min.	sec.	
Mean of clock error is .			44·86	Slow; clock gaining 0·2 sec. daily.
Times of comparison: clock	7	48	35	On February 22.
			44·86	
True time by clock . .	7	49	19·86	Sid. time.
Chronometer . . .	9	32	30·5	M. solar time.
Sid. time at mean noon preceding at Greenwich was	22	6	29·82	

	h.	min.	sec
Change of R.A. for 1 hour = 9·528, change for 2 min. 10 sec. of E. longitude = 9·528 × ·036 . . .			·34
Sid. time at mean noon locally	22	6	29·48
Compl. of 24 hours . .	1	53	30·52
	7	49	19·86
Sid. time of comparison after mean noon . . .	9	42	50·38
Equivalent in mean solar time	9	41	14·89
	9	32	30·5
Error of chronometer, slow		8	44·39

5. To find the mean time of apparent noon, February 23, 1860. Given, longitude 2 min. 10 sec. E.; chronometer error 8′47″·56; rate losing daily 4″·60.

The last of six readings on a reflecting repeating circle, with an artificial horizon, was 340° 11′ 35″; giving the observed mean altitude of the sun's lower limb 28° 20′ 57″·9; and the corrected altitude 28° 35′ 25″·45. The times of observation and their differences from that of apparent noon were as follows:

	h.	min.	sec.
The time of apparent noon . . .	12	0	0
Equation of time at Greenwich . . .		13	41·22
Correction = 0·353 × 4·6 ÷ 60 . . .			·01
M.T. of apparent noon	12	13	41·23
Chronometer error		8	47·56
App. noon by chronometer . . .	12	4	53·67

	Times			Differences		Versin $h \div \sin 1''$
1st	12	8	48	3	54·33	29·06
2nd	12	10	31	5	37	62·02
3rd	12	11	49	6	55	94·05
4th	12	12	55	8	1·83	126·60
5th	12	14	28·5	9	34·83	180·18
6th	12	15	36	10	42·33	225·01

	min.	sec.	
Mean of intervals or M.T. of app. noon W. .	7	27·49	119·63
Allowance for long. E. . .	2	10	
Time of observation on sun, W. of mer. of Greenwich .	5	17·49	

The Latitude.

1. At sea on November 4, 1863; the latitude was required at 8.20 a.m., at noon, and at 3.20 p.m., dependent on an observation of the sun in the meridian; the sun and the place being on opposite sides of the equator.

The observed meridian altitude	27° 24′	
Corrections for dip, refr. par. and semid.	+ 10	
Corrected meridian altitude	27 34	
	90	
Zenith distance	62 26	N.
Declination	− 15 21	S.
Latitude at noon	47 5	N.
Correction for 3 h. and 40 min.	− 0 20	
Latitude at 8.20 a.m.	46° 45′	N.
Correction for 3 h. 20 min.	+ 0 17	
Latitude at 3.20 p.m.	47° 22′	N.

2. On shore in longitude 2 min. 10 sec. E.; the latitude was required on February 23, 1860; the mean time of apparent noon being 5 min. 17·49 sec. by chronometer.

(See *The Time*, Example 5 for time.)

Declination at Greenwich	9°	59′	39″·5	
Change of Declination, 55·09 per hour, for the time			4·84	
Decln. corrected to M.T. of observation	9	59	34·66	
Mean altitude	28	20	57·90	
Refraction for this alt.		− 1	51·70	
Parallax		+	7·65	
Semi-diameter		+ 16	11·60	
Correction given below		1	23·55	
Colatitude	38	36	23·66	
Latitude	51°	23′	36″·34	N.

Formula for correction.

$$\frac{\cos L \,.\, \cos D}{\sin (L \pm D)} \times \frac{\text{vers } h}{\sin 1''}$$

L. cos approximate latitude	9·79518
L. cos approximate declination	9·99336
Ar. comp. L. sin zen. dist.	·05548
Log 119·63 versin h + sin 1″	2·07784
Log rate losing 4·66	·00005
Log correction 83·55 = 1′23″·55	1·92191

The Azimuth.

1. At sea, at 8 h. 20 min. a.m. on November 4, 1863, in latitude 46°45′ N., the corrected altitude of the sun was 12°59′, and its corrected declination 15°17′35″; required its true azimuth.

L.sec. altitude . .	12° 59′		0·01125
Polar distance	105	17 35″	
L.sec. latitude . . .	46	45	0·16419
L.cos. half sum . . .	82	31	9·11474
L.cos. remainder . . .	22	46 35	9·96480
			19·25498
Log sin $\frac{z}{2}$	25°	5′ 40″	9·62749

True azimuth 50° 11′ 20″ E.

2. At sea, at 3 h. 20 min. p.m. on November 4, 1863, in latitude 47°22′ N., the corrected altitude of the sun was 10°55′, and its corrected declination 15°22′52″; required its true azimuth.

L.sec. altitude . . .	10° 55′		0·00793
Polar distance	105	22 52″	
L.sec. latitude . . .	47	22	0·16922
L.cos. half sum . . .	81	50	9·15245
L.cos. remainder . . .	23	32 52	9·96223
			19·29183
Log sin $\frac{z}{2}$	26°	16′ 0″	9·64591

True azimuth 52° 32′ 0″ W.

The Longitude.

1. Finding the longitude at sea by lunar distance.

(Example taken from Thomson's Lunar Tables.)

Apparent time of ship, March 25, 1 h. 38 min. 18 sec.
Approx. lat. N., 15° 42′·5; approx. long., 72° 40′ E.
Bar. 29·7; therm. 76; height of eye, 17 feet; index error, −5.
Sun's alt. 61° 56′; moon's alt. U.L. 47° 34′ E. of meridian.
Distance of sun west of moon, 69° 45′ 53″.

	h.	m.	sec.
Time by chron., 25th	1	38	18
App. long. in time	−4	50	40
Red. equa. of time	+6	4	
Greenwich M.T., 24th	20	53	42
Red. S.T.M.N., 25th	0	11	23

	°	′	″
Sun's obs. altitude	61	56	0
Semi-diameter		16	3
Dip		−4	1
Sun's app. alt.	62	8	2
Moon's obs. alt. U.L.	47	34	00
Red. semi-diameter		14	59
Dip		−4	1
Moon's app. alt.	47	15	00
Red. parallax		36	1
For therm.			3
Moon's true alt.	47	51	4
Moon's red. P.D.	67	15	40
Latitude	15	42	30
Half sum	65	24	37
Difference	17	33	33

	h.	m.	sec.
Hour-angle, W.	21	3	18
Moon's red. R.A.	4	52	44
Sum less 24 h	1	56	2
Sid. T.M. noon, 25th		11	23
Sid. h from noon	1	44	39
Retard.			−17
Mean time of obs.	1	44	22

	°	′	″
Obs. lunar distance	69	45	53
Index error			−5
Moon's red. semi.		14	59
Sun's semi.		16	3
App. lunar distance	70	16	50

Clearing lunar distance performed in Example 3.

	°	′	″
True lunar distance	69	41	24
Dist. N.A. 18 h. 24th	68	22	58
Difference from N.A.	1	18	26

	h.	m.	sec.
Corresponding interval	2	53	8
Correction N.A.			+1
G^h. time from N.A.	18	0	0
Mean G^h. time, 24th	20	53	9
Mean time of ship, 25th	1	44	22
Longitude in time . E.	4	51	13
Longitude in arc . E.	72	48	15

Logs	
·03514	l.cosec.
·01653	l.sec.
·61923	l.cos.
·47950	l.sin.
9·15046	

(In this example the moon's true altitude should be diminished by 53″ for refraction; the total refraction being 50″. See corrections in Example 3.)

2. Finding the longitude at sea by lunar distance.

(Example taken from Thomson's Lunar Tables.)

Apparent ship time, March 25, 6 h. 52 min. 30 sec.
Approx. lat. N., 15° 53′; approx. long. E., 72° 40′.
Barom. (29″·7; therm. 72°; height of eye, 18 feet; index error, —?)
Star's alt. 49° 4′ E. of meridian; moon's alt. L.L. 58° 41′.
Distance of star Regulus E. of moon, 71° 14′ 25″.

	h.	m.	sec.
App. chron. time, 25th	6	52	30
App. long. in time	−4	50	40
Red. eq. of time	+6	00	
G^h. M.T., 25th	2	7	50
Red. S.T.M.N., 25th	0	11	23

	°	′	″
Stars obs. alt.	49	4	0
Dip	−4	8	
Corr. 50″, therm. −2	−0	48	
Star's true alt.	48	59	0
Moon's obs. alt. L.L.	58	41	0
Red. semid.	+15	2	
Dip	−4	2	
Moon's app. alt.	58	52	0
Red. parallax	54	26	
Moon's true alt.	59	46	26
Star's true alt.	48	59	0
Star's P.D.	77	16	0
Latitude	15	53	0
Half sum	71	4	0
Difference	22	5	0

	h.	m.	sec.
Hour angle W.	21	10	54
Star's red. R.A.	10	0	4
Sum less 24 h.	7	10	58
S.T.M.N., 25th	11	23	
Sid. h from noon	6	59	35
Retardation	−1	9	
Mean time of obs.	6	58	26

	°	′	″
Obs. lunar distance	71	14	25
Semid. remote l.	−15	2	
App. lunar distance	70	59	23

Clearing performed separately.

	°	′	″
True lunar distance	70	33	20
Dist. N.A. noon, 25th	71	33	44
Difference from N.A.	1	3	24

	h.	m.	sec.
Corrg. interval	2	8	1
Correction N.A.			+2
Correct interval	2	8	3
Time at G^h. of dist. 25th	0	0	0
G^h. mean time	2	8	3
Local mean time	6	58	26

		°	′	″
Longitude in time	E.	4	50	23
Longitude in arc	E.	72	35	45

Logs
·01081 l.cosec.
·01691 l.sec.
·51117 l.cos.
·57514 l.sin.

9·11403

(In this Example the refraction of the moon's altitude would amount to about −33″; it appears to have been omitted.)

3. Clearing the lunar distance, Example No. 1.

α. Sun's app. altitude	62° 8′ 2″	l. sec	0·33031
β. Moon's app. altitude	47 15 0	l. sec	0·16826
γ. App. lunar distance	70 16 50		
α'. Sun's true altitude	62 7 37	l. cos	9·66978
β'. Moon's true altitude	47 50 11	l. cos	9·82689
$\sigma = \frac{1}{2}(\alpha+\beta+\gamma)$	89 49 56	l. cos	7·46100
$\sigma-\gamma$ . . .	19 33 6	l. cos	9·97421
D		l. sin^2	7·43045
$\frac{1}{2}(\alpha'+\beta')$ . .	54° 58′ 54″	l. cos	9·75878
		l. cos^2	9·51756
$\frac{1}{2}\gamma'$.		l. sin^2	9·51400
	34 51 0	l. sin	9·75700
γ'. True lunar distance	69 42 0		

Corrections.

	α	β
Deg.	+ 31·″	54·″
Min.	− ·2	− ·5
Bar.	− ·3	− ·5
Ther.	− 1·5	− 2·9
	+ 29·0	50·1
Ref.	− 29·	− 50″
Par.	+ 4·	36′ 1″
	− 25·″	35′ 11

Anti-logs.

·002694

·329280

·326586

(The discrepancy is due to the cause mentioned in Example 1.)

4. Clearing a lunar distance. Data of observation taken from a case in Woodhouse's Astronomy. Using six-figure logarithms.

α. Star's app. alt.	28° 24′ 59″	l. sec	0·055758
β. Moon's app. alt.	61 36 50	l. sec	0·322931
γ. App. lunar dist.	33 30 21		
α'. Star's true alt.	28 23 14	l. cos	9·944362
β'. Moon's true alt.	62 2 0	l. cos	9·671134
$\sigma = \frac{1}{2}(\alpha+\beta+\gamma)$	61 46 5	l. cos	9·674900
$\sigma-\gamma$	28 15 44	l. cos	9·944872
D		l. sin^2	9·613957
$\frac{1}{2}(\alpha'+\beta')$ . .	45 12 37	l. cos	9·847885
		l. cos^2	9·695770
$\frac{1}{2}\gamma'$		l. sin^2	8·930547
	16 58 24	l. sin	9·465274
γ'. True lunar dist.	33 56 48		

Corrections.

	α	β
Ref.	− 105″·	− 31″
Corr. alt. 61° 36′ 19″		
Hor. par. 53′ 59″		
	9·67718	
	8·19602	
	7·87320	
Red par.	+ 25′ 41″	

·411109

·496330

·085221

A result which corresponds with that of Woodhouse.

5. Calculating an altitude for an observation of lunar distance.

The moon's meridian zenith-distance was 19° 20′ 16″ on February 27, in latitude N. 3° 42′ 0″; what was its altitude at 7 h. 1 min. 27 sec. of mean time on the same day; the sid. time of mean noon being 22 h. 24 min. 47 sec.; moon's hor. par. 54′ 57″.

	° ′ ″		° ′ ″
Mean time	7 1 27	True altitude	70 33 42
Equiv. sid. hour from noon	7 2 35	Par. in altitude	−18 17
Sid. T. mean noon	22 24 47	Refraction	+ 20
Sum less 24 h. or R.A. of meridian	5 27 22	App. alt. of moon	70 15 45
Moon's R.A.	5 19 12		
Hour angle	8 10		

Half its equiv. in arc	1 1 15	log sin²	6·50160	
Moon's decl. N.	23 2 16	log cos	9.96391	
Latitude N.	3 42 0	log cos	9·99909	
			6·46460	·000291
Half Mer. Z.D.	9 40 8	sin²		·028209
Half true Z.D.	9 43 9	sin²		·028500
	19 26 18			
True alt. of moon	70 33 42			

6. Finding the longitude on land by lunar transit.

In latitude 51°23′36″·34 N., in February 1860, the following transits of the moon and ♑ Geminorum were observed in sidereal time.

	Moon h. m. sec.	Star h. m. sec.
Observed transits	6 31 36·64	7 10 59·73
Coll.	−·16	−·15
Level	−·05	−·03
Az.	−·07	−·07
Corrected transits	6 31 36·36	7 10 59·48
Interval	0 39 23·12	
R.A. for Greenwich	6 32 30·79	7 11 48·03
Interval	0 39 17·24	
Difference of intervals	0 0 5·88	

Difference for 1 h. from N.A.	160·95
·88 × 60 ÷ 160·95 = 2′·192 =	2′ 11″·52
correction	·28
E	2′ 11″· 8

Corrections for the Transit Instrument.

1. To determine by observation the equatorial intervals of a transit instrument on January 12, 1860, in latitude 51°23′36 N., and longitude 2 min. 10 sec. E.

Vertical circle west, and star above pole.

Wire	Observations			Obs. Intervals		Equat. Intervals
	h.	m.	sec.	m.	sec.	sec.
A	6	21	36·31	−11	47·63	33·87
B	6	27	29·53	− 5	54·41	17·00
C	6	33	23·61		·33	·01
D	6	39	18·18	+ 5	54·24	16·98
E	6	45	12·08	+11	48·14	33·95
Mean	6	33	23·94			

	m.	sec.		°	′	″		
A	11	47·63	=	2	56	54·45	log sin	8·7112806
				87	15	2·4	log cos	8·6809381
		33·87			8	28·09	log sin	7·3922187
B	5	54·41	=	1	28	36·15	log sin	8·4111240
					4	15·03	log cos	8·6809381
		17·00					log sin	7·0928621
C		0·33	=			4·95	log sin	5·3796994
						·13	log cos	8·6809381
		0·01					log sin	4·0606375
D	5	54·24	=	1	28	33·60	log sin	8·4109157
					4	14·80	log cos	8·6809381
		16·98					log sin	7·0918538
E	11	48·14	=	2	57	2·10	log sin	8·7115932
					8	29·28	log cos	8·6809381
		33·95					log sin	7·3925331

Hence, for reduction to mean wire from centre wire add 0·01 when vertical circle is west and star above pole.

2. To determine the equatorial collimation error of

the same instrument at the same place and date, also the level and azimuth errors.

The inequality of pivots, a permanent portion of the level error, n this instrument amounted to o″·03; the remainder of the error will vary with every observation.

The equatorial collimation error (c) is constant for the same instrument.

The azimuthal deviation (d) is the mean result of observations, and is practically a constant quantity.

Observations on δ Ursæ Minoris. Star below pole.

Wires	Obs. transits h.	m.	sec.	Levels (n) sec.	Corr. levels (l) sec.	Transits corr. for level h.	m.	sec.
1st	6	6	31	+·33	4·13	6	6	35·13
2nd	6	11	23	+·34	4·26	6	11	27·26
2nd	6	20	59·5	+·41	5·13	6	21	4·63
1st	6	25	48·5	+·28	3·50	6	25	52·01
Centre	6	16	6·5	+·43 −·03	5·01	6	16	11·51
Mean						6	16	14·75

Collimation error (c) for δ Ursæ Minoris, 3·24

For correcting the levels

$$l = n \frac{\sin 47^\circ\ 59'\ 35''}{\cos 86^\circ\ 35'\ 55''} = n \times 12{\cdot}53.$$

Equ. coll. error $e = 3{\cdot}24 \cos 86^\circ\ 36'$ . . . = 0·19
Adding for reduction to mean wire . . . = 0·01
Equ. coll. error on Jan. 12, was . . . . 0·20

When the vertical circle E., and star above pole, it is −0·20.
When the vertical circle W., and star above pole, it is +0·20.
Next for azimuth error:

	Transits corr. for C. and L. h.	m.	sec.	R.A. from N.A. h.	m.	sec.
δ Ursæ Minoris . .	6	16	14·75	6	17	2·32
Sirius	6	38	11·57	6	39	0·99
Differences . . .		21	57·82		21	58·67

Azimuth error on this 0·85.

$$\text{Azimuth deviation} = \frac{0{\cdot}85}{\frac{\cos A}{\cos D} - \frac{\cos A'}{\cos D'}} = \frac{0{\cdot}85}{11{\cdot}28 - {\cdot}966} = 0{\cdot}07$$

deviation E. of N., star S. of zenith.

3. Determining the corrections for the same instrument on February 10, 1860.

Observations on δ Ursæ Minoris. Star below pole.

Wires	Obs. transits h.	m.	s.	Levels (n)	Corr. levels (l)	Trans. corr. for level h.	m.	s.
E	6	7	43·5	+ ·03	+ ·33	6	7	43·83
D	6	12	32	+ ·05	+ ·56	6	12	32·56
D	6	21	52	+ ·10	+ 1·13	6	21	53·13
E	6	26	42	+ ·28	+ 3·16	6	26	45·16
					Mean	6	17	13·67
Centre	6	17	13	+ ·01 − ·03	− ·22	6	17	12·78

Collimation error (c) for δ Ursæ Minoris . . . 0·89
Equatorial coll. error, $e = 0{\cdot}89 \cos D$ . . . $= 0{\cdot}05$
Adding for reduction to mean wire . . . $= 0{\cdot}01$
Equal coll. error on February 10 . . . . $= 0{\cdot}06$

When the vertical circle is W., and star below pole, it is +
above pole, it is −

Next, for azimuth error:

	Trans. corr. for C. and L.			R.A. from N.A.		
δ Ursæ Minoris . .	6	17	13·67	6	17	6·99
Sirius	6	38	5·84	6	38	0·91
Differences . . .		20	52·17		20	53·92

Azimuth error on this, 1·75.

$$\text{Azimuth deviation} = \frac{1{\cdot}75}{\frac{\cos A}{\cos D} - \frac{\cos A'}{\cos D'}} = \frac{1{\cdot}75}{12{\cdot}58 - {\cdot}966} = 0{\cdot}15$$

deviations E. of N., star S. of zenith.

Taking the permanent means of the preceding corrections for the same instrument, without readjustment, they are

For inequality of pivots, 0″·03.
For equatorial collimation error, $e = \frac{1}{2}(0{\cdot}20 + 0{\cdot}06) = 0{\cdot}13$.
For azimuthal deviation, $d = \frac{1}{2}(0{\cdot}07 + 0{\cdot}15) = 0{\cdot}11$.
In some other examples given, e is taken as $= 0{\cdot}09$ or $0{\cdot}13$.

4. The application of the foregoing corrections is shown in detail in the following example.

On February 10, 1860, it was required to find the clock error by a transit of the sun. The declination being 14°30′; the zenith distance 65°53′40″ altitude; 24°6′20″ vertical circle west.

Wires	Preceding limb			Following limb		
	h.	m.	s.	h.	m.	s.
1	21	30	57	21	33	20
2		31	24		33	37·5
3		31	41		33	55
4		31	59		34	12·5
5		32	16·5		34	30
Means	20	31	39·5	21	33	55

	R.A. at Greenwich			Mean of preceding and foll. limbs			
	h.	m.	sec.	h.	m.	s.	
	21	33	43·67	21	32	47·25	
Corr. for longitude			·35			−·10	*l*
	21	33	43·32			−·06	*c*
						−·14	*a*
Clock error slow			56·37	21	32	46·95	

	Levels E.	Levels W.	
Corrections	9·7	9·45	
	9·9	9·3	
	19·60	−18·75 = 0·85	
Inequality − (0·03 + ¼ 0·85)		= −0·24	

$l = -0{\cdot}24 \cos \text{zenith dist.} \times \sec D = {\cdot}422 \times -{\cdot}24 = -{\cdot}101$

$c = -0{\cdot}06 \sec D = -{\cdot}060$

$a = -0{\cdot}15 \sin \text{zenith dist.} \times \sec D = -{\cdot}141$

Section 4. TABLES FOR CORRECTING ALTITUDES.

ALTITUDE CORRECTIONS—DIP.

(From Pocket Logarithms.)

Height of Eye	Dip of Sea Horizon Dip		Height of Eye	Dip	
Feet	′	″	Feet	′	″
1	0	59	31	5	29
2	1	24	32	5	34
3	1	42	33	5	39
4	1	58	34	5	44
5	2	12	35	5	49
6	2	25	40	6	13
7	2	36	45	6	36
8	2	47	50	6	58
9	2	57	55	7	18
10	3	7	60	7	37
11	3	16	65	7	56
12	3	25	70	8	14
13	3	33	75	8	31
14	3	41	80	8	48
15	3	49	85	9	5
16	3	56	90	9	21
17	4	4	95	9	36
18	4	11	100	9	51
19	4	17	110	10	19
20	4	24	120	10	47
21	4	31	130	11	14
22	4	37	140	11	39
23	4	43	150	12	3
24	4	49	160	12	27
25	4	55	170	12	50
26	5	1	180	13	12
27	5	7	190	13	33
28	5	13	200	13	55
29	5	18	250	15	32
30	5	24	300	16	51

Height of Eye	Dip of Shore Horizon for Distances in miles			
Feet	½ m.	¾ m.	1 m.	1¼ m.
	′	′	′	′
5	6	4	4	3
10	11	8	6	5
15	17	12	9	7
20	22	15	12	9
25	28	19	15	12
30	34	23	17	14
35	39	27	20	16
40	45	30	23	19
50	56	37	27	22
	1½ m.	2 m.	2½ m.	3 m.
	′	′	′	′
5	3	2	2	2
10	4	3	3	3
15	6	5	5	4
20	8	6	6	5
25	10	8	7	6
30	12	10	8	7
35	14	11	9	8
40	15	12	10	8
50	19	15	12	10
	4 m.	5 m.	6 m.	7 m.
	′	′	′	′
5	2	2	2	2
10	3	3	3	3
15	4	4	4	4
20	4	4	4	4
25	5	5	5	5
30	6	5	5	5
35	7	6	6	6
40	7	6	6	6
50	8	7	7	7

Altitude Corrections—Refraction.

App. Altitude	Refraction for Bar. 30″, Th. 50°	Diff. for 10′ of Alt.	Add. Diff. for +1 inch Bar.	Sub. Diff. for +1° Therm.
°	′ ″	″	″	″
0	33 51		74	8·1
1	24 25	74	52	4·7
2	18 29	48	38	3·2
3	14 35	32	30	2·3
4	11 52	22	24	1·7
5	9 58	16	20	1·4
6	8 32	12	17	1·1
7	7 27	10	15	1·0
8	6 35	7	13	·9
9	5 54	6	12	·8
10	5 20	5	11	·7
11	4 51	4	10	·6
12	4 28	3·8	9·0	·5
12½	4 17	3·5	8·6	·53
13	4 8	3·1	8·3	·51
13½	3 58	3·0	8·0	·49
14	3 50	2·8	7 7	·47
14½	3 42	2·6	7·4	·45
15	3 34	2·4	7·2	·44
15½	3 27	2·2	7·0	·42
16	3 21	2·1	6·7	·41
16½	3 14	2·0	6 5	·40
17	3 9	1·9	6·3	·39
17½	3	1·8	6·1	·37

App. Altitude	Refraction for Bar. 30″, Th. 50°	Diff. for 10′ of Alt.	Add. Diff. for +1 inch Bar.	Sub. diff. for
°	′ ″	″	″	″
18	2 58	1·7	6·0	·36
19	2 48	1·6	5·6	·34
20	2 39	1·5	5·3	·32
21	2 30	1·3	5·0	·31
22	2 23	1·2	4·8	·29
23	2 17	1·1	4·6	·28
24	2 10	1·0	4·4	·26
25	2 4	·9	4·2	·25
26	1 59	·9	4·0	·24
27	1 54	·8	3·8	·23
28	1 49	·8	3·7	·22
29	1 45	·7	3·5	·21
30	1 41	·7	3·4	·20
31	1 37	·6	3·2	·19
32	1 33	·6	3·1	·19
33	1 30	·6	3·0	·18
34	1 26	·5	2·9	·17
35	1 23	·5	2·8	·17
36	1 20	·5	2·7	·16
37	1 17	·5	2·6	·16
38	1 14	·5	2·5	·15
39	1 12	·4	2·4	·14
40	1 9	·4	2·3	·14
41			2·2	·13

ALTITUDE CORRECTIONS—REFRACTION (*continued*).

App. Altitude	Refraction for Bar. 30″, Th. 50°	Diff. for 10′ of Alt.	Add. Diff. for +1″ Bar.	Sub. Diff. for +1° Therm.	App. Altitude	Refraction for Bar. 30″, Th. 50°	Diff. for 10′ of Alt.	Add. Diff. for +1″ Bar.	Sub. Diff. for +1° Therm.
°	′ ″	″	″	″	°	″	″	″	″
42	1 5	·38	2·2	·13	66	25·9	·20	·87	·05
43	1 2	·36	2·1	·13	67	24·7	·20	·83	·05
44	1 0	·34	2·0	·12	68	23·5	·20	·79	·05
45	58	·34	1·9	·12	69	22·4	·20	·75	·05
46	56	·33	1·9	·11	70	21·2	·20	·71	·04
47	54	·32	1·8	·11	71	19·9	·20	·67	·04
	″								
48	52·3	·31	1·8	·10	72	18·8	·19	·63	·04
49	50·5	·30	1·7	·10	73	17·7	·18	·59	·04
50	48·8	·29	1·6	·10	74	16·6	·18	·56	·03
51	47·1	·28	1·6	·09	75	15·5	·18	·52	·03
52	45·4	·27	1·5	·09	76	14·4	·18	·48	·03
53	43·8	·26	1·5	·09	77	13·4	·17	·45	·03
54	42·2	·26	1·4	·09	78	12·3	·17	·41	·03
55	40·8	·25	1·4	·08	79	11·2	·17	·38	·02
56	39·3	·25	1·3	·08	80	10·2	·17	·34	·02
57	37·8	·25	1·3	·08	81	9·2	·17	·31	·02
58	36·4	·24	1·2	·07	82	8·2	·17	·27	·02
59	35·0	·24	1·2	·07	83	7·1	·17	·24	·01
60	33·6	·23	1·1	·07	84	6·1	·17	·20	·01
61	32·3	·22	1·1	·07	85	5·1	·17	·17	·01
62	31·0	·22	1·0	·06	86	4·1	·17	·14	·01
63	29·7	·21	1·0	·06	87	3·1	·17	·10	·01
64	28·4	·21	1·0	·06	88	2·0	·17	·07	
65	27·2	·20	0·9	·06	89	1·0	·17	·03	

Altitude Corrections—Parallax.

Lat.	Moon's Equatorial Horizontal Parallax 54′	56′	58′	60′	62′ Subtractive Corrections	Alt.	Sun's Mean Parallax in Alt.
°	″	″	″	″	″	°	″
6	0·1	0·1	0·1	0·1	0·1	0	8·6
12	0·4	0·4	0·4	0·5	0·5	6	8·5
18	0·9	1·0	1·0	1·1	1·1	12	8·4
24	1·6	1·7	1·8	1·9	1·9	18	8·2
30	2·5	2·6	2·7	2·8	2·9	24	7·8
36	3·5	3·6	3·7	3·9	4·0	30	7·4
42	4·5	4·7	4·9	5·1	5·2	36	7·0
48	5·6	5·8	6·0	6·2	6·4	42	6·4
54	6·6	6·9	7·1	7·4	7·6	48	5·7
60	7·6	7·9	8·2	8·5	8·7	54	5·0
66	8·5	8·8	9·1	9·4	9·7	60	4·3
72	9·2	9·5	9·9	10·2	10·6	66	3·5
78	9·7	10·1	10·5	10·8	11·2	72	2·7
84	10·1	10·4	10·8	11·2	11·6	78	1·8
90	10·2	10·6	11·0	11·3	11·7	84	0·9

Parallax in Altitude.

App. Alt.	Planet's Horizontal Parallax 2″	4″	6″	8″	10″	12″	14″	16″	18″	20″	25″	30″
°	″	″	″	″	″	″	″	″	″	″	″	″
15	2	4	6	8	10	12	14	16	18	20	25	29
20	2	4	6	8	10	11	13	15	17	19	24	28
25	2	4	5	7	9	11	13	14	16	18	23	27
30	2	3	5	7	9	10	12	14	15	17	22	26
35	2	3	5	7	8	10	11	13	14	16	20	25
40	2	3	5	6	8	9	11	12	14	15	19	24
45	1	3	4	6	7	8	10	11	13	14	18	21
50	1	3	4	5	6	7	9	10	11	12	16	19
55	1	2	3	5	5	7	8	9	10	11	15	18
60	1	2	3	4	5	6	7	8	9	10	13	15
70	1	1	2	3	4	4	5	5	6		9	10
80	0	1	1	1	2	2	3	3	3	3	4	5
85	0	0	1	1	1	1	1	2	2	2	2	3

ALTITUDE CORRECTIONS—PARALLAX.

THE MOON'S PARALLAX IN ALTITUDE.

Apparent Altitude	Reduced Horizontal Parallax							
	54′	55′	56′	57′	58′	59′	60′	62′
°	′	′	′	′	′	′	′	′
10	48	49	50	51	52	53	54	56
12	48	49	50	51	52	53	54	56
14	49	50	51	52	53	53	54	56
16	49	50	51	52	52	53	54	56
18	48	49	50	51	52	53	54	56
20	48	49	50	51	52	53	54	56
22	48	49	50	51	51	52	53	55
24	47	48	49	50	51	52	53	55
26	47	47	48	49	50	51	52	54
28	46	47	48	49	49	50	51	53
30	45	46	47	48	49	49	50	52
32	44	45	46	47	48	49	49	51
34	43	44	45	46	47	47	48	50
36	42	43	44	45	46	46	47	49
38	41	42	43	44	44	45	46	48
40	40	41	42	43	43	44	45	46
45	37	38	39	40	40	41	42	43
50	34	35	35	36	36	37	38	39
55	30	31	31	32	32	33	34	35
60	26	27	27	28	28	29	29	30
65	22	23	23	24	24	24	25	26
70	18	18	19	19	19	20	20	21
75	14	14	14	14	15	15	15	16
80	2	9	10	10	10	10	10	11
85	5	5	5	5	5	5	5	5
90	0	0	0	0	0	0	0	0

Altitude Corrections—Semi-diameter.

Apparent Altitude	Moon's Semi-diameter 14′ 40″	15′ 0″	15′ 20″	15′ 40″	16′ 0″	16′ 20″	16′ 40″
	Augmentation for Altitude						
°	′	′	′	′	′	′	′
0	0·1	0·1	0·1	0·1	0·1	0·2	0·2
6	1·5	1·6	1·7	1·8	1·9	2·0	2·0
8	2·0	2·1	2·2	2·3	2·4	2·5	2·6
10	2·6	2·7	2·8	2·9	3·0	3·1	3·3
12	3·2	3·3	3·3	3·4	3·6	3·7	3·9
16	3·9	4·1	4·3	4·5	4·7	4·9	5·1
20	4·8	5·0	5·3	5·6	5·8	6·0	6·2
24	5·7	6·0	6·3	6·6	6·8	7·1	7·4
28	6·6	6·9	7·2	7·6	7·9	8·2	8·5
32	7·5	7·8	8·2	8·5	8·9	9·2	9·6
36	8·2	8·6	9·0	9·4	9·9	10·2	10·6
40	9·0	9·4	9·8	10·3	10·7	11·1	11·6
44	9·8	10·2	10·6	11·1	11·6	12·0	12·6
48	10·4	10·9	11·4	11·9	12·4	12·9	13·4
52	11·0	11·5	12·0	12·6	13·1	13·7	14·2
56	11·6	12·1	12·7	13·2	13·8	14·4	14·9
60	12·1	12·7	13·2	13·8	14·5	15·0	15·6
65	12·6	13·2	13·8	14·4	15·1	15·7	16·3
70	13·1	13·7	14·4	15·0	15·7	16·3	16·9
75	13·5	14·1	14·8	15·4	16·1	16·7	17·4
80	13·8	14·4	15·0	15·7	16·4	17·1	17·8
85	13·9	14·5	15·2	15·9	16·6	17·3	18·0
90	14·0	14·6	15·3	16·0	16·7	17·3	18·0

SUBTRACTIVE QUANTITIES FOR CONVERSION OF SIDEREAL INTO MEAN SOLAR TIME.

Hours	m.	s.	Min.	s.	Min.	s.	Sec.	s.
1	0	9·83	1	0·16	31	5·08	2	·01
2		19·66	2	0·33	32	5·24	4	·01
3		29·49	3	0·49	33	5·41	6	·02
4		39·32	4	0·65	34	5·57	8	·02
5		49·15	5	0·82	35	5·73	10	·03
6		58·98	6	0·98	36	5·90	12	·03
			7	1·15	37	6·06	14	·04
7	1	8·81	8	1·31	38	6·22	16	·04
8	1	18·64	9	1·47	39	6·39	18	·05
9	1	28·46	10	1·64	40	6·55	20	·05
10	1	38·29						
11	1	48·12	11	1·80	41	6·72	22	·06
12	1	57·95	12	1·97	42	6·88	24	·07
			13	2·13	43	7·04	26	·07
13	2	7·78	14	2·29	44	7·21	28	·08
14	2	17·61	15	2·46	45	7·37	30	·08
15	2	27·44	16	2·62	46	7·54	32	·09
16	2	37·27	17	2·78	47	7·70	34	·09
17	2	47·10	18	2·95	48	7·86	36	·10
18	2	56·93	19	3·11	49	8·03	38	·10
			20	3·28	50	8·19	40	·11
19	3	6·76						
20	3	16·59	21	3·44	51	8·35	42	·11
21	3	26·42	22	3·60	52	8·51	44	·12
22	3	36·25	23	3·77	53	8·68	46	·13
23	3	46·08	24	3·93	54	8·85	48	·13
24	3	55·91	25	4·10	55	9·01	50	·14
			26	4·26	56	9·17	52	·14
			27	4·42	57	9·34	54	·15
			28	4·59	58	9·50	56	·15
			29	4·75	59	9·67	58	·16
			30	4·91	60	9·83	60	

Additive Quantities for Conversion of Mean Solar into Sidereal Time.

Hours	m.	s.	Min.	s.	Min.	s.	Sec.	s.
1	0	9·86	1	0·16	31	5·09	2	·01
2		19·71	2	0·33	32	5·26	4	·01
3		29·57	3	0·49	33	5·42	6	·02
4		39·43	4	0·66	34	5·58	8	·02
5		49·28	5	0·82	35	5·75	10	·03
6		59·14	6	0·99	36	5·91	12	·03
			7	1·15	37	6·08	14	·04
7	1	8·10	8	1·31	38	6·24	16	·04
8	1	18·85	9	1·48	39	6·41	18	·05
9	1	28·71	10	1·64	40	6·57	20	·05
10	1	38·56						
11	1	48·42	11	1·81	41	6·73	22	·06
12	1	58·28	12	1·97	42	6·90	24	·06
			13	2·14	43	7·06	26	·07
13	2	8·13	14	2·30	44	7·23	28	·08
14	2	17·99	15	2·46	45	7·39	30	·08
15	2	27·85	16	2·63	46	7·56	32	·09
16	2	37·70	17	2·79	47	7·72	34	·09
17	2	47·56	18	2·96	48	7·88	36	·10
18	2	57·42	19	3·12	49	8·05	38	·10
			20	3·28	50	8·21	40	·11
19	3	7·27						
20	3	17·13	21	3·45	51	8·38	42	·11
21	3	26·98	22	3·61	52	8·54	44	·12
22	3	36·84	23	3·78	53	8·71	46	·12
23	3	46·70	24	3·94	54	8·87	48	·13
24	3	56·55	25	4·11	55	9·03	50	·14
			26	4·27	56	9·20	52	·14
			27	4·43	57	9·36	54	·15
			28	4·60	58	9·53	56	·15
			29	4·76	59	9·69	58	·16
			30	4·93	60	9·86	60	·16

FIELD RECORDS.

FIELD - RECORD.

Nº 1.

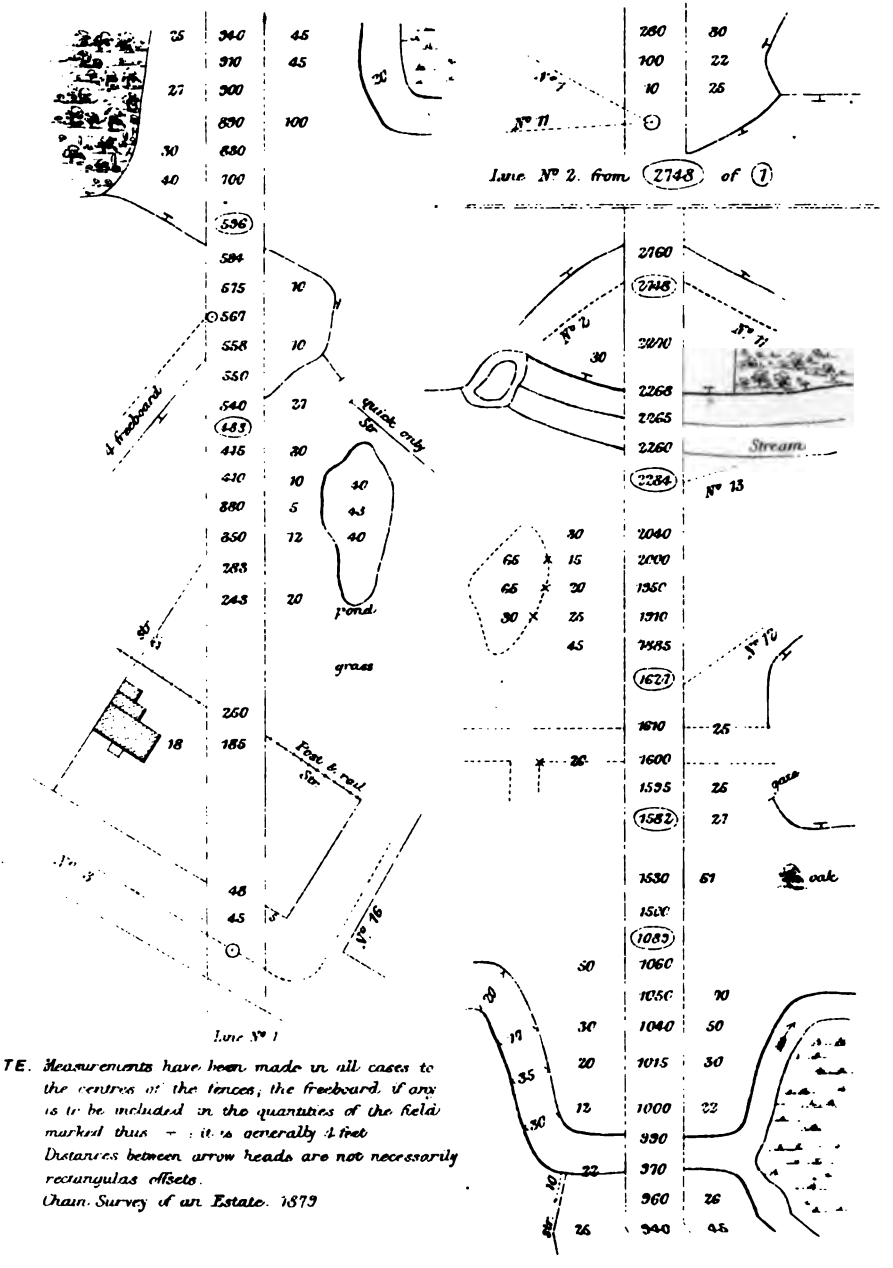
CHAIN SURVEY

OF AN ESTATE.

FOR PLOTTING ON THE SCALE

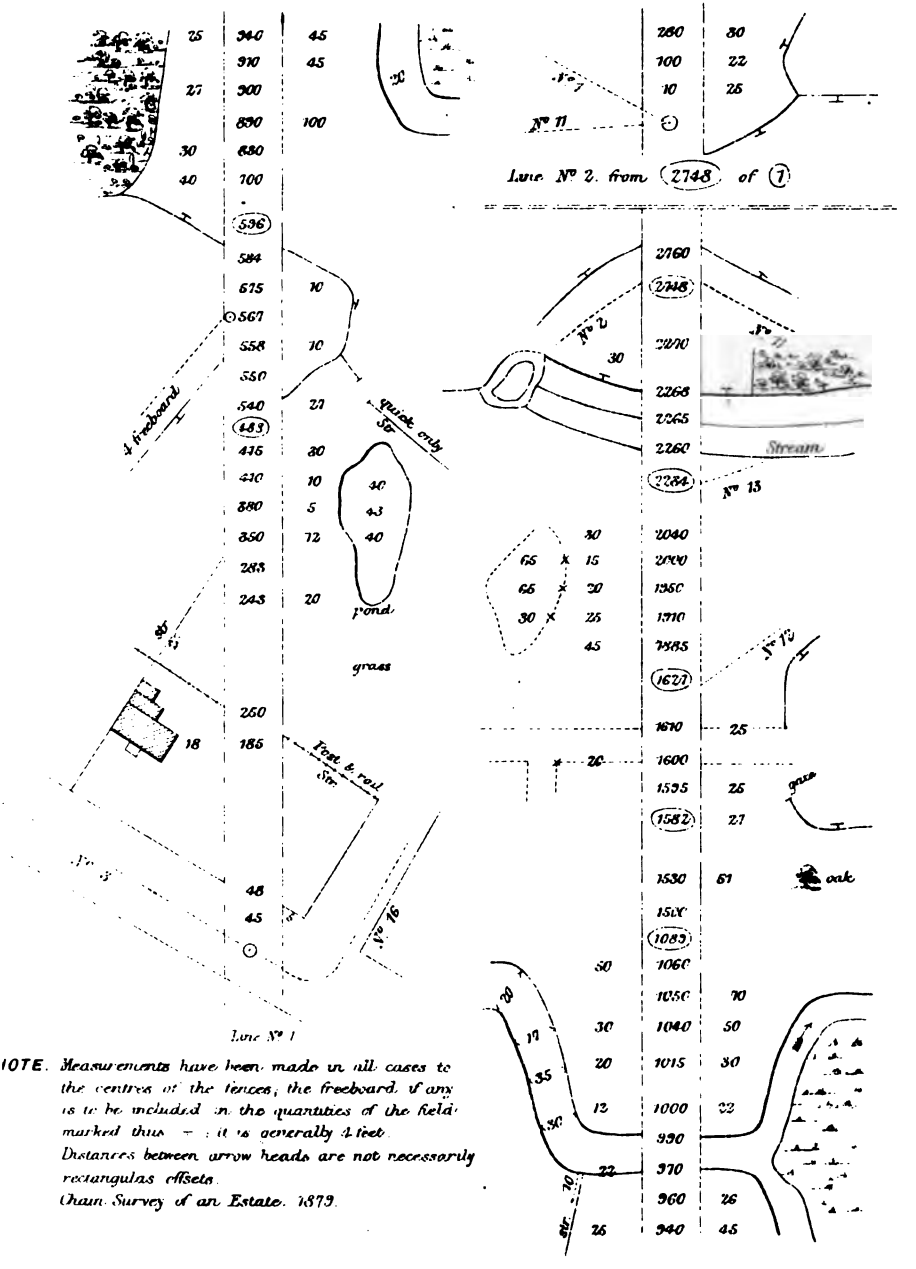
OF ONE CHAIN TO AN INCH.





TE. Measurements have been made in all cases to the centres of the fences, the freeboard, if any is to be included in the quantities of the field marked thus + : it is generally 2 feet
Distances between arrow heads are not necessarily rectangulas offsets.
Chain Survey of an Estate. 1879

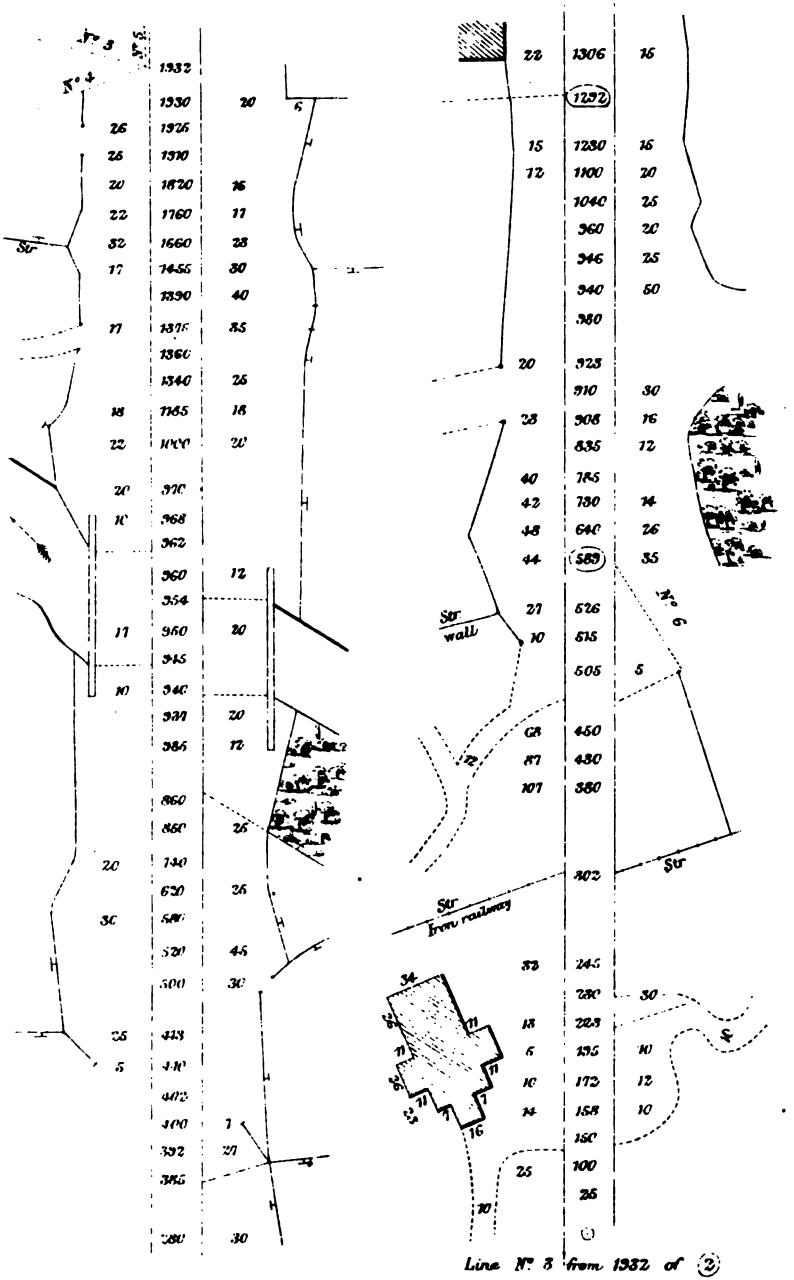




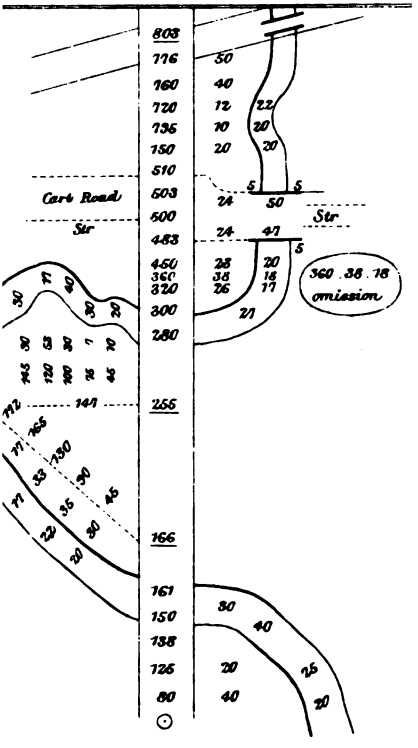
NOTE. Measurements have been made in all cases to the centres of the fences; the freeboard, if any, is to be included in the quantities of the field marked thus — ; it is generally 4 feet.
Distances between arrow heads are not necessarily rectangular offsets.

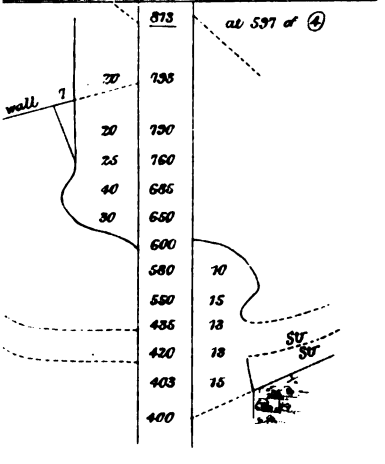
Chain Survey of an Estate. 1879.

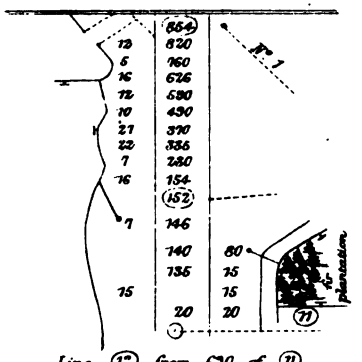


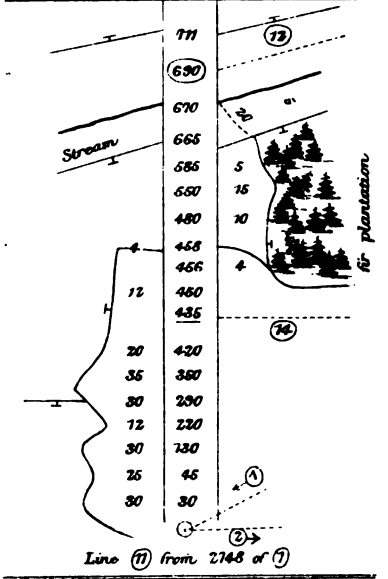

Str
Str wall
Str from railway
No 6
No 5
No 4
No 3
1932
1930
1926
1910
1870
1760
1660
1455
1390
1340
1165
1000
960
940
880
730
620
500
443
402
392
385
280
1306
1292
1230
1100
1040
960
946
940
925
910
908
835
785
730
640
589
526
515
505
450
430
380
302
245
230
228
195
172
158
150
100
Line N° 3 from 1932 of (2)

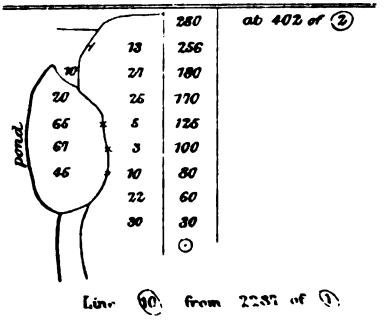



Cart Road
Str
Str
360 . 38 . 18
omission
Line (9) from 488 of (7)

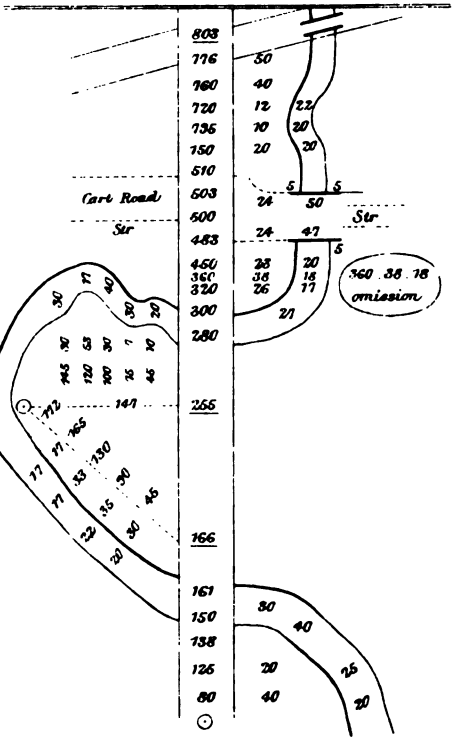

at 597 of (4)
wall
Str
Str

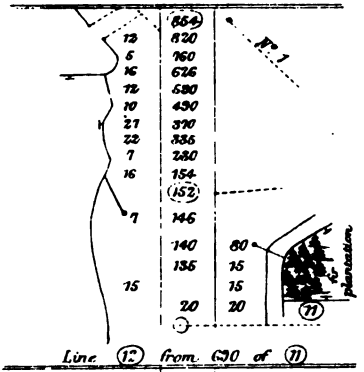

for plantation
Line (12) from 630 of (11)

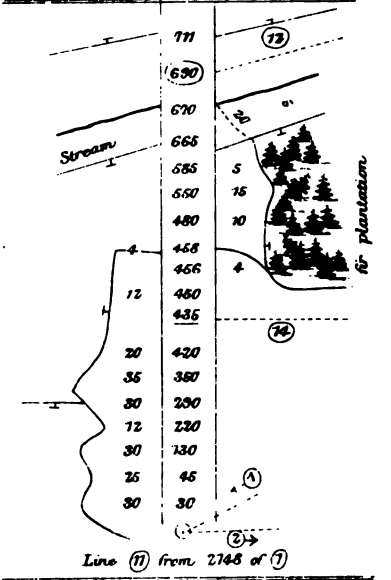

Stream
for plantation
Line (11) from 2148 of (7)

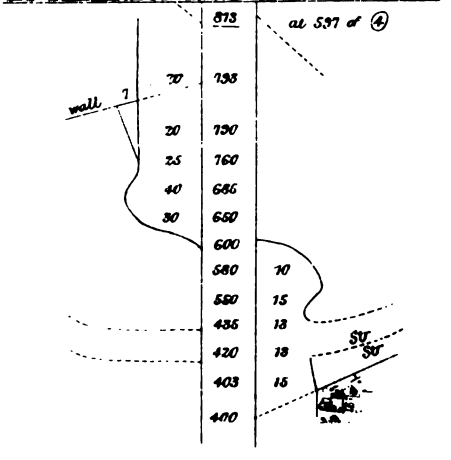

pond
at 402 of (2)
Line (10) from 2237 of (7)

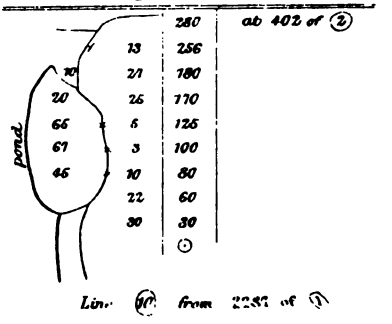



Cart Road
Str
Str
360 38 18
omission
Line (5) from 488 of (7)

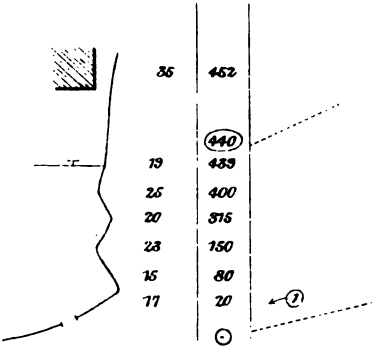

plantation
Line (12) from 690 of (11)


Stream
fir plantation
Line (11) from 2148 of (7)

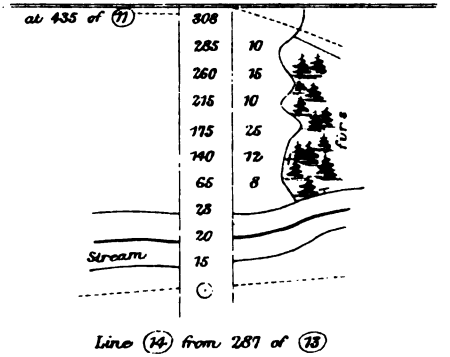

wall
at 597 of (4)


pond
at 402 of (2)
Line (10) from 2287 of (7)

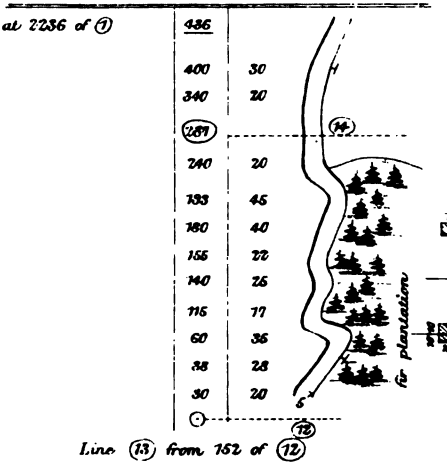




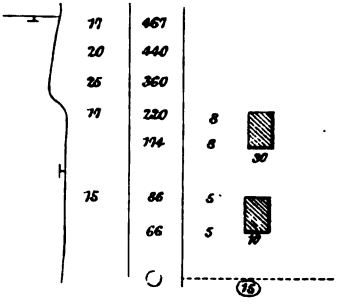
Line (15) from 1582 of (1)



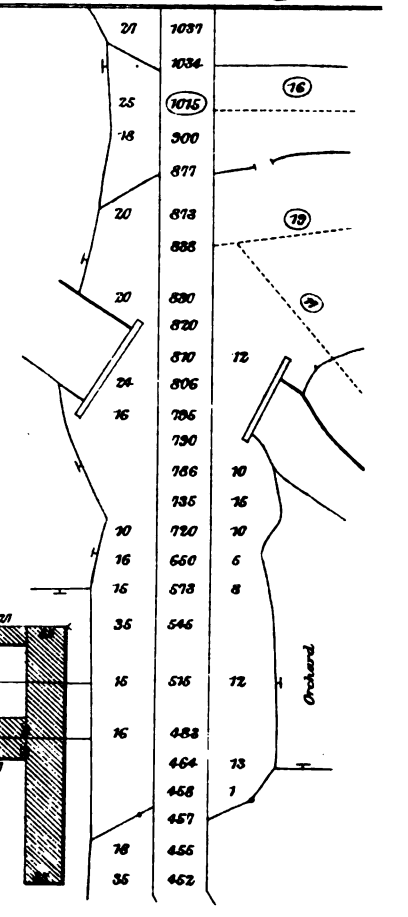
Line (14) from 287 of (13)



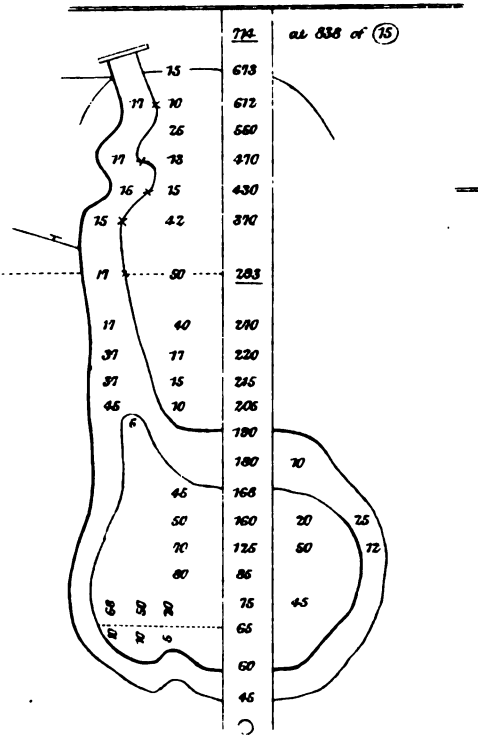
Line (13) from 152 of (12)



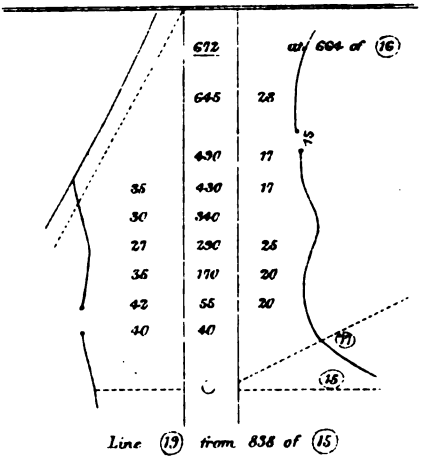
Line (16) from 1015 of (15)



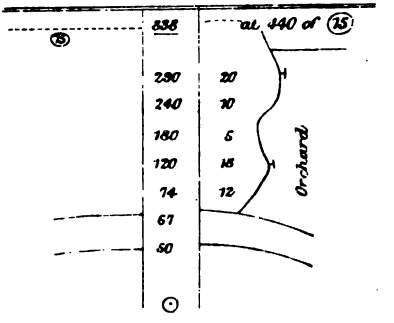




End of Chain Survey of Estate.

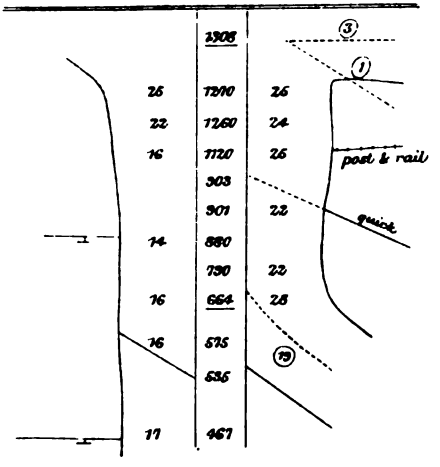


Line (19) from 838 of (15)



Line (18) from 293 of (17)

Line (17) from 930 of (1)





FIELD - RECORD.

Nº II.

TOWN SURVEY.

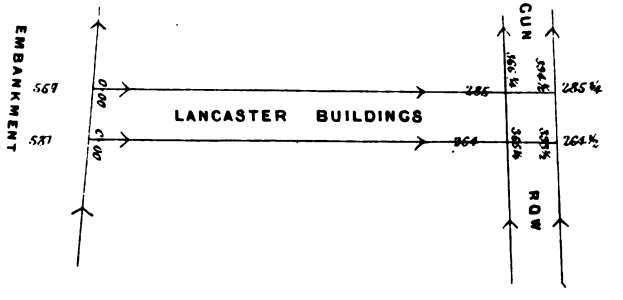
WITH THE CHAIN.

FOR PLOTTING ON THE SCALE OF $\frac{1}{120}$ OR 10 FEET TO THE INCH.

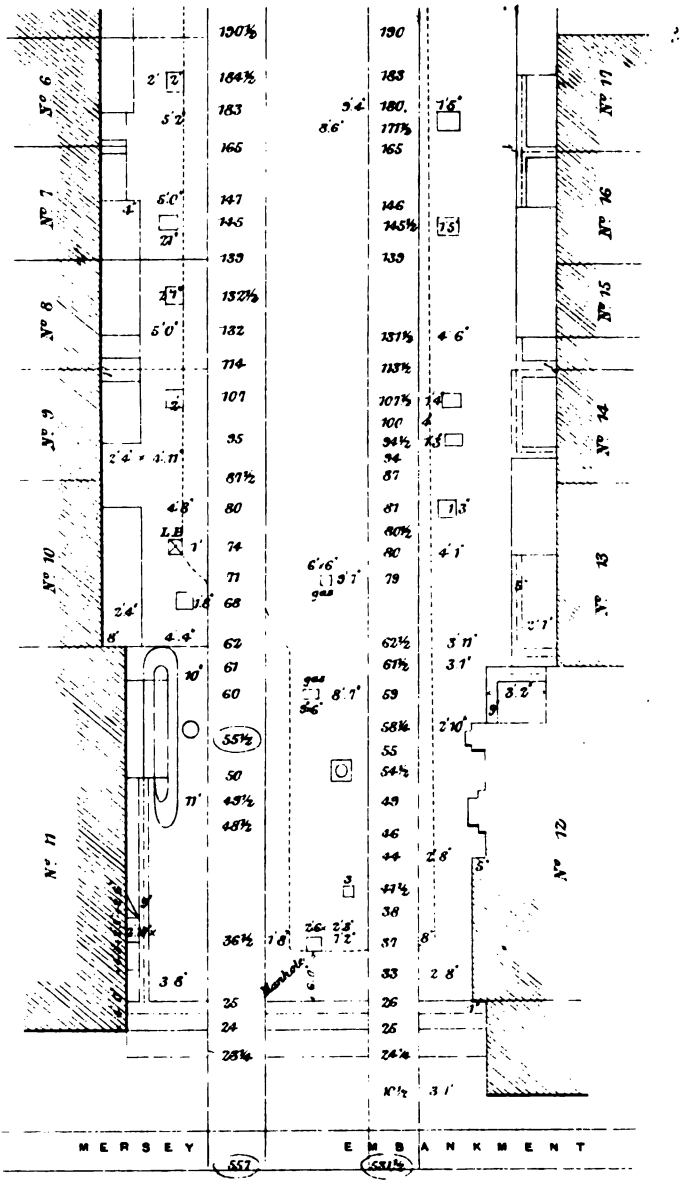
Chain lines are taken in links.
Other dimensions and offsets in feet and inches

The coal Shoots when not marked are
1 foot Sq: or 1 foot in diam.

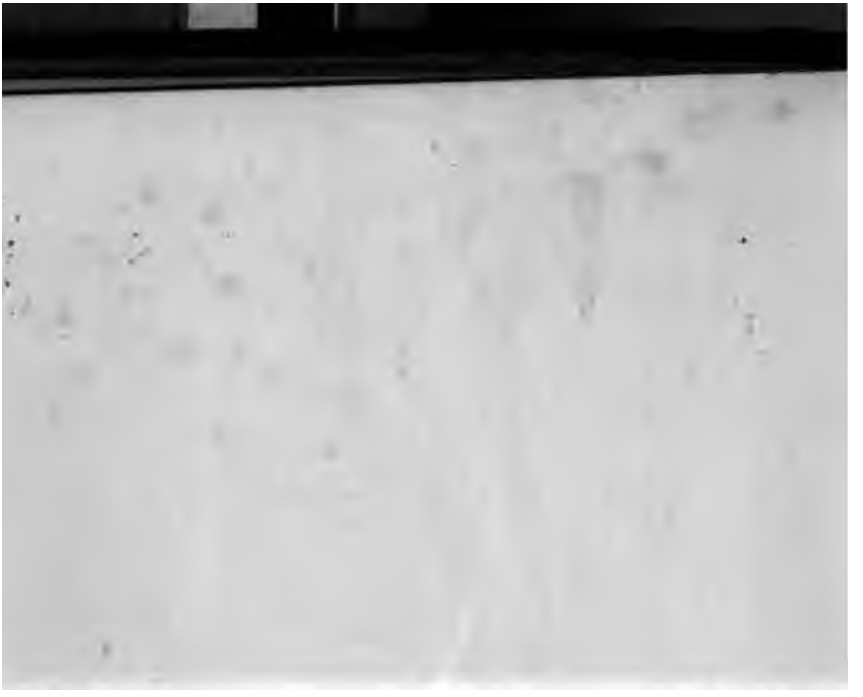
GENERAL SKETCH.



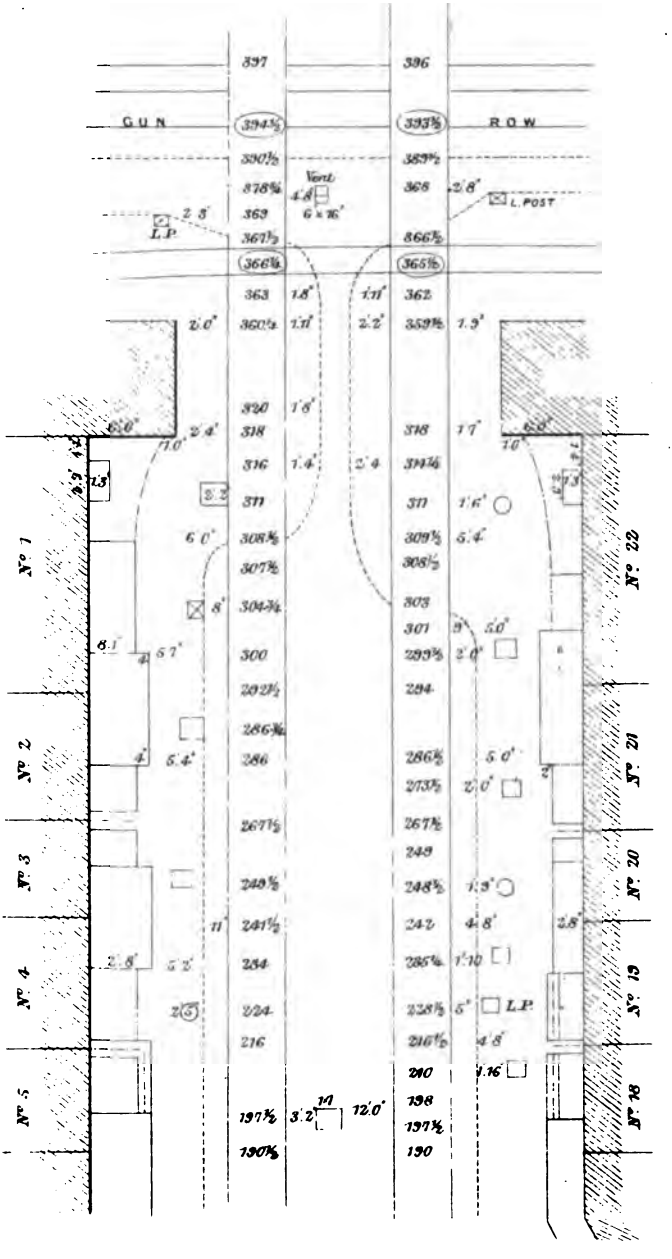




RECORD OF CHAIN SURVEY IN LANCASTER BUILDINGS.



END OF SURVEY.





FIELD-RECORD.

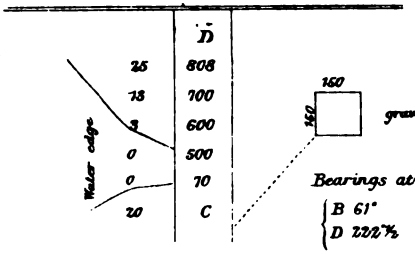
Nº III.

COMPASS SURVEY.

For plotting on the Scale of

$\frac{1}{6336}$ or 8 Chains to the Inch.

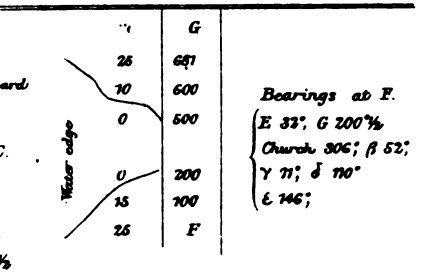




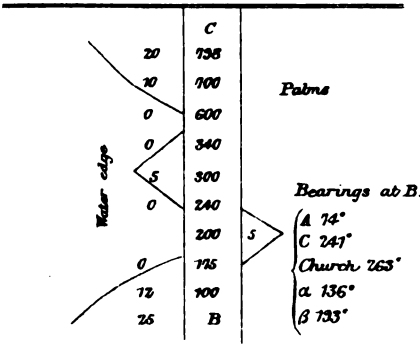
Bearings
α 106°
β 164°
γ 184

Graveyard corner 251°
Graveyard wall 359½
Barrack 286°
Doctors house 266

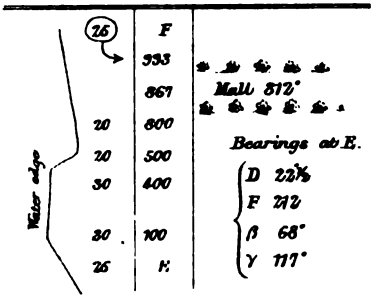
From ○ C near Palms to D.



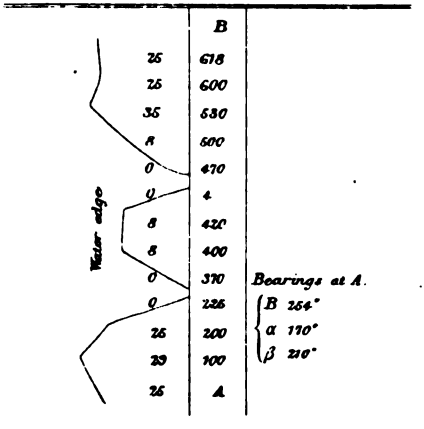
From ○ F near Mall to ○ G.



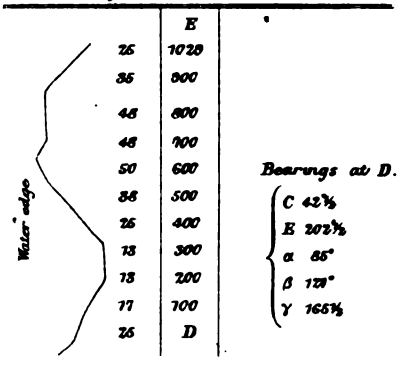
From ○ B near woodstack to C.



From ○ E near woodstack



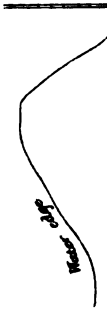
From ○ A near cornstack to B.



Graveyard corner, distance 476
Steward's house 278° distance 1144
Doctor's house 301°
Barrack 343°

From ○ D to ○ E.





	J
25	2028
160	1800
200	1600
240	1400
230	1200
170	1000
110	800
55	600
25	400
25	I

Bearings at I

H 358½, J 155½

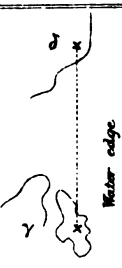
δ 39°, ε 64°

ζ 100° ζ 171

30 lengths

15 well

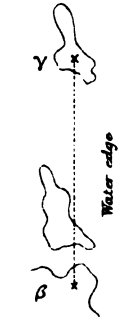
From ⊙ I near well and tree to ⊙ J.



Bearings at γ

A 30°; B 13°; D 346°;
E 297°; G 235°;
β 29°; δ 177°; ε 178°;

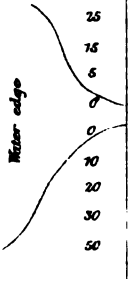
From ⊙ γ to ⊙ δ



Bearings at B.

A 30°, B 13°, C 344°
D 301°, F 248, J 192°
α 58°, γ 208, δ 133°,

From ⊙ β to ⊙ γ



	I
25	858
15	800
5	700
0	600
0	400
10	300
20	200
30	100
50	H

Bearings at H

G 7°; I 173½

δ 55 ε 92½

ζ 171 ζ 138½

Corner of garden 315° dist 836

garden 355°; 600 × 400

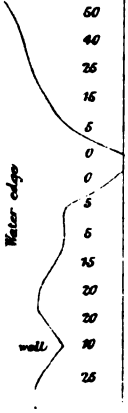
From ⊙ H near house and well to ⊙ I.



Bearings at α

A 350°; B 316°;
C 286°; β 240°;
γ 226°;

From ⊙ α to ⊙ β



	H
50	1165
40	1100
25	1000
15	900
5	800
0	700
0	550
5	500
5	400
15	300
20	200
20	100
10	30
25	G

Bearings at G

F 20½, H 187°

Church 316°; γ 54½

δ 90°; ε 127½

From ⊙ G near well to ⊙ H.

Bearings at J.

I 335½, H 340, Church 332°

ζ 30°, ζ 14½

At ⊙ J close to a well.



FIELD-RECORD.

Nº IV.

RAILWAY SURVEY.

For plotting on the Scale of

$\frac{1}{4752}$ *or 6 Chains to the Inch.*



FIELD-RECORD.

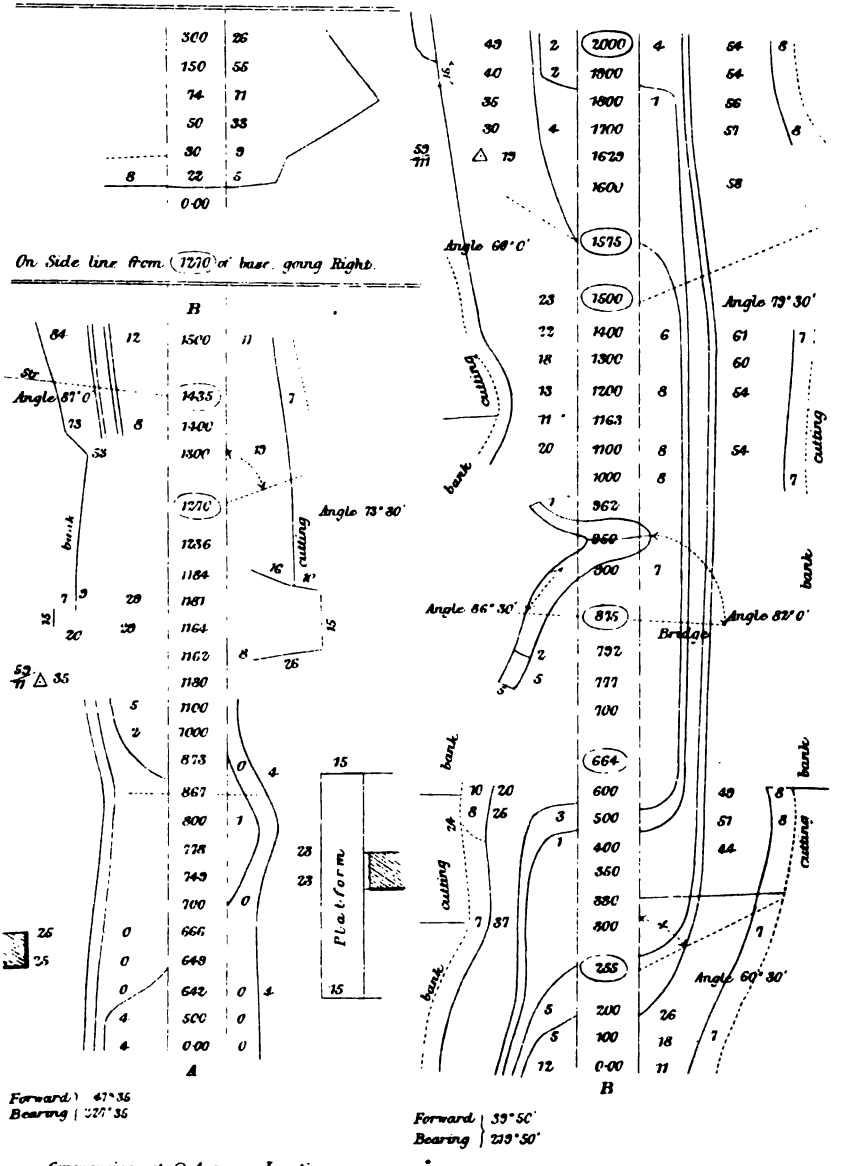
Nº IV.

RAILWAY SURVEY.

For plotting on the Scale of

$\frac{1}{4752}$ *or 6 Chains to the Inch.*

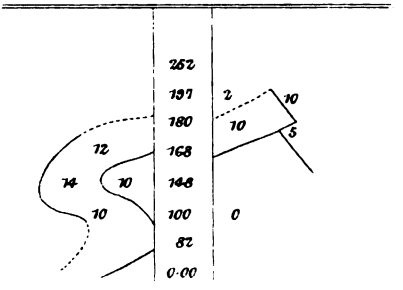




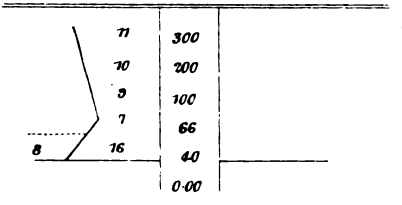
Commencing at ⊙ A. near Junction.
Surveying on base line A. B.

From ⊙ B. on base line B C.

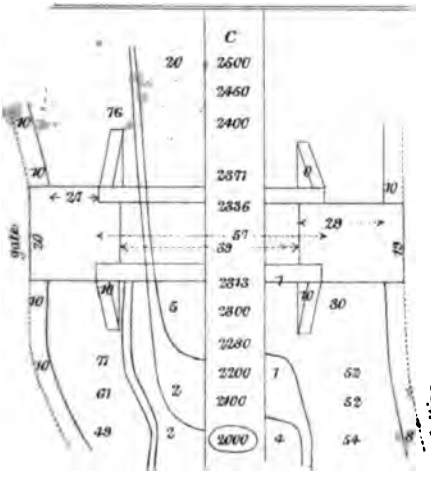




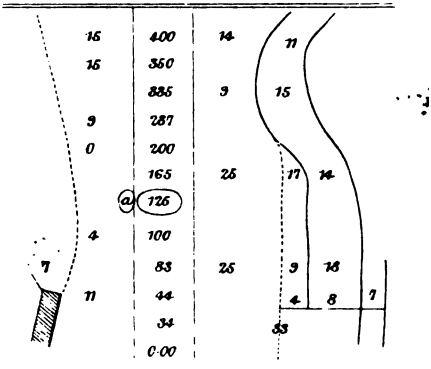
On Side line from (664) of B C going left.



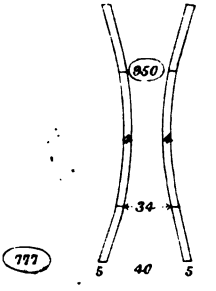
On Side line from (255) of B C.



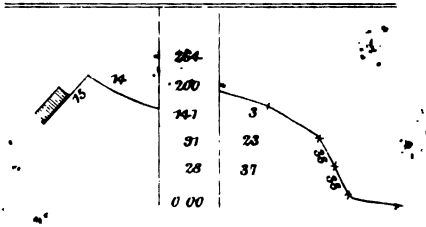
Continuing on base line B C.



Side line from (875) of B.C. going right

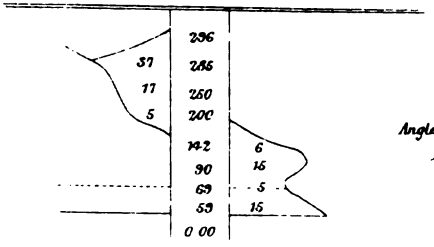


Sketch of bridge at (875) of B C.

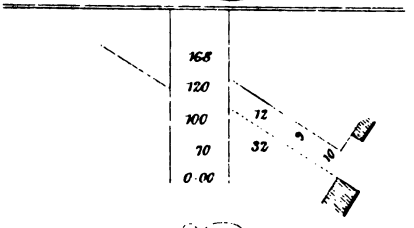


On Side line from (664) of B C going right.

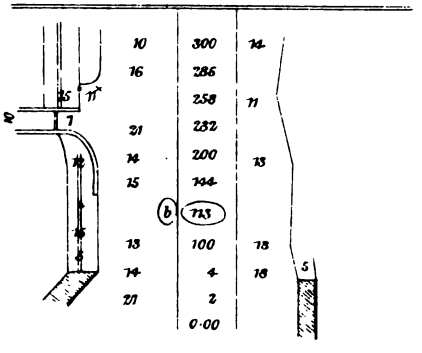




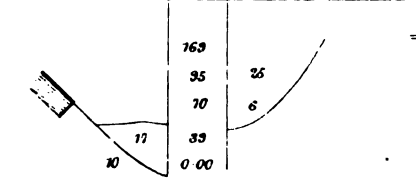
On Side line from (1500) of B.C.



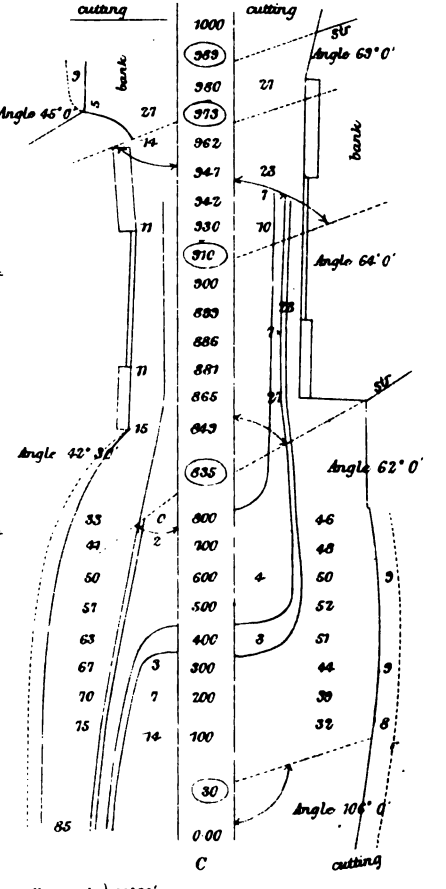
On Side line from (b) (713) of last Side line



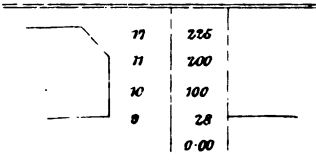
On Side line from (875) of B.C going left



On Side line from A (725) of last Side line

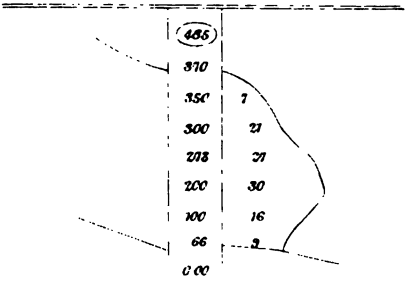


On base line C.D.

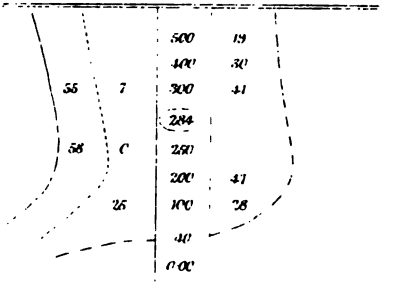


On Side line from (1575) of Base B.C.

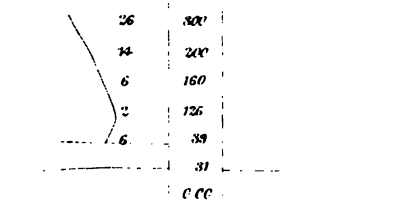




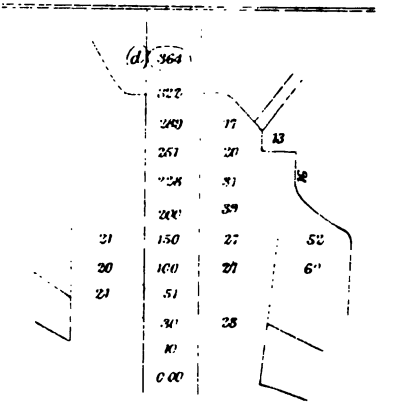
On Side line from (835) of Base C D



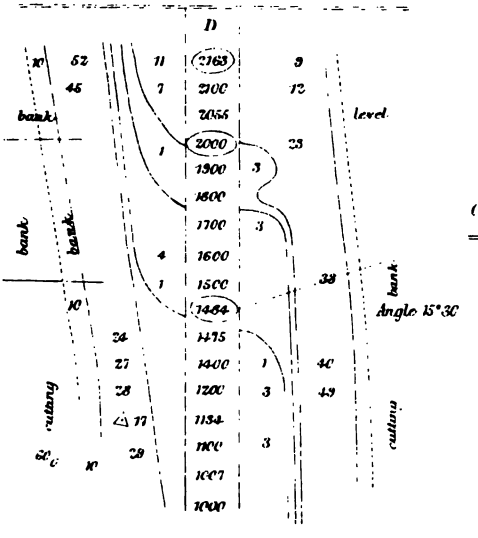
On Side line from d (364) of last Side line



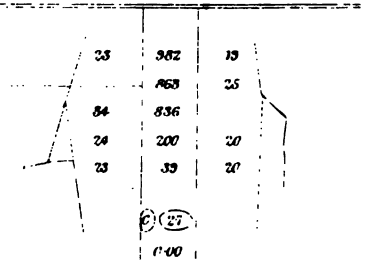
On Side line from (30) of Base C D.



On Side line from (c) (27) of last Side line

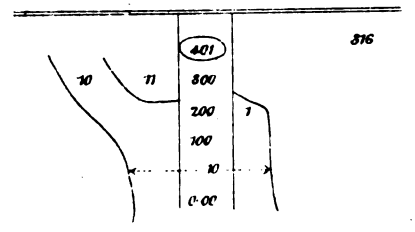


Continuing on base line C. D.

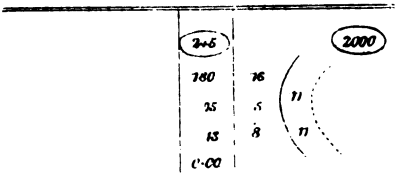


On Side line from (390) of base C D.

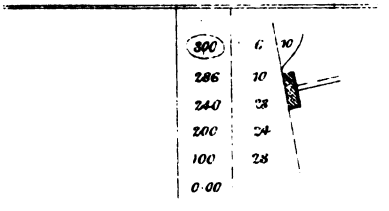




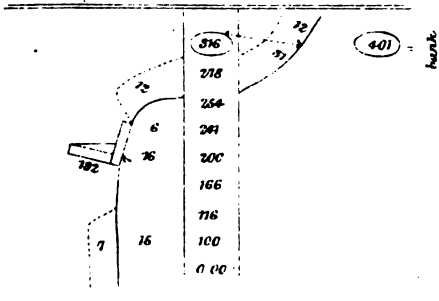
On Side line from (2163) of base C.D. going back left



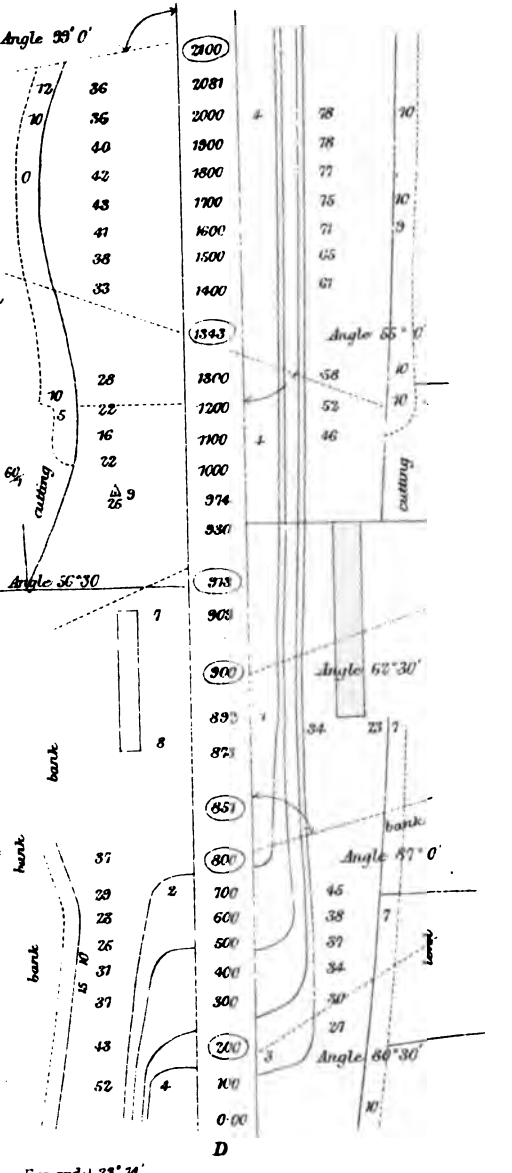
On Side line from (300) of last side line



On Side line from (1484) of base C.D going R.

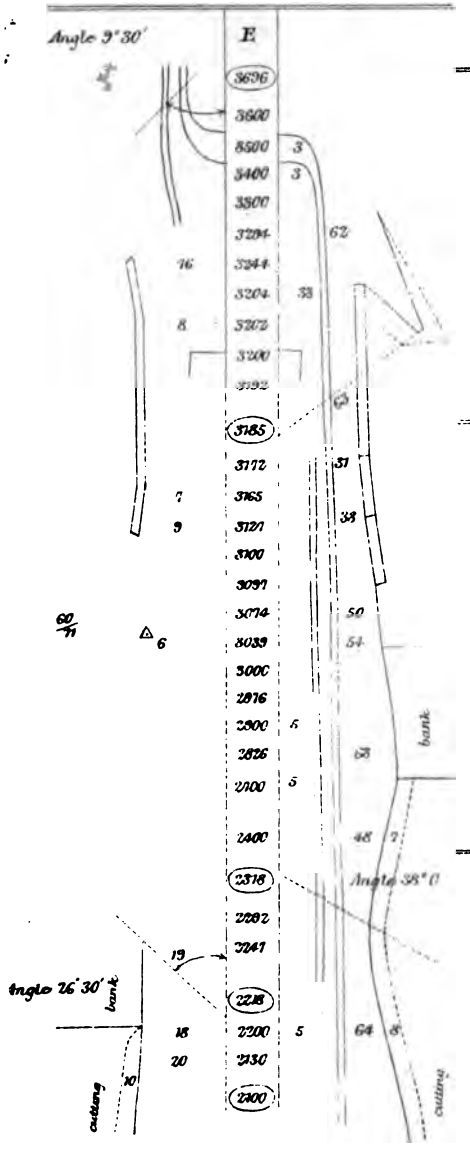


On Side line from (1484) of base C.D going L.

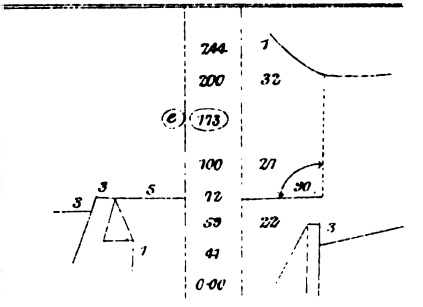


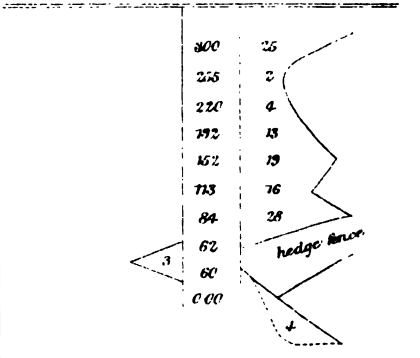
Forward Bearing | 23° 14' 203° 14'

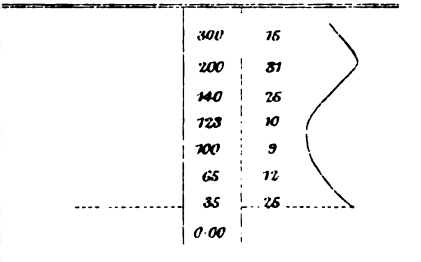
On Base line DE.



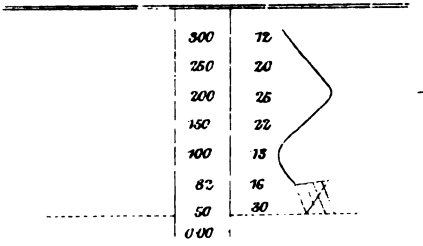
Continuing on Base line DE.



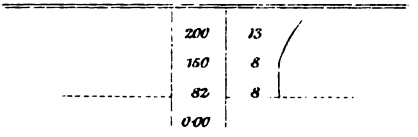




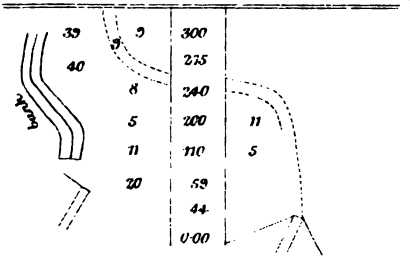




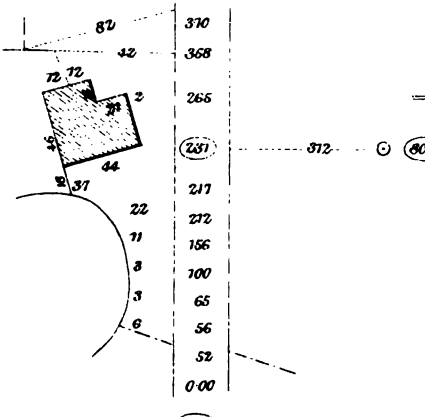
On Side line from (1343) of base DE going L



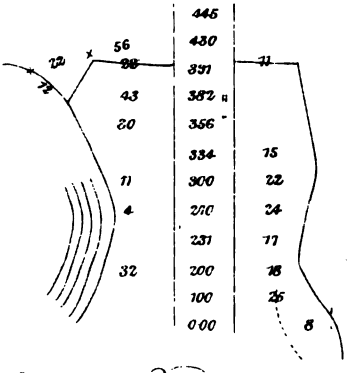
On Side line from (1343) of base DE going R.



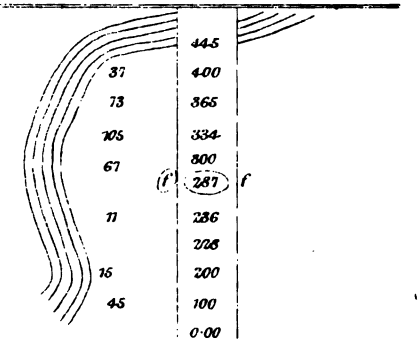
On Side line from (713) of base DE.



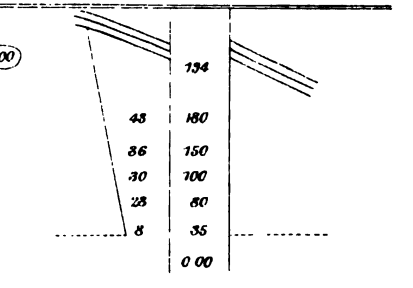
On Side line from c (713) of last Side line.



Side line from (f) 287 of last

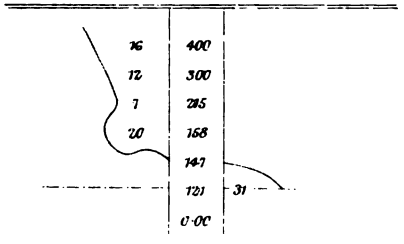


Side line from (2218) of base DE.

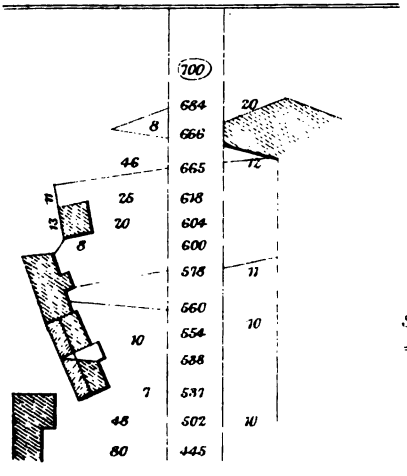


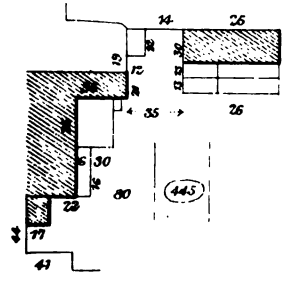
On Side line from (2100) of base DE.



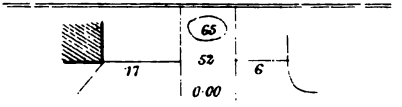


Side line from (2318) of base D.E.

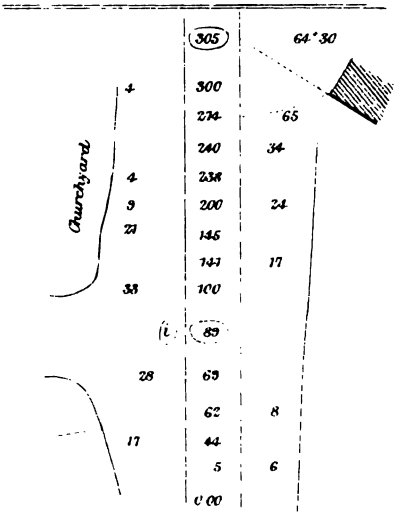




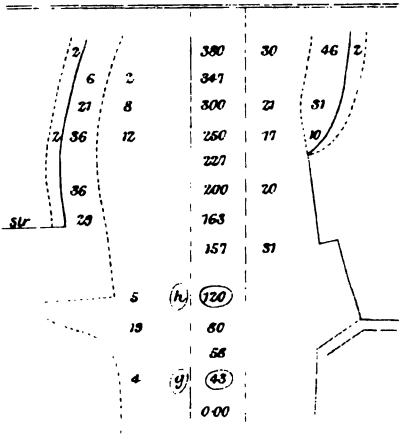
Continued Sketch at 445.



Side line from (i) (89) in last Side line.



Side line from g (43) of last Side line going back.



Side line from (3185) of base D.E.



RECORD

Nº V.

EIGHT DAYS' COURSE AT SEA

IN 1863.

[To follow Litho. pages.]

Detail of Daily Traverse.

Sept. 9, 1863. At noon; latitude observed 29°41 S.; Greenwich time by chron. 20 h. 02 m. 59 s.; slow 18 m. 32 s., losing 3″·9 daily. Longitude by chronometer 9° 19′ 30″ E.

The latitude and longitude at noon on the day before were 31° 50 S. and 11° 29′ E.

The courses during the 24 hours were:

Points	Miles	N.	W.		° ′
N. 5 W.	49	27·2	40·7	Lat. left	31 50 S.
N. 6 W.	4	1·5	3·7	diff.	1 48 N.
S. 5½ W.	5	S. 2·4	4·4	Lat. in	30 02 S.
N. 3½ W.	16	12·4	10·1	Mid. lat.	30 56
N. 3 W.	10	8·3	5·6	Long. left	11 29 E.
N. 4½ W.	25½	19·6	16·1	diff.	1 57 W.
N. 3½ W.	30	26·5	14·1	Long. in	9 32 E.
N. 2 W.	16	14·8	6·1		
Pts.		N. 110·3	100·8		
Var. 2½ W.		S. 2·4			
		N. 107·9	117′		

Course N. 43° W. Distance 148 miles.

Sept. 10, 1863. At noon; latitude observed 27° 58′. Greenwich time by chron. 20 h. 11 m. 40 s., slow 18 m. 36 s. losing 3″·9. Longitude by chron. 7°44 45 E.

Points	Miles	N.	W.		°
N. 4 W.	44	31·1	31·1	Lat. left	29 41 S.
N. 5 W.	31	17·2	25·8	diff.	1 23 N.
N. 5½ W.	11	5·2	9·7	Lat. in	28 18 S.
N. 4½ W.	47	29·8	36·3	Mid. lat.	28 59
Var. 2½ W.		83·3	102·9	Long. left	9 20 E.
			117′	diff.	1 57 W.
				Long. in	7 23 E.

Course N. 51° W. Distance 132 miles.

Sept. 11, 1863. Latitude obs. 26° 34′ S. Greenwich time by chron. 20 h. 07 m. 32 s. slow 18 m. 39 s.; long. by chron. 54° 49′ E.

Courses in the last 24 hours,

Points	Miles	N.	W.		° ′
N. 4½ W.	79	53	58·5	Lat. left	27 58 S.
N. 4½ W.	54	32·2	43·4	diff.	1 25 N.
Pts.		85·2	101·9	Lat. in	26 33 S.
Var. 2½ W.			114′	Mid. lat.	27 15
				Long. left	7 45 E.
				diff.	1 54 W.
				Long. in	5 51 E.

Course N. 50° W.
Distance 133 miles.

Sept. 12, 1863. Lat. obs. 25° 11′. Chronom. time 20 h. 50 m. 42 s. Slow 18 m. 43 sec. Longitude by chron. 3° 57′ 45″ E.

Points	Miles	N.	W.		° ′
N. 4½ W.	116	77·9	85·9	Lat. left	26 34 S.
				diff.	1 20 N.
Var. 2½ W.			95′	Lat. in	25 14 S.
				Mid. lat.	25 54
				Long. left	5 49 E.
				diff.	1 35 W.
				Long. in	4 14 E.

Course N. 47° 49′ W.
Distance 116 miles

Sept. 13, 1863. At noon; lat. obs. 24° 02′ S. Greenwich time by chron. 20 h. 43 m. 25 s. Slow 18 m. 47 s. Longitude by chron. 2° 21′ 15″ E.

Points	Miles	N.	W.		° ′
N. 4 W.	52½	37·1	37·1	Lat. left	25 11 S.
N. 4½ W.	49	31·1	37·9	diff.	1 08 N.
		68·2	75·0	Lat. in	24 03 S.
			83′	Mid. lat.	24 37
				Long. left	3 58 E.
				diff.	1 23 W.
				Long. in	2 35 E.

Course N. 48° W.
Distance 102 miles.

Sept. 14, 1863. At noon; lat. obs. 22° 15′ S. Greenwich time by chron. 20 h. 37 m. 40 s. Slow 18 m. 51 s. Longitude by chronometer 0° 3′ 15″ W.

Points	Miles	N.	W.		° ′
N. 4 W.	171	120·9	120·9	Lat. left	24 02 S.
			131′	diff.	2 02 N.
				Lat. in	22 00 S.
				Mid. lat.	23 01
				Long. left	2 21 E.
				diff.	2 11 W.
				Long. in	0 10 E.

Course N. 45° W.
Distance 171 miles.

Sept. 15, 1863. At noon; lat. obs. 20° 27 S. Greenwich time by chron. and long. not noted. Very stormy day.

Points	Miles	N.	W.		° ′
N. 4½ W.	157	105·4	0·04	Lat. left	22 15 S.
			2·06	diff.	1 45 N.
			2·10	Lat. in	20 30 S.
			116·3	Mid. lat.	21 22
			126′	Long. left	0 3 E.
				diff.	2 6 W.
				Long. in	2 10 W.

Course N. 47° 49′ W.
Distance 157 miles.

Sept. 16, 1863. At noon; lat. obs. 18° 56 S. Greenwich time by chron. 20 h. 27 m. 43 s., slow 18 m. 59 s. Longitude by chronom. 4° 45′ 30″ W.

Points	Miles	N.	W.		° ′
N. 4½ W.	6	4	4·4	Lat. left	20 27 S.
N. 4¾ W.	37	22	29·7	diff.	1 28 N.
N. 5½ W.	120	61·7	102·9	Lat. in	18 59 S.
Var. 2½ W.		87·7	137	Long. left	2 10 W.
			146′	diff.	2 26 W.
				Long. in	4 36 W.

Course N. 57° W.
Distance 163 miles.

Reduction of Longitude	h. m. s.
App. local time	20 34 01
Equation of time	5 01
True mean local time	20 29 00
Greenwich time for same moment	20 46 42
Longitude in time	17 42
	° ′ ″
Longitude in degrees W.	4 25 30
Correction ,,	20 00
True longitude	4 45 30

Summary of Eight Days' Courses.

Sept.	Lat. Obs.	Long. Chron.	Lat. by account	Long. by account
	° ′	° ′	° ′	° ′
8	31 50	11 29 E.	30 02	9 32 E.
9	29 41	9 20 E.	28 18	7 23 E.
10	27 58	7 45 E.	26 33	5 51 E.
11	26 34	5 49 E.	25 14	4 14 E.
12	25 11	3 58 E.	24 03	2 35 E.
13	24 02	2 21 E.	22 00	0 10 E.
14	22 15	0 3 W.	20 30	2 10 W.
15	20 27	2 10 W.	18 59	4 36 W.
16	18 56	4 46 W.		

Sept.	Course	Distance
	°	
9	NW. 43	148 miles
10	NW. 51	132 ,,
11	NW. 50	133 ,,
12	NW. 47 49′	116 ,,
13	NW. 48	102 ,,
14	NW. 45	171 ,,
15	NW. 47 49′	157 ,,
16	NW. 57	163 ,,



RECORD

No. VI.

ROUTE-SURVEY IN THIBET.

Route—
Badrinath to Thokjalung.

Number of Station	Forward Bearing	Paces Forward	Places where latitude was observed
	° ′		
Badrinath	9 45	3400	L. O. †
2	338 0	1200	
3	313 0	700	
4	3 30	3300	
5	351 30	6100	
6	330 0	4000	
7	344 0	10000	
8	315 30	12700	
9	24 30	11700	Crest of pass
10	22 45	11800	
11	67 0	11700	
12	358 0	2760	L. O.
Lumarti	" "	4700	
13	28 0	6000	
14	6 30	6800	L. O. Chirkong
15	25 0	12400	
16	27 0	12000	
17	37 0	5500	L. O. Barku
18	76 45	10000	
19			L. O. Totling
Totling	52 0	12300	L. O. Nairding
		38200	
20	68 0	32300	L. O. Khangiah
		50340	L. O. Gugti camp
		50600	
21	47 0	13700	
22	85 0	5300	L. O. Gugti-la
23	62 0	4500	
24	44 30	11300	
25	88 15	4100	
25	250 0	2000	L. O. Chojo-Gugti
26	98 30	8000	
27	68 0	9300	
28	20 45	13300	
29	48 0	15300	L. O. Giamchichu
		50800	L. O. Kiangma
		56000	
30	20 0	3700	
31	44 30	5000	L. O. Giachuruf
		30600	
32	70 0	19900	
33	94 0	4000	L. O. Thokjalung
		6000	End of Bazar

Route—Giachuruf to Gartok.

Giachuruf	316 30	22700	
2	300 0	19100	L. O. Shildung
3	290 0	4100	
4	246 0	2000	L. O. Giamchupku
		5700	
5	291 0	9400	
6	323 0	4400	
7	289 0	8000	L. O. Thankar
		17100	
8	272 0	9800	
9	225 0	3500	
10	276 0	6300	L. O. Pika
11	308 0	2300	
12	242 30	2000	
13	290 0	3300	
14	258 0	3000	
15	265 30	900	L. O. Burkung
		16500	
16	258 0	11100	
17	287 0	10000	L. O. Marku
18	209 0	6700	
19	255 0	7890	
20	226 15	5900	L. O. Dak-Maru
21	260 30	19900	L. O. Ralajung
		29000	
22	227 45	9500	
23	143 0	19500	L. O. Lujanchumik
24	130 0	24800	L. O. Ju camp
25	150 0	30400	L. O. Loa-Gong
		47200	L. O. Gargunsa
26	129 0	26700	L. O. Gartok

† The observed latitudes follow on the route record.

Route of third Pandit—Lujanchumik to Totling.

Number of Station	Forward Bearing	Paces Forward	Places where latitude was observed
	° ′		
Lujan Chumik	319 15	16700	
2	314 30	6300	L. O. Tashikang
3	309 30	18400	
4	326 42	3500	L. O. Damakolok
5	311 0	6700	
6	312 30	1700	L. O. Demchok
Demchok	312 30	1000	
7	224 30	3900	
8	194 30	5390	L. O. Demchok-phu
		8800	
9	186 30	6900	
10	169 30	2100	
11	178 0	5500	
12	158 30	4000	L. O. Deboche
		21900	
13	150 30	12300	
14	197 30	5300	L. O. Medokding
15	117 0	7200	
16	160 30	3500	L. O. Dilchachini
		8800	
17	150 30	20500	
18	153 0	6200	L. O. Lamoche
		7500	
19	167 30	2500	
20	134 0	2600	L. O. Jiasumdo
21	206 0	12300	
22	223 0	800	L. O. Chokche
		4100	
23	317 0	2700	
23 A.	308 0	5600	L. O. Rabgialing
23	143 0	1800	
24	139 0	1500	
25	141 30	7600	L. O. Shangche
		6200	
26	142 0	3600	
27	160 0	4100	
28	114 30	4000	
29	131 0	4900	L. O. Tibuphu
		10200	
30	102 0	4000	
31	69 30	1700	
32	104 0	4100	
33	187 30	600	L. O. Dunkhar
		4800	
34	181 0	24300	L. O. Totling

Route of second and third Pandits—Barku to Shipki.

Number of Station	Forward Bearing	Paces Forward	Places where latitude was observed
Barku	229 0	10400	
2	224 30	9500	
3	244 0	9500	
4	287 0	2000	
5	305 0	1700	
6	290 30	1700	
7	234 0	2700	
8	284 30	2000	
9	340 0	3300	L. O. Puling
10	294 30	3000	
11	289 0	7900	
12	292 0	2500	
13	325 0	4500	
14	305 0	2900	
15	323 0	1900	
16	323 0	13200	
17	345 0	3300	
18	322 0	4500	L. O. Rildighang
19	325 0	2200	
20	301 0	3300	
21	298 0	9500	
22	308 0	7200	L. O. Ri
23	355 0	8500	L. O. Bridge
		19300	
24	324 30	4000	

Number of Station	Forward Bearing	Paces Forward	Places where latitude was observed
	° ′		
25	278 30	3100	L. O. Dongkhang
		8800	
26	282 0	2400	L. O. Miang
		11500	
27	216 0	4000	
28	302 0	500	L. O. Tiak
		2900	
29	315 0	3300	
30	264 0	8550	L. O. Kuak
		13900	L. O. Shipki

Route of second and third Pandits—Shipki to Mukpa.

Shipki	84 0	13900	
2	135 0	3300	
3	122 0	2900	
4	302 0	500	
5	167 30	6600	L. O. Tiak
6	172 30	2800	L. O. Kuang
		4500	
7	222 30	3800	
8	265 0	1600	
9	170 30	4000	L. O. Sang
		12300	
10	160 0	2000	
11	90 0	5000	L. O. Sumna
		8600	
12	133 0	3200	
13	106 0	3200	
14	36 0	2200	
15	53 30	1300	
16	101 30	4700	L. O. Biar
		6200	
17	157 30	2700	
18	126 30	4500	
19	129 0	800	L. O. Sarang
		2800	
20	152 45	3700	
21	139 0	1700	
22	127 30	2000	
23	130 0	2300	
24	141 0	1300	
25	205 0	2300	
26	156 0	10000	
27	177 0	6900	
28	167 0	10700	
29	202 30	190	
30	193 30	3000	
31	235 0	1900	
32	188 0	1280	L. O. Changjum
		1900	
33	231 0	1100	
34	176 0	1100	
35	203 0	2200	
36	196 0	1200	
37	153 0	4300	
38	212 30	1800	
39	160 0	1200	
40	192 0	1600	L. O. Nonam
		4400	
41	196 30	3800	
42	158 0	1400	
43	211 0	2800	
44	251 30	2700	
45	263 30	4300	
46	217 0	1100	
47	255 30	5600	L. O. Nilung
48	182 0	2300	
49	213 0	5700	
50	229 0	3700	
51	259 0	5300	
52	245 0	2000	
53	257 0	2000	
54	288 0	5700	
55	266 0	4700	
56	259 0	3200	L. O. Mukpa

Route of second and third Pandits—Gartok to Chirkong.

Number of Station	Forward Bearing	Paces Forward	Places where latitude was observed
	° ′		
Gartok	281 0	9900	L. O.
2	259 0	7500	
3	213 30	5800	
4	259 0	8000	
5	327 0	5200	
6	292 30	5200	
7	315 0	8000	
8	285 0	4400	
9	315 0	3200	
10	270 0	2800	
11	309 0	10500	
12	230 0	3500	
13	306 30	3100	
14	270 0	13500	
15	146 0	17300	
16	169 0	16000	
17	95 0	4000	L. O. Dunkhar
18	185 1	5200	
19	179 0	23700	L. O. Totling
20	123 30	9100	
21	200 0	17300	
22	252 30	6000	L. O. Manguang
23	225 0	9000	
24	242 0	7700	
25	230 0	7500	L. O. Chirkong

Route of third Pandit—Giachuruf to Jiachan.

Giachuruf	155 0	7900	L. O.
2	90 0	2600	
3	170 0	5200	
4	208 0	2400	
5	198 0	5600	
6	135 0	9600	
7	124 0	10200	L. O. Lapta Rebo
8	193 0	7000	L. O. Nagpo
		10000	
9	245 0	8900	
10	216 0	11600	
11	190 0	18900	L. O. Jiachan

Route—Milam to Gartok.

Milam	33 0	6400	
2	5 0	9000	
3	22 30	2800	
4	333 10	8200	
5	30 0	5000	
6	26 20	6100	
7	52 0	21500	
8	48 40	3600	
9	90 0	1400	
10	71 40	2900	
11	38 30	2500	
12	52 30	12500	
13	46 0	15400	
14	26 0	15400	
15	350 20	11800	
16	358 10	7600	
17	26 0	6700	
18	28 20	2100	
19	72 20	6000	
20	32 20	6800	
21	318 20	22600	
22	322 10	10500	
23	315 40	10800	
24	335 0	22900	
25	319 0	20200	
26			L. O. Garto

Route—Gartok to Milam.

Number of Station	Forward Bearing	Paces Forward	Places where latitude was observed
	° ′		
26	170 20	10300	L. O. Gartok
27	141 20	8800	
28	155 30	4200	Namochi
29	155 30	10500	
30	196 10	12100	
31	235 0	2200	
32	235 20	21300	
33	231 30	6000	
34	171 0	3500	
35	202 0	2500	
36	203 10	3500	
37	191 40	10500	
38	170 50	3600	
39	200 0	11000	Dongpu
40	196 0	3000	Naglo
41	197 30	7300	
42	194 30	9700	
43	177 30	13800	
44	176 0	13000	
45	176 0	5700	
46	130 30	16100	
47	172 0	12400	
48	166 0	7900	
5	153 10	8200	As on previous route, from Milam
	202 30	2800	
3	175 0	9000	
2	213 0	6400	
1	— —	—	Milam

Mean Latitudes of Various Stations.

	°	′	″
Chojo-Gugti	31	54	17·5
Giamchuphu	32	26	57·5
Thankar	32	30	47·6
Burkung	32	32	53·1
Marku	32	21	3·6
Dakmaru	32	30	3·0

	°	′	″
Ralajung	31	27	52·6
Ju camp	32	18	22·8
Gargunsa	32	8	45·6
Loagong	31	57	45·6
Dunkhar	31	41	26·9
Totling Monastery	31	30	4·7
Mangnang	31	21	47·2
Tashikang	32	30	13·1
Damakolok	32	38	9·7
Demchok	32	41	38·5
Demchok-phu	32	38	30·7
Deboche	32	29	8·7
Medokding	32	20	14·0
Dilchachini	32	13	3·7
Lamoche	32	0	47·8
Giasumdo	31	59	43·0
Chokche	31	52	58·2
Rabjialing	31	53	37·0
Shangche	31	49	16·0
Tibuphee	31	42	0·7
Barku	31	25	52·0
Puling	31	20	20·2
Rildighang	31	31	3·1
Ri	31	39	58·5
Bridge Satlaj	31	41	3·0
Dongkhang	31	45	4·7
Miang	31	47	17·3
Tiak	31	46	59·5
Shipki	31	48	25·0
Kuang	31	43	52·0
Sumna	31	36	45·2
Biar or Bekar	31	37	32·1
Sarang	31	34	19·3
Changjam	31	17	18·6
Nonam	31	10	33·4
Nilung	31	6	17·9
Mukpa	31	2	19·3
Lapta rebo	32	0	23·0
Jiachan	32	5	27·3
Nagpo	31	53	13·2

Abstract from Observations for Latitude taken with 6-inch Sextants.—Nos. 44 and 45.

No.	Astron. Date 1867	Watch Time	Station	Object on Meridian	Double Altitude	Index Error	Deduced Latitudes	Mean Latitudes
					° ′ ″	′ ″	° ′ ″	° ′ ″
1	May 29	6 30 P.M.	Near Badrinath	Polaris	58 46 0	+3 0	30 44 29·4	30 44 29·4
3	,, 31	6 30 ,,	At ,,	,,	58 41 10		30 45 4·7	30 45 4·7
4	June 7	11 30 ,,	Mana	Antares	66 12 10	+3 10	30 45 20·6	30 45 17·8
5	,, 27	10 0 ,,	,,	,,	66 12 40	+2 50	30 45 15·0	
6	July 27	3 30 ,,	Lumarti	Polaris	65 10 50	+2 30	31 11 29·6	31 12 9·7
7	,, 27	4 30 ,,	,,	,,	65 18 20	−7 10	31 10 24·6	
8	,, 30	3 15 ,,	,,	Fomalhaut	57 4 0	−7 10	31 13 22·3	31 12 52·0
11	Aug. 1	4 15 ,,	Chirkong	Polaris	65 23 10	−7 10	31 12 52·0	
13	,, 3	4 15 ,,	Chaprang	,,	65 43 0	+2 30	31 27 35·6	31 27 35·6
15	,, 5	4 30 ,,	Totling	,,	65 43 30	+2 30	31 27 48·9	31 27 48·9
16	,, 7	11 0 ,,	Nairding	Altair	134 5 30	−7 10	31 32 36·7	31 32 36·7
17	,, 10	2 30 A.M.	Khangiah	Fomalhaut	56 5 0	−7 0	31 42 52·7	31 42 52·7
18	,, 11	5 40 ,,	Gugti camp	Polaris	66 25 10	−7 0	31 44 0·3	
19	,, 11	5 40 ,,	,,	,,	66 17 0	+3 0	31 44 65·3	31 44 47·0
21	,, 12	5 40 ,,	Gugti-la	,,	66 23 30	+3 0	31 48 18·7	
22	,, 12	5 40 ,,	,,	,,	66 30 30	−7 0	31 46 48·7	31 47 33·7
26	,, 16	Stopped	Giamchicho?	,,	67 0 40	−7 0	32 1 54·1	32 1 54·1
27	,, 19	4 0 A.M.	Kiangma	,,	67 9 30	+3 0	32 11 20·0	32 11 20·0
28	,, 20	4 0 ,,	Giachuruf	,,	67 16 0		32 14 34·5	32 14 34·5
29	,, 26	2 42 ,,	Thokjalung	,,	67 37 0		32 25 5·6	
30	,, 27	9 30 P.M.	,,	Altair	132 13 0		32 13 46·5	32 24 26·5
33	,, 30	9 5 ,,	,,	,,	132 13 10		32 23 41·8	
34	,, 30	12 15 A.M.	,,	Fomalhaut	54 33 30		32 23 36·9	
35	,, 30	2 30 ,,	,,	Polaris	67 37 20		32 25 16·8	
43	Sept. 4	8 40 P.M.	Shildung	Altair	132 27 10	+3 0	32 16 42·9	32 19 27·0
44	,, 4	2 0 A.M.	,,	Polaris	67 31 10		32 22 11·0	
49	,, 7	8 30 P.M.	Pika	Altair	131 58 0		32 31 18·2	
50	,, 7	11 30 ,,	,,	Fomalhaut	54 18 50		32 30 59·0	32 31 43·1
51	,, 7	1 30 A.M.	,,	Polaris	67 51 20		32 32 17·5	
59	,, 12	8 15 P.M.	Lujanchumik	Altair	132 11 40	+2 30	32 24 44·5	32 24 35·1
60	,, 12	1 25 A.M.	,,	Polaris	67 36 10		32 24 25·6	
67	,, 16	1 30 ,,	Gartok	Altair	66 22 30	−7 0	31 42 50·6	31 43 54·0
68	,, 16	7 30 P.M.	,,	,,	133 40 30		31 45 4·2	
69	,, 18	7 30 ,,	,,	,,	133 40 40		31 44 59·3	
70	,, 19	1 0 A.M.	,,	Polaris	66 22 10		42 41·6	

RECORD

Nº VII.

ABSTRACT FROM OBSERVATIONS OF THE BOILING POINT TAKEN IN THE ROUTE-SURVEY OF THIBET.

No. of Station	Astron. Date	Watch Time	Place	Therm.		Therm.		Height above Sea
				No.	Boiling Point	No.	In Air	
1	June 28	5 42 P.M.	Badrinath	22	195·40	30	60·0	10284
2	July 4	8 30 A.M.	Mana	,,	195·10	,,	64·0	10510
7	,, 28	9 0 ,,	Crest of pass	30	182·0	38	37·0	18576
8	,, 30	Noon	Lumarti	,,	185·5	,,	57·5	16660
11	Aug. 2	7 40 A.M.	Chirkong	,,	187 0	,,	53·5	15708
13	,, 5	4 0 P.M.	Totling Monastery	,,	192·75	,,	69·0	12295
16	,, 6	9 0 A.M.	Bogola crest pass	,,	181·00	,,	40·0	19220
18	,, 10	6 30 P.M.	Khangiah	,,	188·00	,,	58·25	15129
19	,, 11	5 10 ,,	Gugti camp	,,	188·0	,,	67·50	15205
20	,, 12	4 25 ,,	Gugti Sumdo	,,	184·50	,,	62·50	17324
21	,, 13	9 0 A.M.	Gugti, crest pass	,,	180·50	,,	37·00	19490
22	,, 13	5 45 P.M.	Chogo Gugti	,,	185·00	,,	58·25	16968
26	,, 19	4 23 ,,	Kiangma	,,	185·50	,,	59·00	16669
28	,, 20	5 30 ,,	Giachuruf	,,	187·00	,,	57·00	15732
32	,, 29	2 50 ,,	Thokjalung	,,	186·00	30	55·00	16327
34	Sept. 4	6 15 ,,	Shildung	,,	188·75	,,	55·00	14652
35	,, 5	4 23 ,,	Giamchu	,,	188·50	,,	62·50	14861
36	,, 6	5 30 ,,	Thankar	,,	188·75	,,	59·75	14688
37	,, 7	5 52 ,,	Pika	,,	188·80	,,	56·75	14637
39	,, 8	3 36 ,,	Burkung	,,	189·40	,,	64·0	14324
40	,, 9	4 15 ,,	Marku	,,	189·75	,,	57·5	14071
41	,, 10	5 12 ,,	Dakmaru	,,	190·00	,,	58·25	13920
43	,, 11	5 35 ,,	Ralajung	,,	191·50	,,	58·50	13022
44	,, 12	5 55 ,,	Lujanchumik	,,	191·50	,,	54·50	12999
48	,, 17	5 30 A.M.	Gartok Monastery	38	188·75	,,	30·00	14241
50	,, 30	6 55 ,,	Totling Monastery	,,	192·50	,,	42·00	12101
6	Oct. 8	5 59 A.M.	Bridge Suspension	,,	196·00	,,	39·75	10039
11	,, 12	6 25 ,,	Shipki Village	,,	196·00	,,	35·00	10027
23	,, 27	6 30 P.M.	Nilung		194·00		32·00	11181
25	Nov. 7	4 45 ,,	Mukpa Dharmsala		199·25		48·00	8172

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A very portable ditto, 1½ inch in diameter, in silver case	5	5	0
Best make Pocket-Size Aneroid Barometer, 2¼ inches in diameter, with metallic dial, and compensated, in case...	2	15	
Ditto ditto, with altitude scale	3	0	
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